TME085 - Compressible Flow

2024-06-04, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

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Results available no later than 2024-06-27

Good luck!

Part I - Theory Questions (20 p.)

T1. (3.0 p.) Total flow conditions:

(a) (1.0 p.) How are total flow properties such as **total temperature** (T_o) and **total pressure** (p_o) defined?

(b) (0.5 p.) How does total pressure and total temperature change over a **normal shock**?

(c) (0.5 p.) How does total pressure and total temperature change in a one-dimensional flow with **friction**?

(d) (0.5 p.) How does total pressure and total temperature change in a one-dimensional flow with **heat addition**?

(e) (0.5 p.) How does total pressure and total temperature change in an **isentropic** flow?

T2. (3.0 p.) Shocks:

(a) (0.5 p.) The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the **weak** type or the **strong** type. What is the main difference between these two shock types and which type is usually seen in reality?

(b) (0.5 p.) Describe what happens when a **moving normal shock** hits a solid wall.

(c) (1.0 p.) Assume a steady-state 1D flow with a **stationary normal shock**. The fluid particles crossing the shock are subjected to

i. a pressure drop
ii. a density increase
iii. an entropy increase
iv. a temperature drop
v. a deceleration

Which statements are true and which are false?

(d) (1.0 p.) Can a **moving normal shock** travel at a speed lower than the **speed of sound**? Explain why/why not.

T3. (3.0 p.) The area-velocity relation:

(a) (2.0 p.) Derive the **area-velocity relation** in quasi-one-dimensional flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

(b) (1.0 p.) What can we learn about the flow through a convergent-divergent nozzle from the **area-velocity relation** for quasi-one-dimensional flow?

T4. (2.0 p.) Describe the concept of **maximum flow deflection** in relation to oblique shocks. What happens in a case where the need for flow deflection exceeds the maximum flow deflection?
it might be of help to use a schematic representation of a shock polar or the θ-β-M-relation to support your explanation

T5. (3.0 p.) Gas models

(a) (1.0 p.) What do we mean by **thermally perfect gas** and **calorically perfect gas**, respectively. What is the main difference between these gas models and for which temperature ranges do they apply?

(b) (1.0 p.) Explain what the **Boltzmann distribution** describes and what sparsely populated implies.

(c) (1.0 p.) Using **equilibrium gas** assumption in the analysis of chemically reacting nozzle flow will lead to higher exhaust temperatures than if calorically perfect gas assumption is used for the same analysis. Explain why.

T6. (4.0 p.) Compressible CFD:

(a) (0.5 p.) What is meant by the term **density-base** when discussing CFD codes for compressible flow?

(b) (0.5 p.) When the governing equations are discretized using a **finite-volume** approach, cell face values of flow properties appears in the equations. How are these values approximated?

(c) (1.0 p.) What do we mean when we say that a CFD code for compressible flow is **conservative**? Why is it important that the numerical scheme is conservative?

(d) (1.0 p.) When applying a **time-marching** flow solution scheme the so-called **CFL number** is an important parameter. Define the **CFL number** and describe its significance.

(e) (1.0 p.) How can we use our knowledge of **characteristics** (and their speed of propagation) to guide us when determining suitable **boundary conditions** for compressible flows?

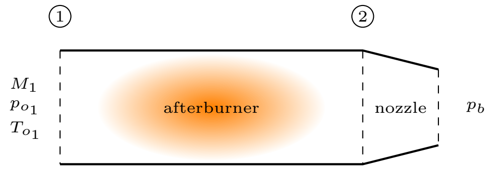
T7. (1.0 p.) In shock tubes, unsteady **contact discontinuities** are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p, T, ρ, u, s is/are necessarily continuous across such a **contact discontinuity**?

T8. (1.0 p.) Explain the consequence of **free-boundary reflection** for the external flow of a nozzle operating at **overexpanded** conditions.

Part II - Problems (40 p.)

Problem 1 - AFTERBURNER (10 p.)

An aeroengine intended to be used for high-subsonic/low-supersonic propulsion is tested mounted on a stationary test stand, The engine is equipped with an afterburner which in essence is a constant area duct where fuel can be added and combusted. Activating the afterburner is an effective, but not very efficient, way of generating extra power. Downstream of the afterburner, the hot gas leaves the engine through a convergent nozzle with adjustable exit area. Upstream of the afterburner duct the mach number is $M_1 = 0.3$ and the total pressure and total temperature of the flow are $p_{o_1} = 500\ kPa$ and $T_{o_1} = 900\ K$, respectively. With the afterburner activated, the total temperature is increased to $T_{o_2} = 1900\ K$ at the end of the afterburner. The massflow through the engine is $\dot{m} = 90\ kg/s$ and the surrounding pressure downstream of the nozzle exit is $p_b = 100\ kPa$.



calculate:

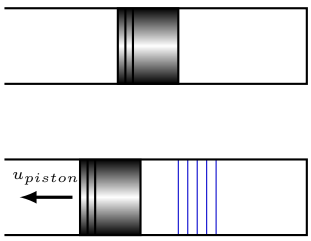
(a) the total pressure loss in the afterburner due to the heat addition

(b) the nozzle-exit area for flow without afterburner activated and flow with afterburner activated assuming that the flow is sonic at the nozzle exit in both cases

(c) the engine thrust with and without afterburner (again assume sonic flow at the nozzle exit)

Problem 2 - PISTON (10 p.)

The piston in the tube depicted below is suddenly moved to the left at a constant velocity (acceleration and initial transients are discarded). As the piston starts to move to the left, an expansion region moving to the right will be formed over which the velocity and thermodynamic flow properties are gradually changed from the initial flow conditions to the conditions just behind the piston. Before the piston starts to move the temperature and pressure in the cylinder are $T = 300\ K$ and $p = 100\ kPa$, respectively.



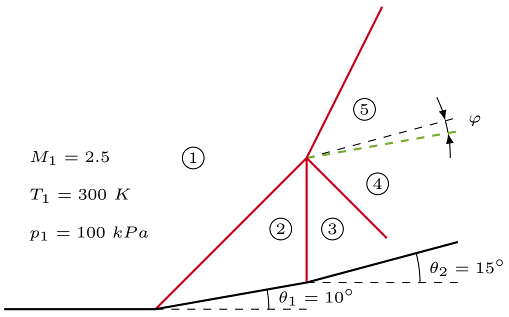
calculate:

(a) the piston speed for which the tail of the expansion (the left-most end) stands still in the tube

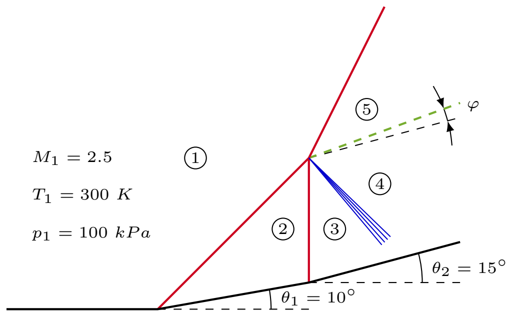
(b) the pressure and temperature just behind the piston for the calculated piston velocity

Problem 3 - SHOCK SYSTEM (10 p.)

The double compression corner represented in the figures below will lead to the formation of two oblique shocks of different shock angle and shock strength in order to deflect the flow in two steps. Due to the nature of shock generation the shock angles will be such that the two oblique shocks will eventually meet at a point away from the wall and from that point outwards, the two shocks will be replaces by a single stronger oblique shock. Since the generation of entropy will be higher through the stronger shock than by the two weaker shocks combined, a slip line will be formed at the shock intersection points separating the flow going through the single shock and the flow passing the lower shock system. Over the slip line pressure and flow angle must be continuous, other flow quantities may be discontinuous. It is rather unlikely that one would get a perfect match of both pressure and flow direction without the generation of additional shock (case 1 in the figure below) or expansion (case 2 in the figure below). If an extra shock will be generated between region 3 and 4 or if flow deflection will be accomplished by an expansion depends on whether the flow needs to be deflected towards the wall (case 1) or away from the wall (case 2).



case 1: negative flow deflection φ
shock between region 3 and 4

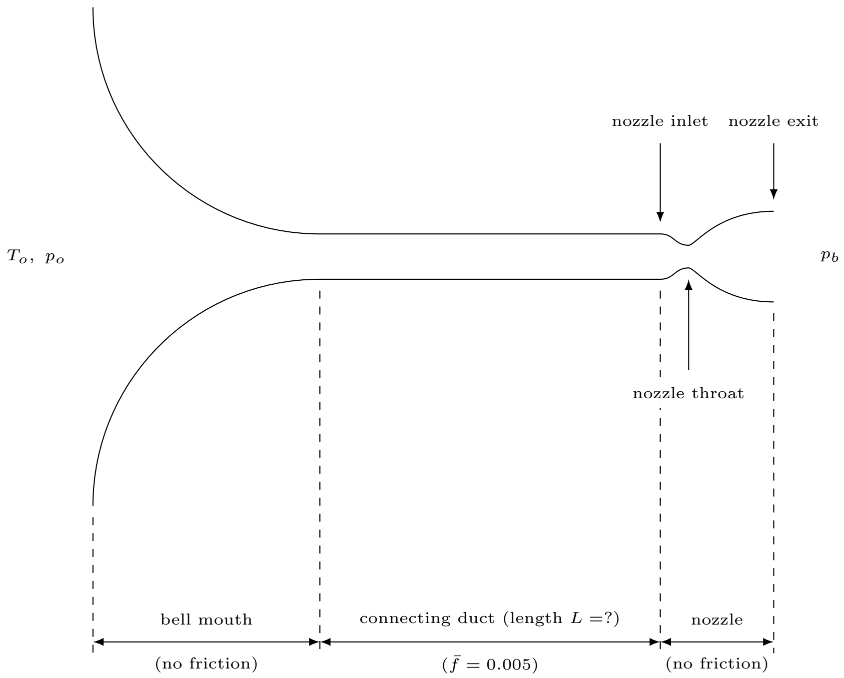


case 2: positive flow deflection φ
expansion between region 3 and 4

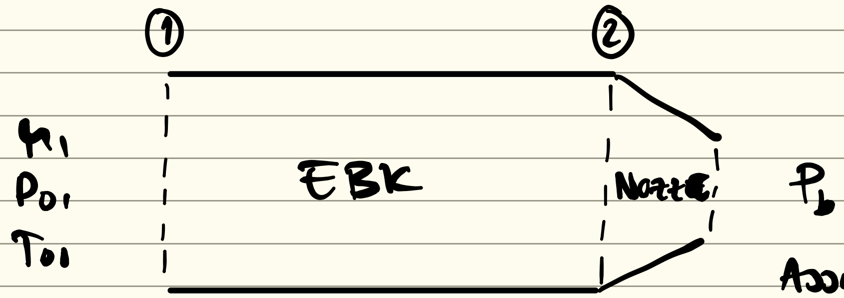
Calculate the flow-deflection angle φ (the angle between the slip line and the direction of the wall)

Problem 4 - WALL FRICTION(10 p.)

A convergent divergent nozzle is connected to a large reservoir via a duct for which the wall friction may be represented by an average friction factor of $\bar{f} = 0.005$. The nozzle inlet area area (and thus the duct area) is $A_i = 0.30\ m^2$, the nozzle throat area is $A_t = 0.25\ m^2$, and the nozzle exit area is $A_e = 0.50\ m^2$. The pressure and temperature in the upstream reservoir are $p_o = 120\ kPa$ and $T_o = 300\ K$, respectively. The pressure downstream of the nozzle is $p_b = 100\ kPa$. Find the length of the connecting duct that will lead to choked subsonic flow, i.e. the condition where sonic flow is reached at the nozzle throat and the flow is subsonic in the diverging part of the nozzle.



P1



ASSUME CALORICALLY PERFECT AIR ALTHOUGH IT IS NOT CORRECT...

$M_1 = 0.3$, $P_{01} = 500$ kPa , $T_{01} = 900$ K

AFTERBURNER ACTIVATED $\Rightarrow$ $T_{02} = 1900$ K

$\dot{m} = 90$ kg/s

$P_b = 100$ kPa

a) CALCULATE THE TOTAL-PRESSURE LOSS IN THE AFTERBURNER DUE TO THE HEAT ADDITION.

(3.77) $\quad q_{12} = c_p (T_{02} - T_{01})$

$$q^*_1 = c_p T_{01} \left(\frac{T_0^*}{T_{01}} - 1 \right)$$

(3.88)

$$\frac{T_{01}}{T_0^*} = \frac{(\gamma+1)M_1^2}{(1+\gamma M_1^2)^2} \left(2 + (\gamma-1)M_1^2\right)$$

$$q^*_2 = c_p T_{02} \left(\frac{T_0^*}{T_{02}} - 1 \right) = q^*_1 - q_{12}$$

(3.89)

$$\frac{T_{02}}{T_0^*} = \frac{(\gamma+1)M_2^2}{(1+\gamma M_2^2)^2}\left(2+(\gamma-1)M_2^2\right)$$

$$\Rightarrow \boxed{M_2 = 0.53}$$

(3.88)

$$\frac{P_{01}}{P_0^*} = \left(\frac{1+\gamma}{1+\gamma M_1^2}\right)\left(\frac{2+(\gamma-1)M_1^2}{(\gamma+1)}\right)^{\gamma/(\gamma-1)}$$

(3.30)

$$\frac{P_{01}}{P_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)}$$

(3.88)

$$\frac{P_{02}}{P_0^*} = \left(\frac{1+\gamma}{1+\gamma M_2^2}\right)\left(\frac{2+(\gamma-1)M_2^2}{(\gamma+1)}\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow \quad P_{01} - P_{02} = 40.176\,Pa$$

TOTAL PRESSURE LOSS:

$$\boxed{\frac{P_{01}-P_{02}}{P_{01}} \approx 8\%}$$

b) CALCULATE THE NOZZLE-EXIT AREA FOR ACTIVE AND INACTIVE AFTERBURNER ASSUMING SONIC FLOW AT THE EXIT IN BOTH CASES.

#1. INACTIVE AFTERBURNER:

SONIC FLOW @ EXIT $\Rightarrow$ CHOKED..

(5.21)

$$\dot{m} = \frac{P_{01} A^*}{\sqrt{T_{01}}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$\Rightarrow \boxed{A^* = A_e = 0.13\,m^2}$$

#2. ACTIVE AFTERBURNER:

(5.21)

$$\dot{m} = \frac{P_{02} A^*}{\sqrt{T_{02}}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$\Rightarrow \boxed{A^* = A_e = 0.21\,m^2}$$

c) CALCULATE THE ENGINE THRUST FOR BOTH CASES.

$M_e = 1.0$ IN BOTH CASES..

#1. INACTIVE AFTERBURNER.

$$T_{0e} = T_{01}$$
$$M_e = 1.0$$

(3.28)

$$\frac{T_{0e}}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 = \frac{T_{0e}}{T^*}$$

$$\Rightarrow T_e = T^* = T_{0e}\left(\frac{2}{\gamma+1}\right)$$

$$\Rightarrow T_e = 750K$$

$$u_e = M_e a_e = a_e = \sqrt{\gamma R T_e}$$
$$= 548.95 \ m/s$$

$$F = \underbrace{(P_b - P_e)}_{=0} A_e + \dot{m} u_e \Rightarrow$$

$$\boxed{F = \dot{m} u_e = 49.91 kN}$$

#2. ACTIVE AFTERBURNER

$$T_e = T^* = T_{0e}\left(\frac{2}{\gamma+1}\right)$$

WHERE $T_{0e} = T_{02}$

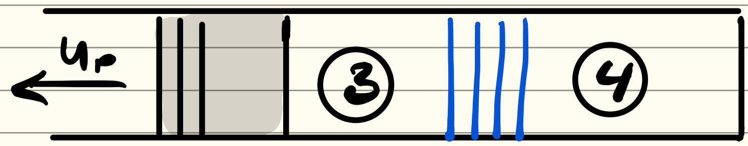
$$\Rightarrow T_e = 1583.33\ K$$

$$u_e = M_e a_e = a_e = \sqrt{\gamma R T_e}$$
$$= 797.61\ m/s$$

$$F = \underbrace{(p_b - p_e)}_{=0} A_e + \dot{m} u_e \Rightarrow$$

$$\boxed{F = \dot{m} u_e = 71.78\ kN}$$

P2



$T_4 = 300\,K$, $P_4 = 100\,kPa$, $u_4 = 0$

a) CALCULATE THE PISTON SPEED FOR WHICH THE TAIL OF THE EXPANSION STANDS STILL IN THE CYLINDER..

TAIL VELOCITY:

$$u_t = u_p - a_3 = u_p - \sqrt{\gamma R T_3}$$

(7.85)

$$\frac{T_3}{T_4} = \left(1 - \frac{\gamma - 1}{2}\left(\frac{u_3}{a_4}\right)\right)^2$$

$$\Rightarrow a_3 = \sqrt{\gamma R T_3} = \underbrace{\sqrt{\gamma R T_4}}_{a_4}\left(1 - \frac{\gamma-1}{2}\left(\frac{u_3}{a_4}\right)\right)$$

$$u_t = u_p - a_4\left(1 - \frac{\gamma - 1}{2}\left(\frac{u_3}{a_4}\right)\right)$$

$u_3 = u_p$ (THE FLOW IN REGION 3 MUST FOLLOW THE PISTON)

$$\Rightarrow \quad u_t = u_p - a_4 + \frac{\gamma - 1}{2} u_p$$

$$\Rightarrow \boxed{u_t = u_p \frac{\gamma+1}{2} - a_4}$$

$$u_t = 0 \Rightarrow u_p \frac{\gamma+1}{2} = a_4$$

$$\Rightarrow \boxed{u_p = \frac{2a_4}{\gamma+1}}$$

$$\Rightarrow \boxed{u_p = 289.3 \text{ m/s}}$$

b) CALCULATE THE PRESSURE AND TEMPERATURE JUST BEHIND THE PISTON (p_3 AND T_3)

(7.85)

$$\frac{T_3}{T_4} = \left(1 - \frac{\gamma-1}{2}\left(\frac{u_3}{a_4}\right)\right)^2$$

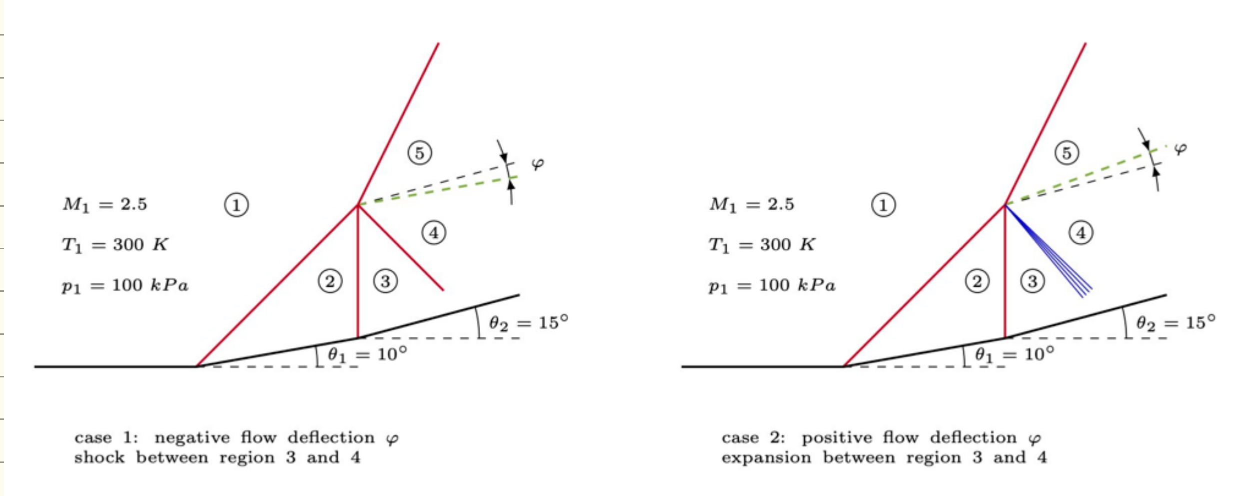
$$\Rightarrow \boxed{T_3 = 208.3 \text{ K}}$$

(7.86)

$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow \boxed{p_3 = 27.9 \text{ kPa}}$$

P_3



$1 \rightarrow 2$

$$\left.\begin{array}{l} M_1 = 2.5 \\ \theta_1 = 10° \end{array}\right\} \theta - \beta - M \Rightarrow \beta_2 = 31.85°$$

(4.7)

$$M_{n2_1} = M_1 \sin \beta_2$$

(4.9)

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n2_1}^2 - 1\right)$$

(4, 10)

$$M_{n_{22}}^2 = \frac{M_{n2_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_{21}}^2 - 1}$$

(4.12)

$$M_2 = \frac{M_{n22}}{\sin(\beta_2 - \theta_1)}$$

$$\Rightarrow \boxed{\frac{p_2}{p_1} = 1.86\ ,\quad M_2 = 2.09}$$

$2 \rightarrow 3$

$$\left.\begin{aligned} M_2 &= 2.09 \\ \theta = \theta_2 - \theta_1 &= 5° \end{aligned}\right\} \Rightarrow \beta_3 = 32.8°$$

(4.7)

$$M_{n_{31}} = M_2 \sin \beta_3$$

(4.9)

$$\frac{p_3}{p_2} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{n_{31}}^2 - 1\right)$$

(4.10)

$$M_{n_{32}}^2 = \frac{M_{n_{31}}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_{31}}^2 - 1}$$

(4.12)

$$M_3 = \frac{M_{n_{32}}}{\sin(\beta_3 - \theta_2)}$$

$$\Rightarrow \boxed{M_3 = 1.90 \quad , \quad \frac{p_3}{p_2} = 1.33}$$

FIND SLIPLINE ANGLE :

φ DEFINED AS THE DEVIATION FROM THE 15° FLOW DIRECTION (POSITIVE UPWARD..)

1. GUESS A VALUE OF φ

2.

$\underline{\varphi < 0}$:

SHOCK BETWEEN REGIONS 3 AND 4

$$\left.\begin{matrix} M_3 \\ \varphi \end{matrix}\right\} \theta-\beta-M \Rightarrow \beta_4$$

(4.7) $M_{n_{4,1}} = M_3 \sin(\beta_4)$

(4.9) $\dfrac{p_4}{p_3} = 1 + \dfrac{2\gamma}{\gamma+1}\left(M_{n_{4,1}}^2 - 1\right)$

$\underline{\varphi > 0}$:

EXPANSION BETWEEN REGIONS 3 AND 4

(4.44) $\Rightarrow \nu(M_3)$

$\nu(M_4) = \varphi + \nu(M_3)$

$\Rightarrow M_4$

$$\frac{p_3}{p_4} = \left(\frac{1 + \frac{\gamma-1}{2} M_4^2}{1 + \frac{\gamma-1}{2} M_3^2}\right)^{\gamma/(\gamma-1)}$$

3. 1 → 5

$$\left.\begin{matrix} \theta = \theta_2 + \varphi \\ M_1 = 2.5 \end{matrix}\right\} \theta-\beta-M \Rightarrow \beta_5$$

(4.7)

$$M_{n_{s1}} = M_1 \sin\beta_s$$

(4.9)

$$\frac{p_s}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_{s1}}^2 - 1\right)$$

4.

$$\text{if}\left(\left|\frac{p_4}{p_3}\frac{p_3}{p_1} - \frac{p_s}{p_1}\right| > \text{TOLERANCE}\right)$$

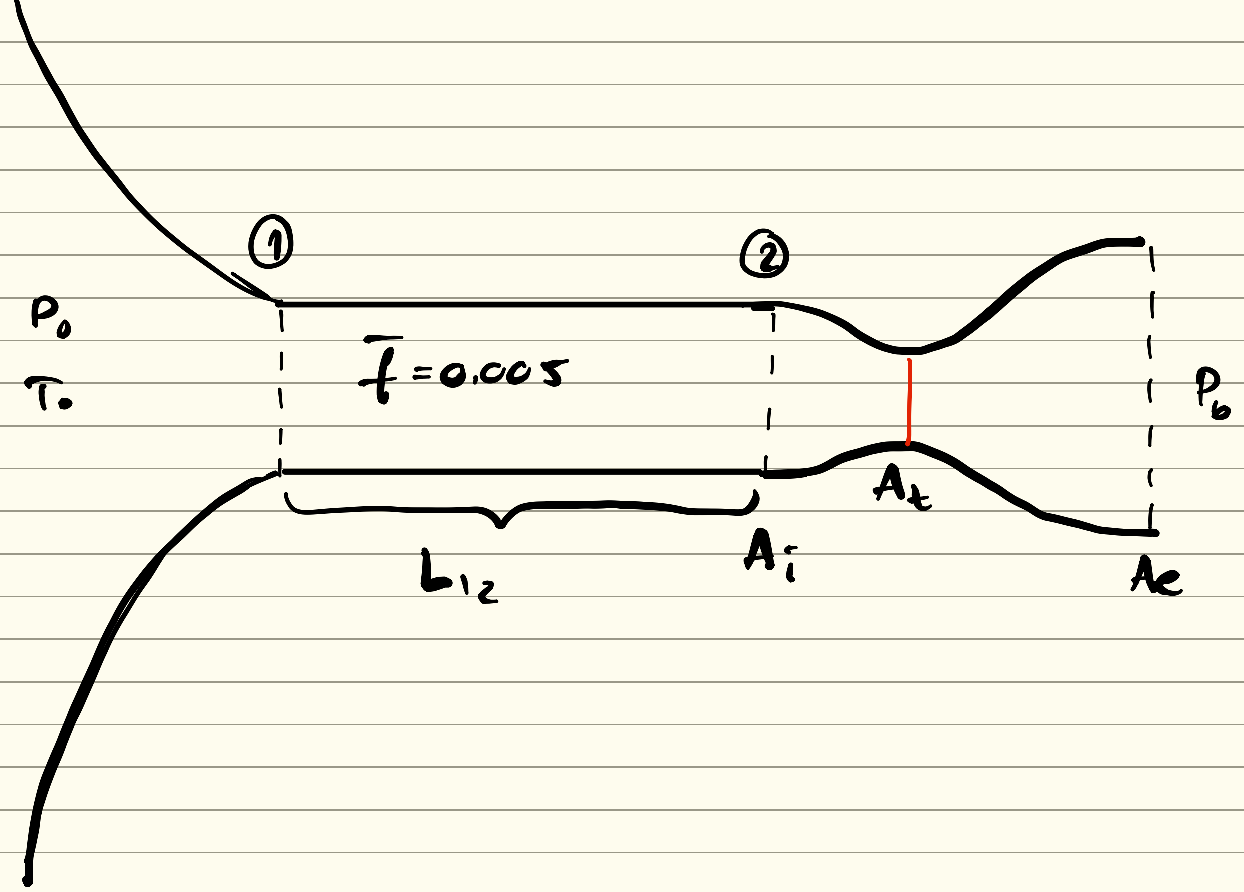
UPDATE φ AND REPEAT #2 → #4.

else

DONE!

$$\Rightarrow \boxed{\varphi = 0.01^\circ}$$

P4



FIND THE PIPE LENGTH RESULTING IN CRITICAL FLOW (SONIC FLOW @ THROAT, SUBSONIC FLOW @ EXIT)

$A_i = 0.30\,m^2$, $A_t = 0.25\,m^2$, $A_e = 0.50\,m^2$

$P_0 = 120\ kPa$

$T_0 = 300\ K$

$P_b = 100\ kPa$

(5.20)

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUBSONIC SOLUTION $\Rightarrow M_e = 0.31$

$$\frac{P_{0e}}{P_e} = \left(1 - \frac{\gamma-1}{2}M_e^2\right)^{\gamma/(\gamma-1)}$$

$$p_e = p_0 \Rightarrow \boxed{p_{0e} = 106.7\ kPa}$$

(5.20)

$$\left(\frac{A_i}{A_t}\right)^2 = \frac{1}{M_i^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_i^2\right)\right)^{\frac{\gamma+1}{\gamma-1}}$$

$$\Rightarrow \boxed{M_i = 0.59}$$

(3.107)

$$\frac{4\bar{f}L_2^*}{D_2} = \frac{1-M_2^2}{\gamma M_2^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2}\right)$$

$$\Rightarrow \{M_2 = M_i\} \Rightarrow \boxed{L_2^* = 16.9\ m}$$

(3.106)

$$\frac{p_{02}}{p_0^*} = \frac{1}{M_2}\left(\frac{2+(\gamma-1)M_2^2}{\gamma+1}\right)^{(\gamma+1)/(2(\gamma-1))}$$

$$\Rightarrow \boxed{p_0^* = 88.9\ kPa}$$

$$\frac{p_{01}}{p_0^*} = \frac{1}{M_1}\left(\frac{2+(\gamma-1)M_1^2}{\gamma+1}\right)^{(\gamma+1)/(2(\gamma-1))}$$

$$\Rightarrow \boxed{M_1 = 0.50}$$

(3.107)

$$\frac{4\bar{f}L_1^*}{D_1} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

$$\Rightarrow \boxed{L_1^* = 34.32\ m}$$

$$\Rightarrow \boxed{L_{12} = L_1^* - L_2^* = 17.92\ m}$$