TME085 - Compressible Flow

2025-03-20, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs*
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Responsible teacher: Niklas Andersson tel.: 031-772 3788

Results available no later than 2025-04-10

Good luck!

Part I - Theory Questions (20 p.)

T1. (1 p.) Clasification

(a) What are the criteria for the classifications:

subsonic flow
transsonic flow
supersonic flow
hypersonic flow

(b) What are the criteria for an **isentropic** process, i.e. what conditions must be satisfied for a **steady-state** compressible flow to be **isentropic**?

T2. (1 p.) What is the physical interpretation of each of the terms in the **momentum equation** on integral form

$$\frac{d}{dt}\iiint_{\Omega} \rho \mathbf{v} d\mathscr{V} + \oiint_{\partial\Omega} [\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}]\, dS = \iiint_{\Omega} \rho \mathbf{f} d\mathscr{V}$$

T3. (1 p.) Reference flow states

(a) How do we define **total conditions** in a steady-state isentropic flow?

(b) How do we define **critical conditions** in a steady-state isentropic flow?

(c) Does the total temperature T_o change due to **friction**? Explain why/why not.

T4. (7 p.) Flow discontinuities

(a) The **normal shock relations** actually allow two solutions, one that corresponds to a discontinuous compression (a sudden pressure increase) and one that corresponds to a discontinuous expansion (a sudden pressure decrease). However, only one of these solutions is physically valid. What thermodynamic principle guides us in the choice of the physically correct solution, and which solution is the correct one?

(b) How come that the control volume approach applied to the governing equations on **adiabatic** form gives us the normal-shock relations? *i.e.*, how do the equations "know" that there is a shock inside of the control volume?

(c) An object placed in a supersonic freestream will for some Mach numbers generate a **detached shock** in front of the object and for other Mach numbers **oblique shocks** attached to the object will be formed - explain why.

(d) What is a **slip line**. What conditions must be fulfilled across slip line? When is a slip line generated?

(e) Which types of waves or discontinuities are generated in **shock tubes**?

(f) Describe what happens when a **moving normal shock** hits a **solid wall**.

(g) Can a **moving normal shock** travel at a speed lower than the **speed of sound**? Explain why/why not.

T5. (2 p.) Nozzle flow

(a) Explain the concept **choked** nozzle flow?

(b) Explain the concept **underexpanded** nozzle flow?

T6. (2 p.) Acoustic waves

(a) Which flow equations and what assumptions are used to derive the **acoustic equations**?

(b) What is the difference between **acoustic waves** and other types of waves such as **shock waves** and **expansion waves**?

T7. (3 p.) Computational Fluid Dynamics

(a) What is meant by the term **density-based** when discussing CFD codes for compressible flow?

(b) What is meant by the term **fully-coupled** when discussing CFD codes for compressible flow?

(c) What do we mean when we say that a CFD code for compressible flow is **conservative**?

(d) How can we use our knowledge of **characteristics** (and their speed of propagation) to guide us when determining suitable **boundary conditions** for compressible flows?

T8. (3 p.) Gas model

(a) Explain the concept **zero-point energy**.

(b) Try to explain what the **Boltzmann distribution** describes and what sparsely populated implies.

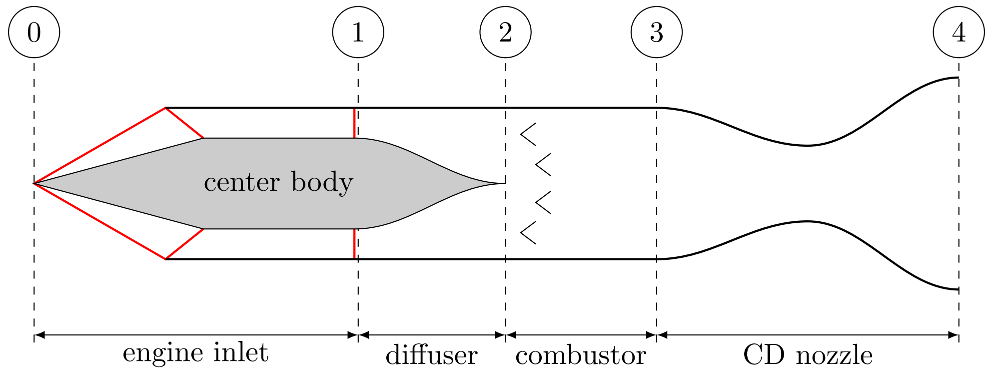
(c) What is the difference between a **calorically perfect** gas and a **thermally perfect** gas?

(d) Using **equilibrium gas** assumption in the analysis of chemically reacting nozzle flow will lead to higher exhaust temperatures than if calorically perfect gas assumption is used for the same analysis. Explain why.

Part II - Problems (40 p.)

Problem 1 - RAMJET ENGINE (20 p.)

The figure below (not to scale) gives a schematic representation of an axisymmetric ramjet engine divided into four sections (engine inlet, diffuser, combustor, and exhaust nozzle). A ramjet engine is an engine constructed without moving parts. The flow is compressed through shocks instead of using compressors. In a ramjet engine, the flow through the combustor is subsonic. Air at a pressure of 101325 Pa, a temperature of 293 K and a Mach number of 3.0 enters the engine inlet and is compressed by two oblique shocks followed by a normal shock standing at the end of the inlet section. The first oblique shock is generated as the flow deflects an angle of $\theta = 15°$ by the center body. In the diffuser section, the radius of the center body is reduced from 0.063 m to zero. In the combustor heat is added such that the temperature reaches 950 K at the end of the combustor. After the combustor, the gas is expanded through a CD-nozzle.



More detailed description of each section on next page

Calculate:

(a) Temperature, pressure and Mach number in station 1 (just downstream of the normal shock)

(b) Temperature, pressure and Mach number in station 2 (at the end of the diffuser)

(c) Temperature, pressure and Mach number in station 3 (at the end of the combustor)

(d) The throat area of the nozzle

(e) Temperature, pressure and Mach number in station 4 (at the nozzle exit)

(f) The engine thrust

Note:

The gas can be assumed to be calorically perfect air through the entire engine although that is not appropriate considering the temperature range.

This problem involves quite a few steps. If you get stuck somewhere along the way or if you do not know how to solve one of the involved subproblems, you can make an educated guess of flow parameters entering the next section and continue from that point.

0-1 **Shock compression:**

- In the engine inlet, the incoming air is compressed by **two oblique shocks** followed by **a normal shock**
- The angle of the front part of the center body is 30°, which means that the **flow deflection angle** is $\theta = 15°$
- Note: since the engine is axisymmetric the front part of the center body is a cone and not a wedge but you can use the methods that we have used for wedge-flows in the course anyway

1-2 **Isentropic diffusion:**

- The radius of the axisymmetric center body is reduced from 0.063 m at station 1 to zero at station 2

2-3 **Heat addition:**

- **Constant cross-section area** - radius of axisymetric combustor: 0.1 m
- The heat addition can be assumed to be evenly distributed between 2 and 3
- The combustor will be operated such that the maximum allowed temperature is reached at the end of the combustor ($T_3 = 950\ K$)
  the maximum temperature is set by material constraints
- Additional mass added in the combustor in the form of fuel does not have to be accounted for
- Although fuel is added as part of the combustion, the gas can be treated as air (calorically perfect)

3-4 **Flow expansion:**

- Expansion through a convergent-divergent nozzle
- The nozzle flow can be assumed to be **supercritical**
  perfectly expanded supersonic flow

Problem 2 - BURSTING CAR TIRE (10 p.)

A shock wave is generated as a car tire bursts for a car coasting along a high way. Just before the accident, the driver stopped at a gas station and filled the tires with air. After filling, the tire pressure was 2.4 bar. The shock wave will expand spherically from the location where it was initiated but you can treat the wave front as a normal shock (the more you zoom in on the wave front, the more it will resemble a normal shock).

Calculate:

(a) The propagation speed of the shock wave generated by the tire burst

(b) The induced air velocity behind the shock wave

Problem 3 - PIPE FLOW WITH FRICTION (10 p.)

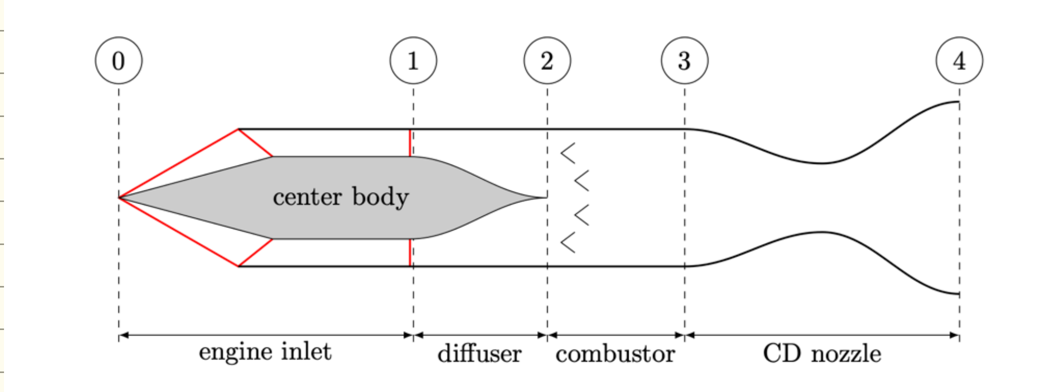
Air is transported through a 20 cm diameter pipe with an average friction factor of $\bar{f} = 0.005$ at a massflow rate of 2.0 kg/s. At the pipe inlet, the temperature of the air is 293 K. Calculate the maximum possible pipe length if the massflow rate at which air is transported through the pipe must not be affected and the static pressure at the inlet of the pipe is

(a) $p_1 = 78\ kPa$

(b) $p_1 = 7.8\ kPa$

P1

M_∞
P_∞
T_∞



CALCULATE STATION DATA ACCORDING TO SPECIFICATIONS, NOZZLE THROAT AREA AND ENGINE THRUST.

CALORICALLY PERFECT GAS CAN BE ASSUMED IN ALL ENGINE SECTIONS. ($\gamma = 1.4$)
OBLIQUE SHOCKS ARE TREATED AS 2D - SHOCKS.

0 → 1 : ENGINE INTAKE -

2 OBLIQUE SHOCKS AND 1 NORMAL SHOCK.

OBLIQUE SHOCK 1:

$M_\infty = 3.0$
$\theta = 15°$

$\theta - \beta - M$ (4.17) $\Rightarrow \beta = 32.2°$

$$M_{n_\infty} = M_\infty \sin\beta \qquad (4.7)$$

$$\frac{p_x}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_\infty}^2 - 1\right) \qquad (4.9)$$

$$\frac{\rho_x}{\rho_\infty} = \frac{(\gamma+1)M_{n_\infty}^2}{(\gamma-1)M_{n_\infty}^2 + 2} \qquad (4.8)$$

$$\frac{T_x}{T_\infty} = \frac{p_x}{p_\infty}\frac{\rho_\infty}{\rho_x} \qquad (4.11)$$

$$M_{n_x}^2 = \frac{M_{n_\infty}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_\infty}^2 - 1} \qquad (4.10)$$

$$M_x = \frac{M_{n_x}}{\sin(\beta-\theta)} \qquad (4.12)$$

$$(4.7) \rightarrow (4.12) \Rightarrow$$

$$M_x = 2.25$$

$$\frac{p_x}{p_\infty} = 2.82$$

$$\frac{T_x}{T_\infty} = 1.39$$

OBLIQUE SHOCK 2: (the flow is turned back 15°)

$$\left.\begin{aligned} M_x &= 2.25 \\ \theta &= 15^\circ \end{aligned}\right\} \quad (4.7) \Rightarrow \quad \beta = 40.35^\circ$$

$$M_{n_x} = M_x \sin\beta \qquad (4.7)$$

$$\frac{p_y}{p_x} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_x}^2 - 1\right) \qquad (4.9)$$

$$\frac{\rho_y}{\rho_x} = \frac{(\gamma+1) M_{n_x}^2}{(\gamma-1) M_{n_x}^2 + 2} \qquad (9.8)$$

$$\frac{T_y}{T_x} = \frac{P_y}{P_x} \frac{\rho_x}{\rho_y} \qquad (9.11)$$

$$M_{n_y}^2 = \frac{M_{n_x}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1)) M_{n_x}^2 - 1} \qquad (9.10)$$

$$M_y = \frac{M_{n_y}}{\sin(\beta - \theta)} \qquad (9.12)$$

$(9.7) \rightarrow (9.12) \Rightarrow$

$$M_y = 1.67$$

$$\frac{P_y}{P_x} = 2.32$$

$$\frac{T_y}{T_x} = 1.28$$

Normal shock:

$$M_1^2 = \frac{1 + ((\gamma-1)/2) M_y^2}{\gamma M_y^2 - (\gamma-1)/2} \qquad (3.51)$$

$$\frac{P_1}{P_y} = 1 + \frac{2\gamma}{\gamma+1} (M_y^2 - 1) \qquad (3.57)$$

$$\frac{T_1}{T_y} = \left(1 + \frac{2\gamma}{\gamma+1} (M_y^2 - 1)\right) \left(\frac{2 + (\gamma-1) M_y^2}{(\gamma+1) M_y^2}\right) \qquad (3.59)$$

$$\Rightarrow M_1 = 0.65$$

$$\frac{P_1}{P_y} = 3.09 \quad , \quad \frac{T_1}{T_y} = 1.44$$

Σ: 0 → 1 ENGINE INLET

$M_1 = 0.65$

$$\frac{P_1}{P_\infty} = \frac{P_x}{P_\infty}\frac{P_y}{P_x}\frac{P_1}{P_y} = 20.25$$

$$\frac{T_1}{T_\infty} = \frac{T_x}{T_\infty}\frac{T_y}{T_x}\frac{T_1}{T_y} = 2.58$$

$P_\infty = 101325\ Pa \Rightarrow P_1 = 2025.3\ kPa$

$T_\infty = 293\ K \Rightarrow T_1 = 756.8\ K$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2 \quad (3.28) \Rightarrow T_{01} = 820.9K$$

$$\frac{P_{01}}{P_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \quad (3.30) \rightarrow$$

$\Rightarrow P_{01} = 2721.5\ kPa$

1 → 2 : AXISYMMETRIC DIFFUSER

ISENTROPIC FLOW ⇒ T_0 & P_0 PRESERVED

(5.20)

$$\left(\frac{A_1}{A^*}\right)^2 = \frac{1}{M_1^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_1^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\left(\frac{A_2}{A^*}\right)^2 = \frac{1}{M_2^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_2^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$A^* =$ CONST ; A_1, A_2 & M_1 KNOWN ⇒

$$\Rightarrow M_2 = 0.32$$

$$T_{02} = T_{01}$$

$$P_{02} = P_{01}$$

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2 \qquad (3.28)$$

$$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/(\gamma-1)} \qquad (3.30)$$

Σ 1 → 2 AXISYMMETRIC DIFFUSER

$M_2 = 0.32$, $T_2 = 803.4\ K$, $P_2 = 2529.6\ kPa$

$T_{02} = 820.4\ K$, $P_{02} = 2721.5\ kPa$

2 → 3 : COMBUSTOR (CONST. AREA)

WE ARE GIVEN THE COMBUSTOR EXIT TEMPERATURE : $T_3 = 950\ K$

$$\frac{T_2}{T^*} = M_2^2 \left(\frac{1+\gamma}{1+\gamma M_2^2}\right)^2 \qquad (3.86)$$

$$\Rightarrow T^* = 1741.0\ K > 950K$$

⇒ FLOW IS NOT CHOKED IN THE COMBUSTOR

$T^* = $ CONST ⇒

$$\frac{T_3}{T^*} = M_3^2 \left(\frac{1+\gamma}{1+\gamma M_3^2}\right)^2 \qquad (3.86)$$

SOLVE FOR M_3 $\Rightarrow$ $M_3 = 0.37$

$$\frac{P_2}{P^*} = \frac{1+\gamma}{1+\gamma M_2^2} \quad (3.85) \Rightarrow P^* = 1209.7\ kPa$$

$$\frac{P_3}{P^*} = \frac{1+\gamma}{1+\gamma M_3^2} \quad (3.85) \Rightarrow P_3 = 2446.4\ kPa$$

$$\frac{T_{o3}}{T_3} = 1 + \frac{\gamma-1}{2} M_3^2 \quad (3.28) \Rightarrow T_{o3} = 975.4\ K$$

$$\frac{P_{o3}}{P_3} = \left(1 + \frac{\gamma-1}{2} M_3^2\right)^{\gamma/(\gamma-1)} \quad (3.30) \Rightarrow P_{o3} = 2682.6\ kPa$$

Σ 2 $\rightarrow$ 3 : COMBUSTOR

$M_3 = 0.37$, $T_3 = 950\ K$, $P_3 = 2446.4\ kPa$

$T_{o3} = 975.4\ K$, $P_{o3} = 2682.6\ kPa$

3 $\rightarrow$ 4 : NOZZLE EXPANSION

SUPERCRITICAL EXPANSION IS ASSUMED :

SONIC FLOW IN THROAT (CHOKED)

NO SHOCKS

EXIT PRESSURE = AMBIENT PRESSURE.

$$\left(\frac{A_3}{A^*}\right)^2 = \frac{1}{M_3^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_3^2\right)\right)^{(\gamma+1)/(\gamma-1)} \quad (5.20)$$

A_3 & M_3 KNOWN

$$\left(A_3 = \frac{\pi D_3^2}{4} \quad \text{WHERE} \quad D_3 = 0{,}1\,m \right)$$

$$\Rightarrow A^* = A_{THROAT} = 183.24\,cm^2$$

$$(R_{THROAT} = 0{,}076\,m)$$

ISENTROPIC EXPANSION $\Rightarrow$ P_0 & T_0 CONST.

$$\frac{P_{04}}{P_4} = \frac{P_{03}}{P_\infty} = \left(1 + \frac{\gamma - 1}{2} M_4^2\right)^{\gamma/(\gamma-1)} \qquad (3.30)$$

$$\Rightarrow M_4 = 2.78$$

$$\frac{T_{04}}{T_4} = \left(1 + \frac{\gamma - 1}{2} M_4^2\right) \qquad (3.28)$$

$$\Rightarrow T_4 = 382.5\,K$$

NOZZLE EXIT AREA:

$$\left(\frac{A_4}{A^*}\right)^2 = \frac{1}{M_4^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_4^2\right)\right)^{(\gamma+1)/(\gamma-1)} \qquad (5.20)$$

$$\Rightarrow A_4 = 631.5\,cm^2 \qquad (R_4 = 0.14\,m)$$

ENGINE THRUST:

UNDER STEADY STATE FLOW CONDITIONS AND WITH THE MASSFLOW ASSUMPTION GIVEN, THE

ENGINE THRUST IS CALCULATED AS:

$$F = \dot{m}\,(u_4 - u_\infty)$$

(THE MOMENTUM EQUATION ON INTEGRAL FORM FOR A CONTROL VOLUME SURROUNDING THE ENGINE.)

THERE WILL BE NO CONTRIBUTION FROM PRESSURE FORCES AS THE EXIT PRESSURE EQUALS THE AMBIENT PRESSURE.

$$\dot{m} = u_4\, \rho_4\, A_4$$

$$u_4 = M_4\, a_4 = M_4 \sqrt{\gamma R T_4} = 1.09 \text{ km/s}$$

$$\rho_4 = p_4 / (R T_4)$$

$$\Rightarrow \dot{m} = 63.6 \text{ kg/s}$$

$$u_\infty = M_\infty\, a_\infty = M_\infty \sqrt{\gamma R T_\infty} = 1{,}03 \text{ km/s}$$

$$\Rightarrow F = \dot{m}\,(u_4 - u_\infty) = 3.94 \text{ kN}$$

a

$M_1 = 0.65$

$T_1 = 756.8\ K$

$P_1 = 2052.3\ kPa$

b

$M_2 = 0.32$

$T_2 = 809.4\ K$

$P_2 = 2529.6\ kPa$

c

$M_3 = 0.37$

$T_3 = 950\ K$

$P_3 = 2446.4\ kPa$

d

$A_t = 183.2\ cm^2$

e

$M_4 = 2.78$

$T_4 = 382.5\ K$

$P_4 = 101.325\ kPa$

f

$F = 3.94\ kN$

P_2

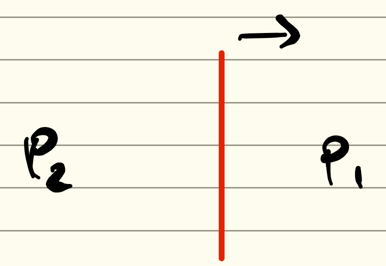
Bursting car tire.

To be able to solve the problem we need to make assumptions about the surrounding conditions.

$P_\infty = 101325\ Pa$, $T_\infty = 293\ K$

The car is coasting on the highway as one of the tires bursts, which generates a shock wave.
Assume that the shock wave propagates into air that is standing still.

The shock wave is approximated as a moving normal shock.



$P_1 = P_\infty$

$P_2 = 2.9$ bar

Mach number of moving shock:

$$(7.13) \qquad M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1}$$

(WHERE $\gamma = 1.4$)

$$\Rightarrow \boxed{M_s = 1.47}$$

WE ARE ASKED TO CALCULATE THE PROPAGATION VELOCITY OF THE MOVING SHOCK

$$W = M_s a_1 = M_s \sqrt{\gamma R T_1} = 505.8 \text{ m/s}$$

NEXT, WE ARE ASKED TO CALCULATE THE INDUCED FLOW VELOCITY BEHIND THE SHOCK.

$$(7.16) \qquad u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

$$\Rightarrow u_p = 227.5 \text{ m/s}$$

a) $W = 505.8$ m/s

b) $u_p = 227.5$ m/s

P_3

PIPE FLOW WITH FRICTION

AIR IS TRANSPORTED THROUGH A 20 cm DIAMETER PIPE AT THE MASSFLOW RATE 2.0 kg/s

THE AVERAGE FRICTION FACTOR IS $\bar{f} = 0.005$ AND THE INLET TEMPERATURE IS 293 K.

CALCULATE THE MAXIMUM POSSIBLE PIPE LENGTH IF THE MASSFLOW RATE IS TO BE KEPT CONSTANT.

a) $P_1 = 78\ kPa$

$$\left.\begin{aligned} P_1 &= 78\ kPa \\ T_1 &= 293\ K \end{aligned}\right\} \Rightarrow \rho_1 = \frac{P_1}{RT_1} = 0.93\ kg/m^3$$

$$\dot{m} = \rho_1 u_1 A = \rho_1 u_1 \frac{\pi D^2}{4} \Rightarrow u_1 = 68.6\ m/s$$

$$M_1 = \frac{u_1}{a_1} = \frac{u_1}{\sqrt{\gamma R T_1}} = 0.2$$

$M_1 = 0.2$: SUBSONIC INFLOW $\Rightarrow$

IF THE PIPE IS LONGER THAN L^*, THE MASSFLOW RATE WILL CHANGE

(THE INLET STATIC CONDITIONS WILL CHANGE SUCH THAT $L = L^*$ AND THUS THE MASSFLOW RATE WILL CHANGE)

(3.107)

$$\frac{4\bar{f}L^*}{D} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

$$\Rightarrow L^* = 145.3 \text{ m}$$

b) $p_1 = 7.8$ kPa

$$\left.\begin{aligned} p_1 &= 7.8 \text{ kPa} \\ T_1 &= 293 \text{ K} \end{aligned}\right\} \Rightarrow \rho_1 = \frac{p_1}{RT_1} = 0.093 \text{ kg/m}^3$$

$$\dot{m} = \rho_1 u_1 A = \rho_1 u_1 \frac{\pi D^2}{4} \Rightarrow u_1 = 686.3 \text{ m/s}$$

$$M_1 = \frac{u_1}{a_1} = \frac{u_1}{\sqrt{\gamma R T_1}} = 2.0$$

FOR SUPERSONIC INFLOW, A PIPE LONGER THAN L^* WILL GENERATE A SHOCK IN THE PIPE. THE LOCATION OF THE SHOCK DEPENDS ON PIPE LENGTH. THE SHOCK DOES NOT AFFECT THE MASSFLOW RATE

THE LONGEST POSSIBLE PIPE WITHOUT CHANGING THE MASS FLOW RATE IS L^* WHEN THE SHOCK STANDS AT THE INLET.

WE KNOW THAT THE INLET MACH NUMBER IS 2.0, THE NORMAL-SHOCK RELATIONS GIVES:

(3.51)

$$M_2^2 = \frac{1 + ((\gamma-1)/2) M_1^2}{\gamma M_1^2 - (\gamma-1)/2}$$

$$\Rightarrow M_2 = 0.58$$

(3.107)

$$\frac{4\bar{f}L^*}{D} = \frac{1-M_2^2}{\gamma M_2^2} + \frac{\gamma+1}{2\gamma} \ln\left(\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2}\right)$$

$$\Rightarrow L^* = 5.9 \text{ m}$$