TME085 - Compressible Flow

2025-06-10, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs*
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

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Results available no later than 2025-07-01

Good luck!

Part I - Theory Questions (20 p.)

T1. (1 p.)
What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T2. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T3. (2 p.)

(a) In shock tubes, unsteady contact discontinuities are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p, T, ρ, u, s is/are necessarily continuous across such a contact discontinuity?

(b) The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the "*weak*" type or the "*strong*" type. What is the main difference between these two shock types and which type is usually seen in reality?

T4. (2 p.)

(a) What simplifications are made when analyzing a convergent-divergent nozzle flow using a quasi-1D approach?

(b) What are the main limitations of such an analys?

(c) What is meant by an under-expanded or over-expanded nozzle flow?

T5. (2 p.)
Derive the special formula for the speed of sound a for a calorically perfect gas:

$$a = \sqrt{\gamma R T}$$

starting from the general formula:

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

T6. (1 p.)
Assume a steady-state 1D flow with a stationary normal shock. The fluid particles crossing the shock are subjected to

(a) a pressure drop

(b) a density increase

(c) an entropy increase

(d) a temperature drop

(e) a deceleration

Which statements are true and which are false?

T7. (1 p.)
In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

T8. (1 p.)
What is the difference between acoustic waves and other types of waves such as shock waves and expansion waves?

T9. (2 p.)

(a) When applying a time-marching flow solution scheme the so-called CFL number is an important parameter. Define the CFL number and describe its significance. What is a typical maximum CFL number for an explicit time stepping scheme.

(b) What is meant by the terms "*density-based*" and "*fully coupled*" when discussing CFD codes for compressible flow?

T10. (1 p.)
What is meant by choking the flow in a nozzle? Describe it.

T11. (2 p.)
Derive the continuity equation in non-conservation form from the corresponding relation on conservation form.

T12. (2 p.)
Prove, by using one of the non-conservation forms of the energy equation, that for steady-state, adiabatic flow with no body forces the total enthalpy is preserved along stream-lines.

T13. (1 p.)
How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - COMBUSTOR (10 p.)

Air enters a constant-cross-section-area combustor at a pressure and temperature of 180.0 kPa and 120.0 °C, respectively. The velocity of the air at the inlet of the combustor is 80.0 m/s. In the combustor, fuel with a heating value of 45.0 MJ/kg is added and combusted. In your calculations, you can neglect the additional mass added in the combustor in form of fuel and the gas can be assumed to be calorically perfect air (although fuel has been added and the temperature is high)

(a) Calculate the air-fuel ratio for which the combustor will be choked

(b) If the air-fuel ratio is 90% of the value calculated in the previous task, the air massflow will be reduced. Calculate the new massflow.

Problem 2 - NOZZLE FLOW (10 p.)

A nozzle is designed (supersonic flow without shocks) to produce a a flow velocity of $u_e = 850.0$ m/s at the nozzle exit plane if the total temperature at the nozzle inlet is 658.0 K. Calculate the Mach number upstream of the normal shock that will appear inside of the nozzle if the nozzle pressure ratio is $NPR = 0.5(NPR_c + NPR_{ns})$ where NPR_c is the critical nozzle pressure ratio (sonic flow in the both the convergent and divergent part of the nozzle and M=1.0 at the throat) and NPR_{ns} is the pressure ratio for which there will be a normal shock standing at the nozzle exit plane.

Problem 3 - MOVING SHOCK (10 p.)

A normal shock wave with a pressure ratio of 4.5 traveling into stagnant air impinges on a plane wall. The static temperature of the air in front of the incident shock wave is 280.0 K.

Calculate:

(a) the Mach number of the incident shock wave

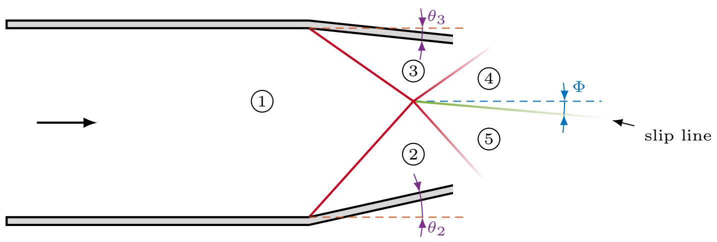
(b) the Mach number of the reflected shock wave

(c) the static pressure ratio for the reflected normal shock wave

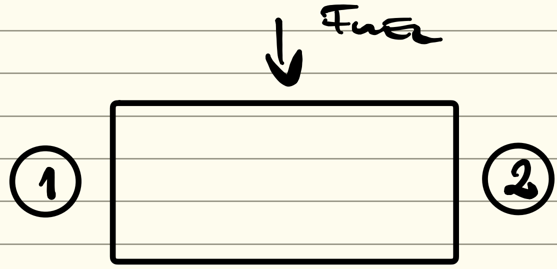
(d) the temperature behind the reflected shock wave

Problem 4 - SHOCK SYSTEM (10 p.)

The figure below depicts a system of oblique shocks. Estimate the slip-line angle Φ if the angles θ_2 and θ_3 are 4.0 and 3.0 degrees, respectively. Upstream of the shock system, the air flow Mach number is $M_1 = 3.0$ and the pressure and temperature are 30.0 kPa and -10 °C, respectively.



P1



$P_1 = 180.0$ kPa

$T_1 = 120.0$ °C

$U_1 = 80.0$ m/s

FUEL HEATING VALUE: 45.0 MJ/kg (HV)

WEIGHT OF ADDED FUEL CAN BE NEGLECTED.

a) CALCULATE THE AIR-FUEL RATIO FOR WHICH THE COMBUSTOR IS CHOKED.

$$q_1^* = c_p \left(T_0^* - T_{0,1} \right) \qquad (1)$$

EQN 8.89

$$\frac{T_{01}}{T_0^*} = \frac{(\gamma+1) M_1^2}{(1+\gamma M_1^2)^2} \left(2 + (\gamma-1) M_1^2 \right) \qquad (2)$$

(1) AND (2) $\Rightarrow$

$$q_1^* = 1868091.4 \text{ J/kg}$$

AIR-FUEL RATIO:

HV / q_1^*

$$\Rightarrow \boxed{24.1 \ \frac{kg_{air}}{kg_{fuel}}}$$

b) AIR-FUEL RATIO 90% OF OPTIMUM. CALCULATE MASSFLOW

$$\Rightarrow q_1 = q_1^* / 0.9$$

q_1 MUST BE THE "NEW q^*"

T_0 WILL NOT CHANGE AT THE INLET

$$q^* = \frac{q_1^*}{0.9} = C_p (T_0^* - T_{0_1})$$

$$= C_p T_{0_1} \left(\frac{T_0^*}{T_{01}} - 1 \right)$$

(8.89)

$$\frac{T_0}{T_0^*} = \frac{(\gamma+1) M_1}{(1+\gamma M_1^2)} (2 + (\gamma-1) M_1^2)$$

SOLVE FOR M_1 $\Rightarrow$ $\boxed{M_1 = 0.17}$

MASSFLOW :

$$\left.\begin{aligned} \dot{m} &= \rho_1 u_1 A \\ u_1 &= M_1 a_1 = M_1 \sqrt{\gamma R T_1} \\ \rho_1 &= \frac{p_1}{R T_1} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \dot{m} = \frac{p_1}{R T_1} M_1 \sqrt{\gamma R T_1}\, A \Rightarrow$$

$$\boxed{\dot{m} = \frac{p_1 \sqrt{\gamma}}{\sqrt{R T_1}} M_1 A}$$

a)

$p_1 = 180$ kPa

$T_1 = 120$ K

$u_1 = 80$ m/s

$M_1 = u_1 / a_1 = u_1 / \sqrt{\gamma R T_1} = 0.20$

$\dot{m}_a = 127.7$ kg/s

b) $M_1 = 0.19$

$$(3.28): \quad \frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$\Rightarrow T_1 = 393.3 \text{ K}$$

(3.30) $$\frac{p_{01}}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow \quad P_1 = 180.5 \text{ kPa}$$

$$\dot{m}_b = 121.9 \text{ kg/s}$$

$$\boxed{\frac{\dot{m}_b}{\dot{m}_a} = 0{,}95}$$

P_2

NOZZLE PRESSURE RATIO

$$NPR = \frac{1}{2}\left(NPR_c + NPR_{ns}\right)$$

MUST BE A NORMAL SHOCK IN THE DIVERGENT SECTION.

CALCULATE MACH NUMBER AHEAD OF THE INTERNAL SHOCK

DESIGN CONDITION:

$u_e = 850.0 \text{ m/s}$

$T_0 = 658.0 \, ^\circ K$

(3.28) $$\frac{T_0}{T_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right) \quad (1)$$

$$M_e = \frac{u_e}{a_e} = \frac{u_e}{\sqrt{\gamma R T_e}} \quad (2)$$

(2) IN (1) $\Rightarrow$

$$\frac{T_0}{T_e} = 1 + \frac{\gamma - 1}{2} \frac{u_e^2}{\gamma R T_e}$$

$$C_p = \frac{\gamma R}{\gamma - 1} \Rightarrow$$

$$T_0 = T_e + \frac{1}{2C_p} u_e^2$$

u_e AND T_0 KNOWN $\Rightarrow T_e$

$$\boxed{T_e = 298.4\ K}$$

$$M_{e_{sc}} = \frac{u_e}{\sqrt{\gamma R T_e}} = 2.45$$

$$NPR_{sc} = \frac{p_0}{p_e} = \left(1 + \frac{\gamma - 1}{2} M_{e_{sc}}^2\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow \boxed{NPR_{sc} = 15.93}$$

(5.20)

$$\left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_{e_{sc}}^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_{e_{sc}}^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow \boxed{\frac{A_e}{A^*} = 2.53}$$

(5.20) SUBSONIC SOLUTION $\Rightarrow$

$$\boxed{M_{e_c} = 0.24}$$

$$NPR_c = \frac{p_0}{p_{e_c}} = \left(1 + \frac{\gamma - 1}{2} M_c^2\right)^{\gamma/(\gamma-1)}$$

$$\boxed{NPR_c = 1.04}$$

$$NPR_{ns} = NPR_{sc} / PR_{ns}$$

(3.57)

$$PR_{ns} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{e_{sc}}^2 - 1\right)$$

$$\boxed{NPR_{ns} = 2.32}$$

$$NPR = \frac{1}{2}\left(NPR_c + NPR_{ns}\right) = 1.68$$

(5.28) EXIT MACH NUMBER FOR FLOWS WITH INTERNAL SHOCK

$$M_e^2 = \frac{-1}{\gamma - 1} + \sqrt{\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{p_{01}}{p_e}\frac{A_t}{A_e}\right)^2}$$

$$\Rightarrow M_e = 0.38$$

$$\frac{p_{oe}}{p_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_o}{p_e} = NPR$$

$\Rightarrow$

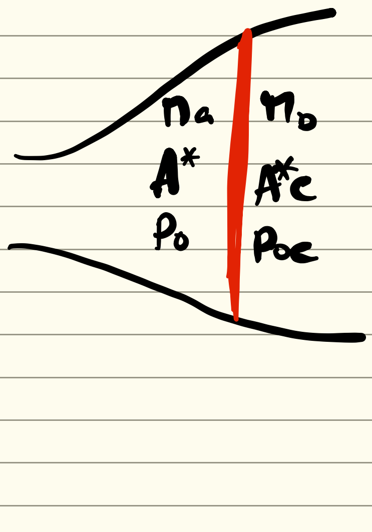
$$\frac{p_{oe}}{p_o} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\gamma/(\gamma-1)} \Big/ NPR$$

THIS CHANGE OF TOTAL PRESSURE IS A CONSEQUENCE OF THE INTERNAL NORMAL SHOCK

$$(5.21) \qquad \dot{m} = \frac{p_o A^*}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$\dot{m}$ AND T_o CONSTANT OVER SHOCK

$$\Rightarrow p_{oe} A^*_e = p_o A^*$$



(5.20) $\Rightarrow$

$$\left(\frac{A_{shock}}{A^*}\right)^2 = f(Ma)$$

$$\left(\frac{A_{shock}}{A^*_e}\right)^2 = f(Mb)$$

(5.21) $\Rightarrow$

$$\frac{A^*_e}{A^*} = \frac{P_0}{P_{0e}}$$

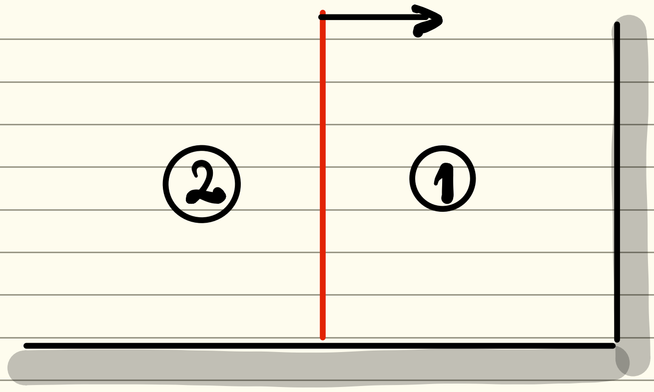
(3.51) $\Rightarrow$

$$Mb^2 = \frac{1 + ((\gamma - 1)/2)Ma^2}{\gamma Ma^2 - (\gamma - 1)/2}$$

SOLVE FOR Ma

$$\Rightarrow \boxed{Ma = 2.14}$$

P3



$$\frac{p_2}{p_1} = 4.5$$

$u_1 = 0$, $u_2 = u_p$

$T_1 = 280\,K$

a) CALCULATE M_s
(SHOCK MACH NUMBER)

(7.13)

$$M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1}$$

$$\Rightarrow \boxed{M_s = 2.0}$$

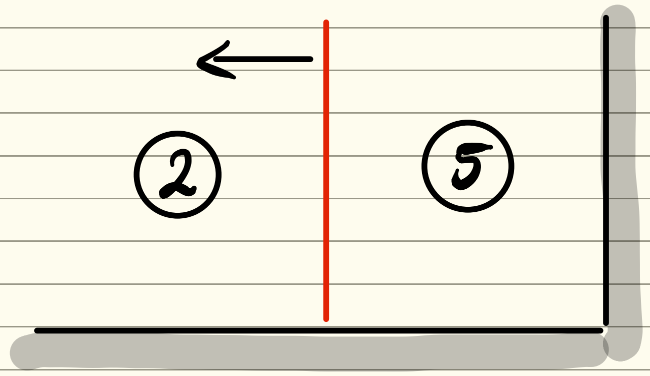
b) CALCULATE THE MACH NUMBER OF THE REFLECTED SHOCK WAVE.

(7.23)

$$\frac{M_r}{M_r^2 - 1} = \frac{M_s}{M_s^2 - 1}\sqrt{1 + \frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2 - 1)\left(\gamma + \frac{1}{M_s^2}\right)}$$

$$\Rightarrow \quad \boxed{M_R = 1.73}$$

c) CALCULATE THE STATIC PRESSURE RATIO OVER THE REFLECTED SHOCK.



(3.57)

$$\frac{p_5}{p_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_R^2 - 1\right)$$

$$\Rightarrow \quad \boxed{\frac{p_5}{p_2} = 3.33}$$

d) CALCULATE THE STATIC TEMPERATURE BEHIND THE REFLECTED SHOCK (T_5)

(7.10)

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left(\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}\right)$$

$$\Rightarrow \quad \frac{T_2}{T_1} = 1.69$$

(3.59)

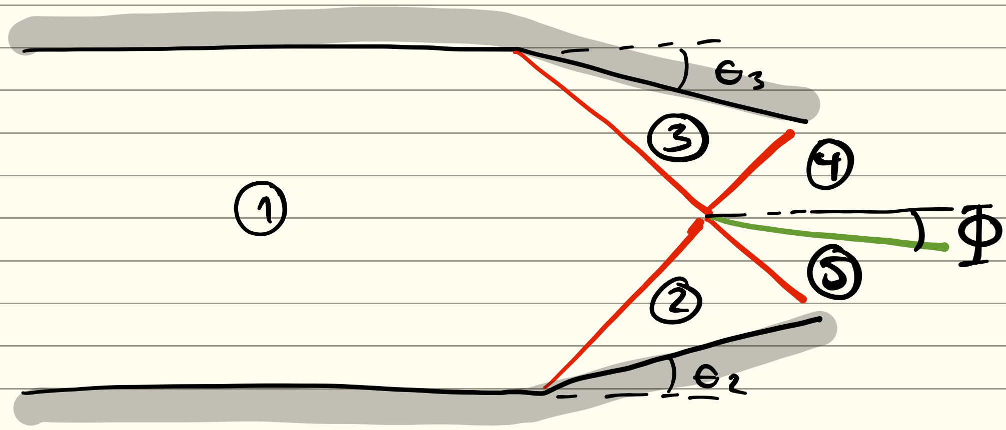
$$\frac{T_5}{T_2} = \left(1 + \frac{2\gamma}{\gamma+1}\left(M_R^2 - 1\right)\right)\left(\frac{2 + (\gamma-1)M_R^2}{(\gamma+1)M_R^2}\right)$$

$$\Rightarrow \quad \frac{T_5}{T_2} = 1.48$$

$$\frac{T_5}{T_1} = \frac{T_5}{T_2}\,\frac{T_2}{T_1}$$

$$\boxed{T_5 = \frac{T_5}{T_2}\,\frac{T_2}{T_1}\,T_1 = 700\,K}$$

P4



$\theta_2 = 4.0^\circ$

$\theta_3 = 3.0^\circ$

$M_1 = 3.0$

$P_1 = 30.0\,kPa$

$T_1 = -10^\circ C$

THE GAS WILL BE ASSUMED TO BE CALORICALLY PERFECT AIR ($\gamma = 1.4$)

ESTIMATE THE ANGLE Φ
FLOW DIRECTION AFTER SHOCK INTERSECTION (SLIP-LINE ANGLE)

1. OBLIQUE SHOCK $1 \rightarrow 2$

$\theta_2 = 3.0^\circ$, $M_1 = 3.0$

$\theta\text{-}\beta\text{-}M \Rightarrow \beta_2 = 22.35^\circ$

(4.7) $M_{n_{21}} = M_1 \sin \beta_2$

(4.9) $$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_{21}}^2 - 1\right)$$

(4.10) $$M_{n_{22}}^2 = \frac{M_{n_{21}}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_{21}}^2 - 1}$$

(4.12) $$M_2 = \frac{M_{n2}}{\sin(\beta_2 - \theta_2)}$$

$$\Rightarrow \boxed{M_2 = 2.8 \qquad \frac{p_2}{p_1} = 1.85}$$

2. OBLIQUE SHOCK $1 \rightarrow 3$

$\theta_3 = 4°$, $M_1 = 3.0$

$\theta - \beta - M \Rightarrow \beta_3 = 21.6°$

(4.7) $M_{n_{31}} = M_1 \sin \beta_3$

(4.9) $$\frac{p_3}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_{31}}^2 - 1\right)$$

(4.10) $$M_{n_{32}}^2 = \frac{M_{n_{31}}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_{31}}^2 - 1}$$

(4.12) $$M_3 = \frac{M_{n_{32}}}{\sin(\beta_3 - \theta_3)}$$

$$\Rightarrow \boxed{M_3 = 2.85 \qquad \frac{P_3}{P_1} = 1.26}$$

\# 8. FIND Φ

CONSTRAINTS :

1) $P_4 = P_5$

2) FLOW DIRECTION SAME IN REGIONS 4 AND 5.

ALGORITHM :

1) GUESS Φ $\Rightarrow$

$$\theta_4 = \theta_3 - \Phi$$

$$\theta_5 = \theta_2 + \Phi$$

2) $\theta - \beta - M$

M_3 AND $\theta_4 \Rightarrow \beta_4$

M_2 AND $\theta_5 \Rightarrow \beta_5$

3) (4.7) AND (4.9)

$$\frac{P_4}{P_1} = f(M_3, \beta_4, P_3/P_1)$$

$$\frac{P_5}{P_1} = f(M_2, \beta_5, P_2/P_1)$$

ITERATE 1 $\rightarrow$ 3 UNTIL

$$\left| \frac{P_4}{P_1} - \frac{P_5}{P_1} \right| < \varepsilon$$

WHERE ε IS AN ACCEPTABLE TOLERANCE.

$\Rightarrow \ldots \Rightarrow$

$$\boxed{\Phi = -1^\circ}$$