TME085 - Compressible Flow

2025-08-20, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs*
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Responsible teacher: Niklas Andersson tel.: 031-772 3788

Results available no later than 2025-09-10

Good luck!

Part I - Theory Questions (20 p.)

T1. Unsteady wave motion (5 p.):

(a) Describe what happens when a moving normal shock hits a solid wall.

(b) The moving shock is an adiabatic process in the same way as a stationary normal shock is adiabatic. Does this mean that total enthalpy is constant over the moving shock? Explain why/why not.

(c) Can a moving normal shock travel at a speed lower than the speed of sound? Explain why/why not.

(d) What types of waves or discontinuities are generated in a shock tube with two initially stagnant regions at different pressure (separated by a thin membrane which is removed very quickly)?

(e) What is the difference between acoustic waves and other types of waves such as shock waves and expansion waves?

T2. Numerical simulation of compressible flows (4 p.):

(a) When the governing equations are discretized using a **finite-volume** approach, cell face values of flow properties appears in the equations. How are these values approximated?

(b) What do we mean when we say that a CFD code for compressible flow is **conservative**?

(c) What is a typical maximum **CFL number** for stable operation when applying an **explicit time stepping scheme**?

(d) How can we use our knowledge of **characteristics** (and their speed of propagation) to guide us when determining suitable **boundary conditions** for compressible flows?

T3. High-temperature effects (4 p.):

(a) Explain what the **Boltzmann distribution** describes and what sparsely populated implies.

(b) What is the difference between a **calorically perfect** gas and a **thermally perfect** gas?

(c) A mixture of chemically reacting perfect gases, where the reactions are always in **equilibrium**, may be thermodynamically described as a single-species gas. How does this thermodynamic description differ from that of a calorically perfect or thermally perfect gas?

(d) Using **equilibrium gas** assumption in the analysis of chemically reacting nozzle flow will lead to higher exhaust temperatures than if calorically perfect gas assumption is used for the same analysis. Explain why.

T4. (1 p.)
What does Crocco's relation say about the flow behind a curved shock?

$$T\nabla s = \nabla h_o - \mathbf{v} \times (\nabla \times \mathbf{v}) + \frac{\partial \mathbf{v}}{\partial t}$$

T5. (2 p.)
Derive the **isentropic relations** for **calorically perfect** gases starting from the **entropy equation**.

$$Tds = de + pd\nu = dh - \nu dp$$

T6. (1 p.)
What is the physical interpretation of each of the terms in the **momentum equation** on integral form

$$\frac{d}{dt}\iiint\limits_{\Omega} \rho \mathbf{v} d\mathscr{V} + \oiint\limits_{\partial\Omega} \left[\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}\right] dS = \iiint\limits_{\Omega} \rho \mathbf{f} d\mathscr{V}$$

T7. (1 p.)
How does the absolute Mach number change after a **weak/strong** stationary oblique shock?

T8. (1 p.)
Assume a steady-state 1D flow with a stationary normal shock. The fluid particles crossing the shock are subjected to

(a) a pressure drop
(b) a density increase
(c) an entropy increase
(d) a temperature drop
(e) a deceleration

Which statements are true and which are false?

T9. (1 p.)
In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

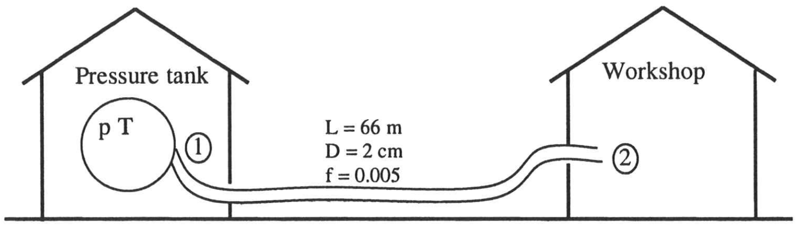
Part II - Problems (40 p.)

Problem 1 - PRESSURIZED AIR (10 p.)

A manufacturing company has built a new workshop. The workshop needs pressurized air, and a tube is connected to another building where they have a pressure tank. The pressure and temperature inside the tank is 10 atm and 288 K, and the tube length and diameter is 66 m and 2 cm.

(a) At some point during the testing of the new facility, the Mach number in the beginning of the tube was $M_1 = 0.1$. What were the conditions at the end of the tube; M_2, p_2, T_2, p_{o_2} assuming adiabatic flow and an average friction coefficient $\bar{f} = 0.005$?

(b) If one would like to make the tube length twice as long, what would the massflow through the tube be?

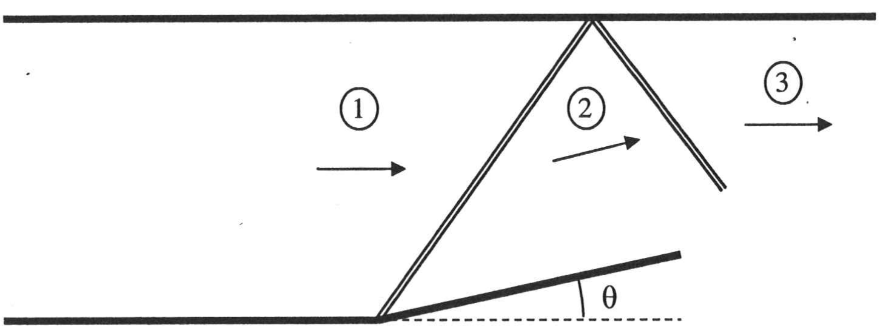


Problem 2 - WIND TUNNEL SHOCK REFLECTION (10 p.)

A compression corner is mounted into a supersonic wind tunnel and an oblique shock is generated that reflects on the top wall. The Mach number, static pressure, and static temperature before the compression corner is $M_1 = 3.0$, $p_1 = 1\ atm$, $T_1 = 300\ K$, and the angle of the corner is $\theta = 8°$.

(a) What is the Mach number and total pressure after the reflected shock, M_3, p_{o_3}?

(b) What is the maximum deflection angle for which there will be an oblique shock formed at the compression corner?

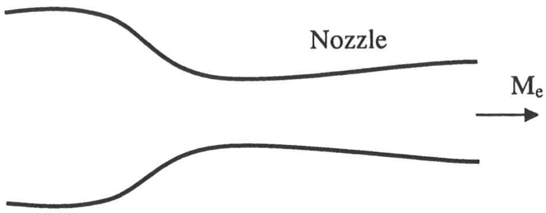


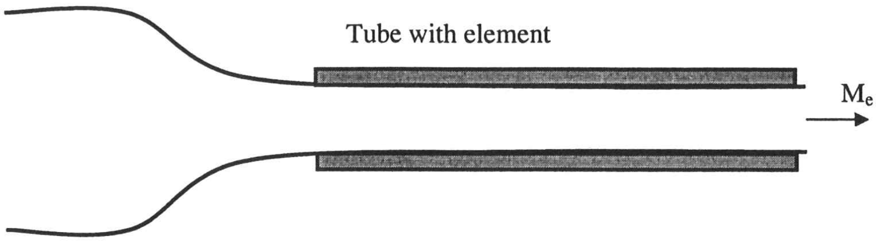
Problem 3 - CONVERGENT-DIVERGENT NOZZLE (10 p.)

A large vessel is connected to a convergent-divergent nozzle, and the pressure and temperature inside is 1 MPa and 300 K. The nozzle has a throat and exit area of 2.0 cm^2 and 2.5 cm^2, respectively. A student wants to study flow with different Mach numbers and comes up with the idea of replacing the divergent part of the nozzle with a constant-area tube equipped with an element that can heat up or cool down the flow inside.

(a) Assuming flow without friction and shocks, and that the cross section area of the tube is the same as the throat area of the nozzle; How much heating or cooling, in J/s, is needed to create the same exit Mach number at the end if the tube as the design exit Mach number of the convergent-divergent nozzle?

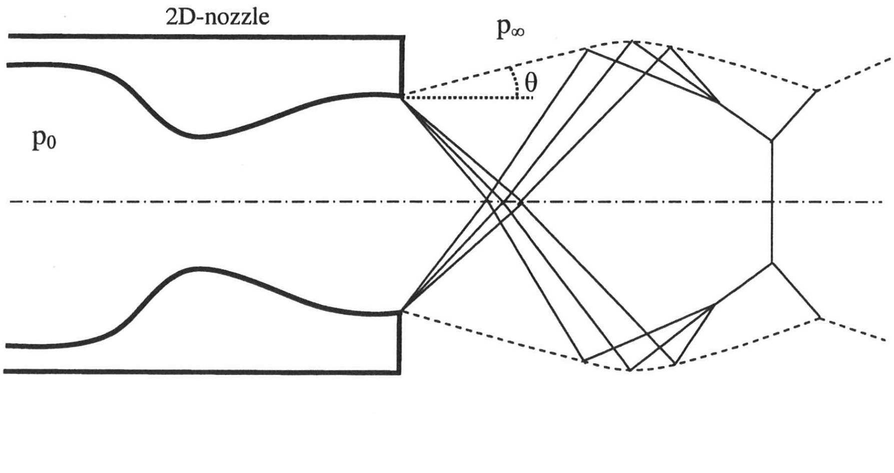
(b) For the calculated exit Mach number, what are the velocities at the exit for the two cases (the nozzle and the tube)?





Problem 4 - UNDER-EXPANDED NOZZLE FLOW (10 p.)

A two-dimensional convergent-divergent nozzle is connected to a stream of air with $p_o = 5\ atm$. The throat and exit dimensions are 2.5 cm and 2.94 cm, and the width of the nozzle is 1.0 m. The surrounding air has a pressure of $p_\infty = 1\ atm$. This means that the nozzle is operated at under-expanded conditions and thus expansion waves will form downstream of the nozzle exit plane. Assuming isentropic flow through the nozzle, what is the deflection angle, θ, of the expansion wave formed just downstream of the nozzle exit?



P_1

PRESSURE TANK:

$P_0 = 10$ atm

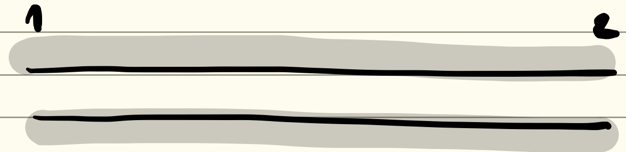
$T_0 = 288$ K

TUBE:

$L = 66$ m

$D = 2$ cm

a)



ADIABATIC TUBE.

$\bar{f} = 0.005$

$M_1 = 0.1$

CALCULATE M_2, P_2, T_2 AND P_{0_2}

(3.28)

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \Rightarrow \boxed{T_1 = 287.43\,K}$$

(3.30)

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1}\right)^{\gamma/(\gamma-1)} \Rightarrow \boxed{P_1 = 9.93\ atm}$$

(3.107)

$$\frac{4\bar{f}L_1^*}{D} = \frac{1 - M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma} \ln\left(\frac{(\gamma+1)M_1^2}{2 + (\gamma-1)M_1^2}\right)$$

$$\Rightarrow \boxed{L_1^* = 66.92}$$

$(L < L_1^* \Rightarrow$ NOT CHOKED$)$

(3.103)

$$\frac{T_1}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_1^2}$$

(3.104)

$$\frac{p_1}{p^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2}$$

$$\boxed{L_2^* = L_1^* - L_{12} = 0.92\ m}$$

(3.107)

$$\frac{4\bar{f}L_2^*}{D} = \frac{1-M_2^2}{\gamma M_2^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2}\right)$$

SOLVE FOR M_2 $\Rightarrow$

$$\boxed{M_2 = 0.52}$$

(3.103)

$$\frac{T_2}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_2^2} \Rightarrow \boxed{T_2 = 273.26\ K}$$

(3.104)

$$\frac{p_2}{p^*} = \frac{1}{M_2}\left(\frac{\gamma+1}{2+(\gamma-1)M_2^2}\right)^{1/2}$$

$$\Rightarrow \boxed{p_2 = 1.867\ atm}$$

(3.30)

$$\frac{P_{o2}}{P_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow \boxed{P_{o2} = 2.241 \text{ atm}}$$

b) TUBE TWICE AS LONG (132 m)

WHAT WILL THE MASSFLOW BE ?

$132 \text{ m} > L_1^*$ $\Rightarrow$ THE MASSFLOW WILL BE REDUCED..

THE MACH NUMBER WILL CHANGE SUCH THAT $L_1^* = 132 \text{ m}$

(3.107)

$$\frac{4\bar{f}L_1^*}{D} = \frac{1 - M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2} \ln\left(\frac{(\gamma+1)M_1^2}{2 + (\gamma-1)M_1^2}\right)$$

SOLVE FOR M_1 $\Rightarrow$ $\boxed{M_1 = 0.07}$

(3.28)

$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \Rightarrow \boxed{T_1 = 287.7 \text{ K}}$$

(3.30)

$$\frac{P_0}{P_1} = \left(\frac{T_0}{T_1}\right)^{\gamma/(\gamma-1)} \Rightarrow \boxed{P_1 = 9.964 \text{ atm}}$$

$$\rho_1 = \frac{p_1}{R T_1}$$

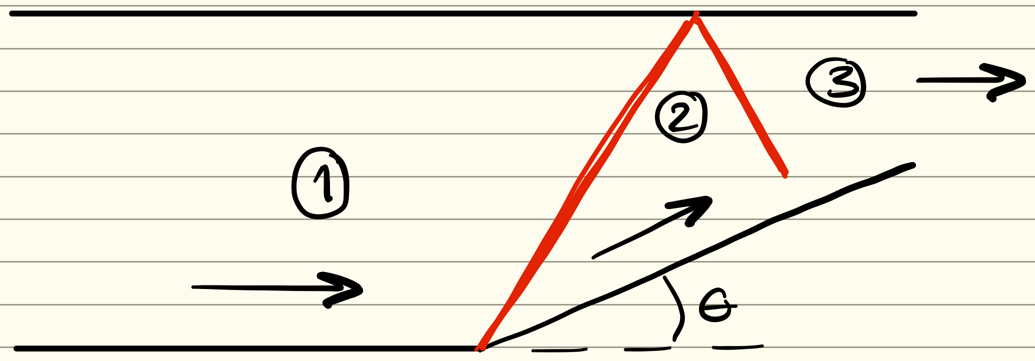
$$u_1 = M_1 a_1 = M_1 \sqrt{\gamma R T_1}$$

$$\dot{m} = \rho_1 u_1 A = \rho_1 u_1 \frac{\pi D^2}{4}$$

$$\Rightarrow \boxed{\dot{m} = 0{,}09 \text{ kg/s}}$$

(IN THE 66 m LONG TUBE, THE MASSFLOW IS 0,13 kg/s ⇒ A REDUCTION OF 0,04 kg/s FOR THE LONGER TUBE.)

P_2



$M_1 = 3.0$

$P_1 = 1$ atm

$T_1 = 300K$

$\theta = 8°$

a) CALCULATE MACH NUMBER AND TOTAL PRESSURE IN REGION 3

$1 \rightarrow 2$:

$$\left.\begin{array}{l} M_1 = 3.0 \\ \theta = 8° \end{array}\right\} \theta-\beta-M \Rightarrow \beta_1 = 25.61°$$

(4.7)

$$M_{n_1} = M_1 \sin \beta_1$$

(4.9)

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)$$

(4.10)

$$M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$$

(4.12) $$M_2 = \frac{M_{n_2}}{\sin(\beta_1 - \theta)}$$

$$\boxed{\begin{aligned} M_2 &= 2.60 \\ P_2 &= 1{,}7953 \text{ atm} \end{aligned}}$$

$2 \rightarrow 3$:

$$\left.\begin{aligned} M_2 &= 2.60 \\ \theta &= 8^\circ \end{aligned}\right\} \theta - \beta - M \Rightarrow \beta_2 = 28.94^\circ$$

(4.7)

$$M_{n_1} = M_2 \sin \beta_2$$

(4.9)

$$\frac{P_3}{P_2} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$$

(4.10)

$$M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$$

(4.12)

$$M_3 = \frac{M_{n_2}}{\sin(\beta_2 - \theta)}$$

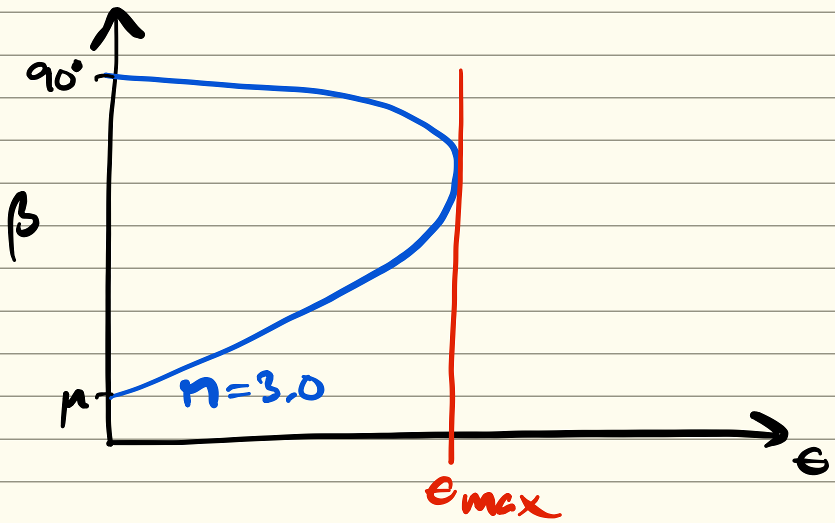
$$\boxed{\begin{aligned} M_3 &= 2{,}26 \\ P_3 &= 3{,}0232 \text{ atm} \end{aligned}}$$

(3.30) $$\frac{P_{o_3}}{P_3} = \left(1 + \frac{\gamma - 1}{2} M_3^2\right)^{\gamma/(\gamma-1)}$$

$$\boxed{\begin{aligned} M_3 &= 2.26 \\ P_{o_3} &= 35.4825 \text{ atm} \end{aligned}}$$

b)

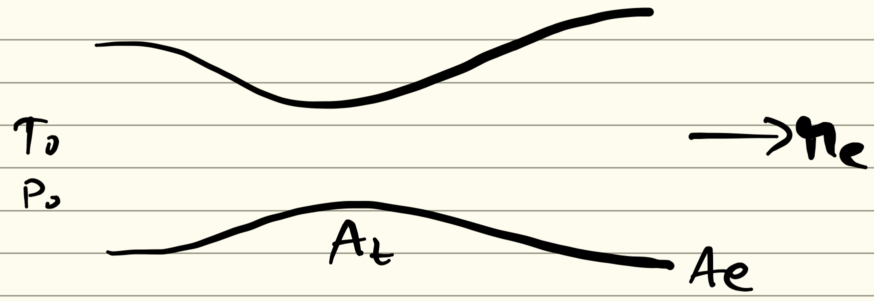
WHAT IS THE MAXIMUM DEFLECTION ANGLE FOR WHICH THERE WILL BE AN OBLIQUE SHOCK



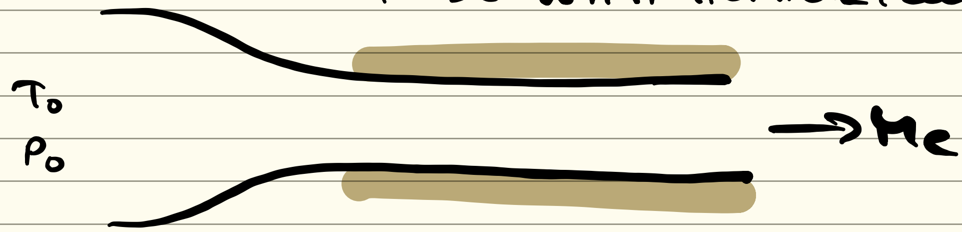
$$\boxed{\theta_{max} = 34°}$$

P3

NOZZLE



TUBE WITH HEATER / COOLER



$T_0 = 300\ K$

$P_0 = 1.0\ MPa$

$A_t = 2.0\ cm^2$

$A_e = 2.5\ cm^2$

a) CALCULATE THE HEATING / COOLING NEEDED TO GET THE SAME EXIT MACH NUMBER FOR THE TUBE AS FOR THE NOZZLE.

(THE NOZZLE IS OPERATED AT DESIGN CONDITIONS)

NOZZLE EXIT MACH NUMBER:

(5.20)
$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUPERSONIC SOLUTION: $\boxed{M_e = 1.6}$

THE TUBE STARTS AT THE NOZZLE THROAT $\Rightarrow M=1$ AT THE TUBE START

$\Rightarrow T_0 = T_0^*$, $P_0 = P_0^*$

TO ACCELERATE THE FLOW TO SUPERSONIC MACH NUMBER, HEAT MUST BE REMOVED FROM THE FLOW.

(3.89)

$$\frac{T_{0e}}{T_0^*} = \frac{(\gamma+1)M_e^2}{(1+\gamma M_e^2)^2}\left(2+(\gamma-1)M_e^2\right)$$

$$\Rightarrow \quad T_{0e} = 265.28 \text{ K}$$

$$\boxed{q = C_p\,(T_{0e} - T_0) = -34878.6 \text{ J/kg}}$$

THE MASSFLOW IS CALCULATED USING THE CHOKED MASSFLOW FUNCTION

(5.21)

$$\dot{m} = \frac{P_0 A_t}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$\dot{m} = 0{,}47 \text{ kg/s}$$

$$\boxed{q\,\dot{m} = -16278.2 \text{ J/s}}$$

b) CALCULATE THE EXIT VELOCITIES FOR THE TWO CASES.

NOZZLE EXIT VELOCITY:

(3.28)

$$\frac{T_0}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2$$

$$u_e = M_e a_e = M_e \sqrt{\gamma R T_e}$$

$$\Rightarrow \boxed{u_e = 451.7 \text{ m/s}}$$

TUBE EXIT VELOCITY:

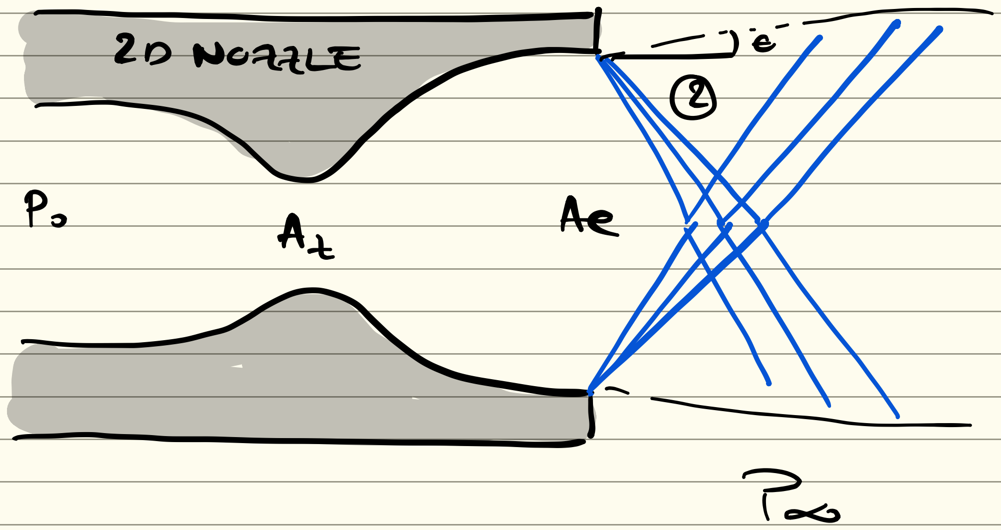
(3.28)

$$\frac{T_{0e}}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2$$

$$u_e = M_e a_e = M_e \sqrt{\gamma R T_e}$$

$$\Rightarrow \boxed{u_e = 424.8 \text{ m/s}}$$

P4



$A_t = 0.025\ m^2$

$A_e = 0.0294\ m^2$

$p_\infty = 1.0$ atm

ASSUME ISENTROPIC FLOW THROUGH THE NOZZLE

THE FLOW IS UNDER EXPANDED $\Rightarrow$ EXPANSION DOWNSTREAM OF THE NOZZLE EXIT.

CALCULATE THE DEFLECTION ANGLE θ

(5.20)

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow \boxed{M_e = 1.5}$$

OVER THE EXPANSION THE FLOW WILL BE EXPANDED SUCH THAT THE STATIC PRESSURE MATCH THE AMBIENT PRESSURE

(3.30)

$$\frac{p_0}{p_\infty} = \left(1 - \frac{\gamma-1}{2} M_2^2\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow \boxed{M_2 = 1{,}71}$$

(4.44)

$$\nu_e = \nu(M_e)$$

$$\nu_2 = \nu(M_2)$$

$$\boxed{\theta = \nu_2 - \nu_e = 6.2°}$$