TME085 - Compressible Flow

2024-03-14, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Responsible teacher: Niklas Andersson tel.: 070 - 51 38 311

Results available no later than 2024-04-04

Good luck!

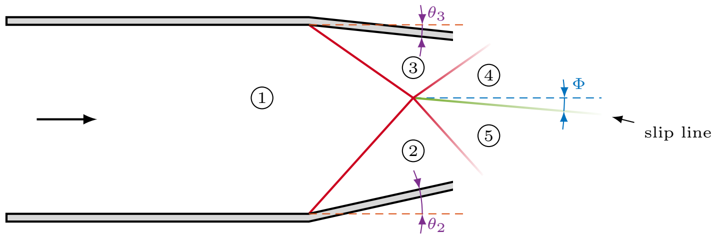
Part I - Theory Questions (20 p.)

T1. (2.0 p.) Shocks:

(a) (0.5 p.) The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the **weak** type or the **strong** type. What is the main difference between these two shock types and which type is usually seen in reality?

(b) (0.5 p.) Are the normal-shock relations mathematically and physically valid for upstream Mach numbers lower than one? Justify your answer.

(c) (1.0 p.) What are the constraints that leads to the generation of the separating line between regions 4 and 5 (represented by a green line in the figure below)? What is the reason for the need for this separating line?



T2. (3.0 p.) Governing equations:

(a) (1.0 p.) What is the physical interpretation of each of the terms in the **momentum equation** on integral form

$$\frac{d}{dt}\iiint_{\Omega} \rho \mathbf{v} d\mathscr{V} + \oiint_{\partial\Omega} [\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}]\, dS = \iiint_{\Omega} \rho \mathbf{f} d\mathscr{V}$$

(b) (1.0 p.) How can the **substantial derivative** operator be interpreted physically?

(c) (1.0 p.) Crocco's theorem says that the flow behind a curved shock must be rotational. Explain the root cause behind this phenomena.

T3. (3.0 p.) Gas models

(a) (1.0 p.) What do we mean by **thermally perfect gas** and **calorically perfect gas**, respectively. What is the main difference between these gas models and for which temperature ranges do they apply?

(b) (1.0 p.) Explain what the **Boltzmann distribution** describes and what sparsely populated implies.

(c) (1.0 p.) Using **equilibrium gas** assumption in the analysis of chemically reacting nozzle flow will lead to higher exhaust temperatures than if calorically perfect gas assumption is used for the same analysis. Explain why.

T4. (4.0 p.) Compressible CFD:

(a) (0.5 p.) What is meant by the term **density-base** when discussing CFD codes for compressible flow?

(b) (0.5 p.) When the governing equations are discretized using a **finite-volume** approach, cell face values of flow properties appears in the equations. How are these values approximated?

(c) (1.0 p.) What do we mean when we say that a CFD code for compressible flow is **conservative**? Why is it important that the numerical scheme is conservative?

(d) (1.0 p.) When applying a **time-marching** flow solution scheme the so-called **CFL number** is an important parameter. Define the **CFL number** and describe its significance.

(e) (1.0 p.) How can we use our knowledge of **characteristics** (and their speed of propagation) to guide us when determining suitable **boundary conditions** for compressible flows?

T5. (2.0 p.) Heat addition:

(a) (1.0 p.) What happens in the flow when heat is added if the flow is initially supersonic and subsonic, respectively

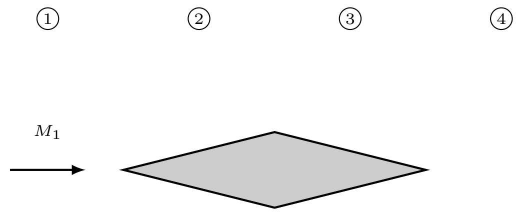
(b) (1.0 p.) q^* defines the heat addition needed to obtain thermal choking. What is thermal choking? What happens if the amount of heat added exceeds q^* for a supersonic and subsonic flow, respectively?

T6. (2.0 p.) For a steady-state **isentropic** flow of a **calorically perfect** gas, derive the formula for T_0/T, making use of the fact that the total enthalpy h_0 is constant along the streamlines.

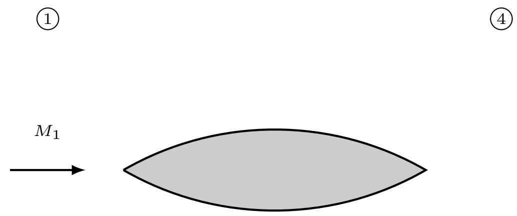
$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

T7. (2.0 p.) Derive the **isentropic relations** for **calorically perfect** gases starting from the **entropy equation**.

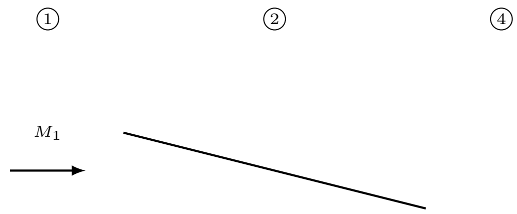
T8. (2.0 p.) For each of the cases a-c below, make a schematic representation showing how the pressure varies from station 1 to 4 (following the flow over the upper side of each "wing"). Any sudden changes in pressure has to be motivated.



(a)
symmetric diamond-wedge airfoil (zero angle of attack)



(b)
double circular arc airfoil (zero angle of attack)



(c)
flat plate at an angle of attack

Part II - Problems (40 p.)

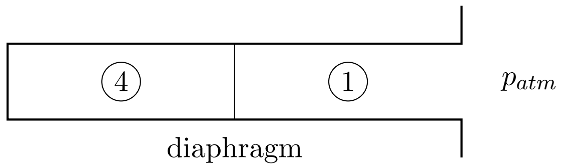
Problem 1 - SHOCK TUBE (10 p.)

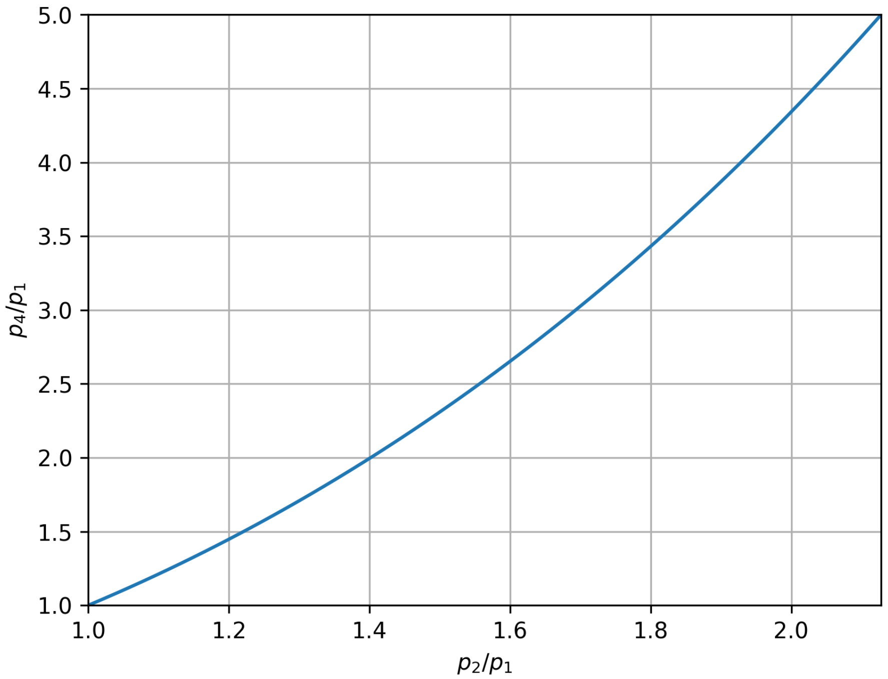
A shock tube is designed as schematically shown in the figure below. Before the shock tube is started up (before removing the diaphragm separating the driver section and the driven section), the pressure in the driver section is 2.5 times the pressure in the driven section and the temperature is the same in the driver section and the driven section. The right end of the driven section is open to the surroundings and thus the pressure and temperature in the driven section is the same as the ambient pressure ($p_{amb} = 101325 Pa$) and temperature ($T_{amb} = 293.0 K$).

(a) Calculate the Mach number of the incident shock wave (the shock wave generated as the diaphragm separating the driver section and driven section is removed) and the induced flow velocity behind the moving shock wave.

A graphical representation of the shock tube relation between the overall pressure ratio (p_4/p_1) and the incident shock pressure ratio (p_2/p_1) for the the specified temperature and gas is given below

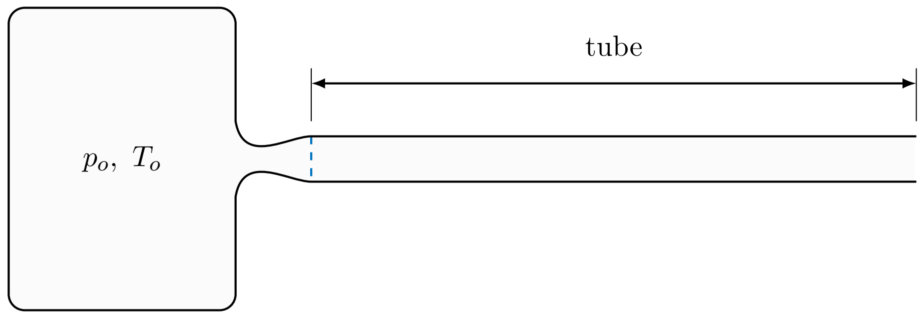
(b) Calculate the pressure, temperature, and flow velocity at the right end of the tube after the event taking place when the shock reaches the right end of the tube.





Problem 2 - FRICTION (10 p.)

Air flows through a nozzle with the throat diameter $d_t = 0.25\ m$ and exit diameter $d_e = 0.3\ m$. The nozzle is operated at its design condition, which implies that flow at the nozzle exit plane is supersonic and that there are no shocks or expansions downstream of the nozzle exit. A tube is mounted downstream of the nozzle exit. The tube diameter is the same as the nozzle exit diameter and the average friction factor for the tube surface is $\bar{f} = 0.005$. The total temperature and total pressure upstream of the nozzle inlet are $T_o = 300.0\ K$ and $p_o = 200.0 kPa$, respectively.



Calculate:

(a) The maximum tube length if no shocks should be generated inside of the tube

(b) The maximum tube length if no shocks should be generated inside of the nozzle

(c) The flow conditions (pressure, temperature, and flow velocity) at the end of the tube for both cases above

Problem 3 - NOZZLE FLOW (10 p.)

Air from a large reservoir (gas container) is expanded through a convergent-divergent nozzle. The gas temperature and pressure inside the reservoir are $p_o = 600.0\ kPa$ and $T_o = 40.0°\ C$, respectively. When the flow is started up, the nozzle pressure ratio (NPR) is increased continuously by reducing the back pressure (the pressure downstream of the nozzle exit). Among the parameters displayed on the monitors that the lab engineers use to control the nozzle startup there is a plot showing the nozzle massflow as a function of nozzle pressure ratio (NPR). When the back pressure reaches $p_b = 524.85\ kPa$, the massflow curve gets flat (the massflow does not change when the back pressure is reduced further).

(a) What is the design back pressure for the nozzle?

(b) Find the range of back pressures for which oblique shocks will form at the nozzle exit plane

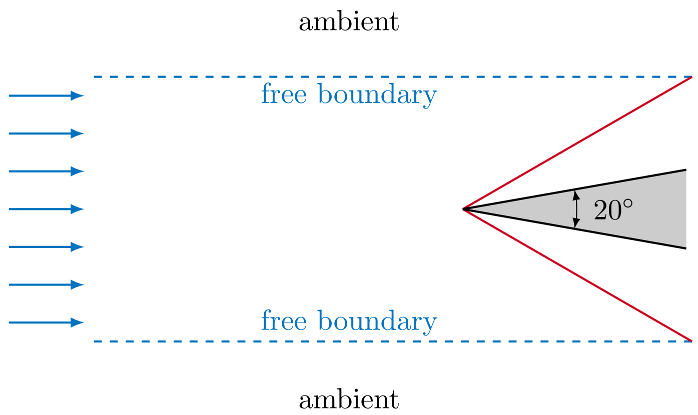
(c) Find the range of back pressures for which shocks will be generated in the divergent part of the nozzle

Problem 4 - WEDGE FLOW (10 p.)

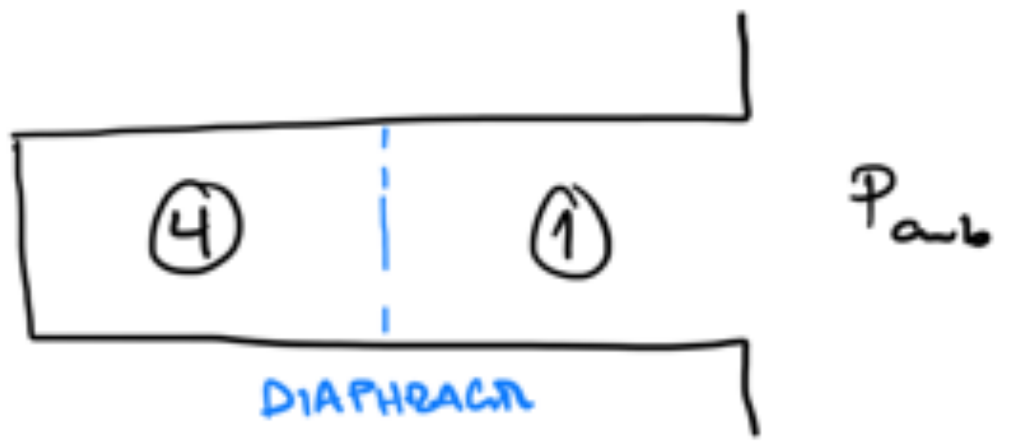
A 20°-degree wedge is placed at the center of a supersonic planar jet. The ambient air outside of the jet does not move, which leads to the formation of a free boundary (shear layer) between the high-speed jet flow and the ambient air. Since the jet flow is supersonic, an oblique shock will be generated at the tip of the wedge. The jet Mach number is $M_j = 2.5$. The pressure and temperature in the jet upstream of the wedge are $p_j = 100.0\ kPa$ and $T_j = 300.0\ K$. The ambient pressure is the same as the jet pressure $p_{amb} = 100.0\ kPa$.

The flow can be assumed to be two-dimensional (*the extent of the planar jet and the wedge in the direction normal to the sketch is enough such that three-dimensional effects can be discarded*)

Calculate the Mach number, temperature, and pressure at a location in the flow downstream of the event taking place when the oblique shock interacts with the free boundary.



P_1 (SHOCK TUBE)



GIVEN: SHOCK TUBE WITH OPEN RIGHT END

$P_1 = P_{amb}$
$T_1 = T_{amb}$

$P_4 / P_1 = 2.5$

ASSUME THAT THE GAS IN THE DRIVEN SECTION IS AIR (ALSO OUTSIDE OF THE TUBE ...)

THE GASES ARE ASSUMED TO BE CALORICALLY PERFECT.

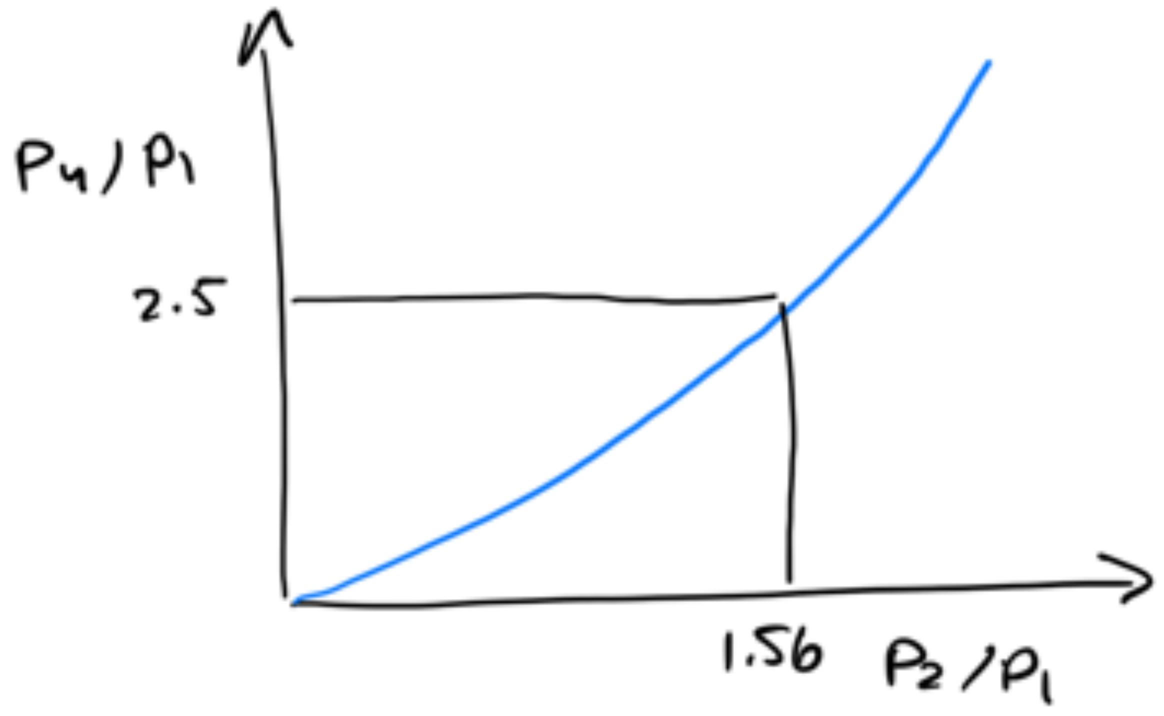
WHEN THE DIAPHRAGM IS REMOVED, A SHOCK TRAVELING TO THE RIGHT (INTO REGION 1) WILL BE GENERATED.

CALCULATE THE MACH NUMBER OF THE INCIDENT SHOCK AND THE VELOCITY OF THE INDUCED FLOW BEHIND THE SHOCK.

(7.13) $$M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right) + 1}$$

WE NEED P_2/P_1 (THE SHOCK PRESSURE RATIO)

WE CAN USE THE PROVIDED GRAPH, WHICH IS BASED ON (7.94) OR SOLVE (7.94)..



$P_2/P_1 = 1.56 \Rightarrow M_s = 1.22$

(7.16) $$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1}-1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

WHERE $a_1 = \sqrt{\gamma R T_1}$

$\Rightarrow u_p = 112.3$ m/s

(7.10) $$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left[\frac{\frac{\gamma+1}{\gamma-1}+\frac{p_2}{p_1}}{1+\frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}\right]$$

$\Rightarrow T_2 = 333.3$ K (WE WILL NEED THAT LATER)

b) CALCULATE THE PRESSURE, TEMPERATURE, AND FLOW VELOCITY AT THE RIGHT END OF THE TUBE AFTER THE EVENT TAKING PLACE WHEN THE SHOCK REACHES THE RIGHT END.

WHAT WILL HAPPEN WHEN THE SHOCK REACHES THE RIGHT END?

SINCE THE FLOW BEHIND THE SHOCK IS SUBSONIC

($M_2 = u_p / a_2 = u_p/\sqrt{\gamma R T_2} = 0.31$)

THE PRESSURE MUST MATCH THE AMBIENT PRESSURE. THEREFORE AN EXPANSION WILL BE GENERATED TRAVELING BACK INTO THE TUBE. THE PRESSURE RATIO OVER THE EXPANSION WILL BE THE SAME AS FOR THE INCIDENT SHOCK.

SINCE THE EXPANSION TRAVELS INTO A REGION WITH A CONSTANT FLOW STATE, THE J^+ INVARIANT WILL BE CONSTANT OVER THE EXPANSION.

$$J^+ = u_p + \frac{2a_2}{\gamma-1} = u_e + \frac{2a_e}{\gamma-1} \quad (1)$$

WHERE $a_2 = \sqrt{\gamma R T_2}$

BOTH u_e AND a_e ARE UNKNOWN

WE CAN USE THE ISENTROPIC RELATION OVER THE EXPANSION.

$$\left(\frac{P_e}{P_2}\right) = \left(\frac{P_1}{P_2}\right) = \left(\frac{T_e}{T_2}\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow T_e = T_2\left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma} = 293.7\text{ K}$$

$$a_e = \sqrt{\gamma R T_e} \quad (2)$$

(2) IN (1) $\Rightarrow u_e = 224.4$ m/s

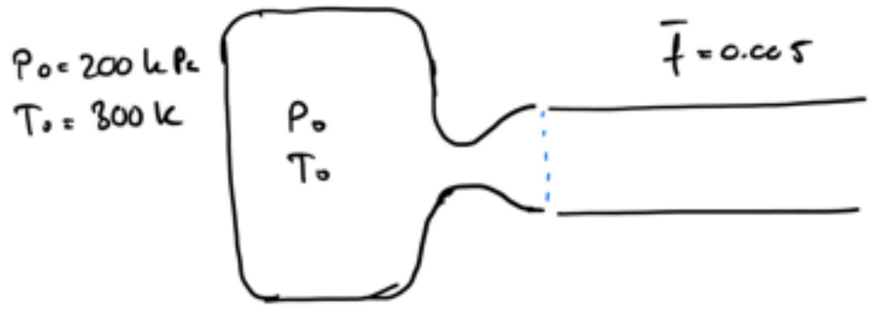
FLOW CONDITIONS AT THE RIGHT END OF THE TUBE:

$P_e = P_{amb}$

$T_e = 293.7$ K

$u_e = 224.4$ m/s

P2 (FRICTION)



$d_t = 0.25\,m$

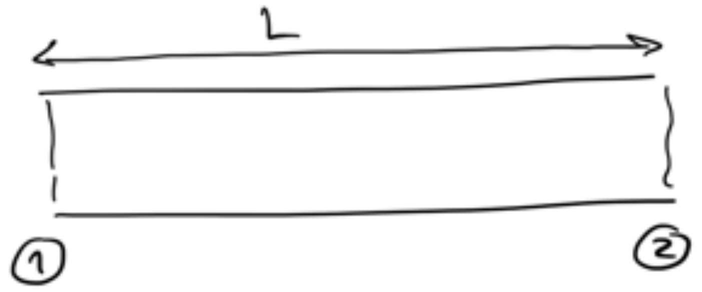
$d_e = 0.3\,m$

THE NOZZLE IS OPERATED AT DESIGN CONDITION (SUPERCRITICAL FLOW)

A PIPE IS ADDED AT THE NOZZLE EXIT

a) CALCULATE THE MAXIMUM TUBE LENGTH IF THERE SHOULD NOT BE ANY SHOCKS IN THE TUBE.

IF THE PIPE LENGTH IS LONGER THAN THE LENGTH THAT CHOKES THE FLOW THERE WILL BE A SHOCK GENERATED IN THE TUBE (OR UPSTREAM OF THE PIPE)



$M_1 = M_{e\ supercritical}$

$M_2 = 1.0$ (choked)

$L^*_1 = L$

$L^*_2 = 0.$

WE MUST FIRST CALCULATE THE NOZZLE EXIT MACH NUMBER.

$$\frac{A_e}{A_t} = \left(\frac{d_e}{d_t}\right)^2 = 1.44$$

(3.20)

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUPERSONIC SOLUTION $\Rightarrow M_{e_{sc}} = 1.8$

(3.107)

$$\frac{4\bar{f}L^*_1}{d_e} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

$\Rightarrow L^*_1 = 3.63\ m$

So, $L = L^*_1 = 3.63\,m$ IS THE LONGEST PIPE POSSIBLE WITHOUT GENERATING A SHOCK IN THE PIPE..

b) CALCULATE THE LONGEST PIPE LENGTH WITHOUT GENERATING A SHOCK INSIDE THE NOZZLE.

IF WE EXTEND THE PIPE MORE THAN THE LENGTH CALCULATED IN THE PREVIOUS TASK THERE WILL BE A SHOCK GENERATED IN THE PIPE. THE LONGER THE PIPE, THE FURTHER UPSTREAM THE SHOCK WILL BE LOCATED AND EVENTUALLY IT WILL MOVE ALL THE WAY INTO THE NOZZLE. THE LONGEST PIPE LENGTH IS THUS THE LENGTH FOR WHICH THE SHOCK WILL STAND AT THE NOZZLE EXIT

A NORMAL SHOCK GENERATED AT THE MACH NUMBER CALCULATED IN THE PREVIOUS TASK GIVES THE NEW INLET MACH NUMBER

(3.51)

$$M_1^2 = \frac{1+((\gamma-1)/2)M_e^2}{\gamma M_e^2-(\gamma-1)/2} \Rightarrow M_1 = 0.62$$

(3.107)

$$\frac{4\bar{f}L^*_1}{d_1} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

$\Rightarrow L^*_1 = 6.45\,m$

c) CALCULATE THE EXIT CONDITIONS FOR THE BOTH CASES ABOVE (T_2, P_2, u_2)

THE STARRED QUANTITIES ARE NOT AFFECTED BY THE SHOCK $\Rightarrow$ $T_2 = T^*$, $P_2 = P^*$, $u_2 = a^*$ WILL BE THE SAME IN BOTH CASES.

WE CAN THUS DO THE CALCULATIONS FOR ONE OF THE CASES.

(a)

(3.28) $$\frac{T_0}{T_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)$$

(3.30) $$\frac{P_0}{P_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)}$$

(3.103) $$\frac{T_1}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_1^2}$$

(3.104) $$\frac{P_1}{P^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2}$$

$T_1 = 278.82\ K$

$P_1 = 125.73\ kPa$

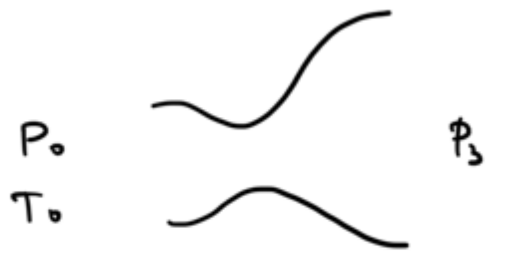
$T^* = 250.0\,K$ $(=T_2)$

$P^* = 73.37\ kPa$ $(=P_2)$

$u_2 = a^* = \sqrt{\gamma R T^*} = 316.9\ m/s$

(you will get the same result if you do the calculations for the other case.)

P3 (NOZZLE FLOW)



$P_o = 600.0\ kPa$

$T_o = 40°C = 313K$

THE NOZZLE PRESSURE RATIO (NPR) IS INCREASED CONTINOUSLY BY REDUCING THE NOZZLE BACK PRESSURE (P_b)

THE MASSFLOW THROUGH THE NOZZLE REACHED A MAXIMUM WHEN THE BACK PRESSURE IS $P_b = 524.85\ kPa$.

$P_b = 524.85\ kPa$ ⇒ CHOKED SUBSONIC FLOW.

CHOKED SUBSONIC FLOW ⇒ ISENTROPIC

(3.30) $$\frac{P_o}{P_{b\,choked}} = \left(1 + \frac{\gamma-1}{2} M_e\right)^{\gamma/(\gamma-1)}$$

$\Rightarrow M_{e\,choked} = 0.44$

(5.20) $$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_e^2\right)\right)^{\frac{\gamma+1}{\gamma-1}}$$

$\Rightarrow A_e/A_t = 1.47$

a) WHAT IS THE DESIGN BACK PRESSURE?

DESIGN CONDITIONS: ISENTROPIC, SUPERSONIC ⇒ SUPERCRITICAL.

(5.20) $$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_e^2\right)\right)^{\frac{\gamma+1}{\gamma-1}}$$

(SUPERSONIC SOLUTION)

$M_{e_{sc}} = 1.83$

(3.30) $$\frac{P_o}{P_{e_{sc}}} = \left(1 + \frac{\gamma-1}{2} M_{e_{sc}}^2\right)^{\gamma/(\gamma-1)} = 6.0$$

$\underline{P_b = 100.0\ kPa}$

b) FIND THE BACK PRESSURE RANGE FOR WHICH OBLIQUE SHOCKS WILL FORM DOWNSTREAM OF THE NOZZLE EXIT.

OVEREXPANDED FLOW:

BETWEEN NORMAL SHOCK AT EXIT AND SUPERCRITICAL (DESIGN)

WE NEED THE BACK PRESSURE CORRESPONDING TO NORMAL SHOCK AT EXIT.

(3.57) $$\frac{P_{b\,nse}}{P_{b\,sc}} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{sc}^2 - 1\right)$$

$\Rightarrow P_{b\,nse} = 373.3\ kPa$

OVEREXPANDED FLOW (OBLIQUE SHOCKS DOWNSTREAM OF NOZZLE EXIT)

$P_{b\,nse} > P_b > P_{b\,sc}$

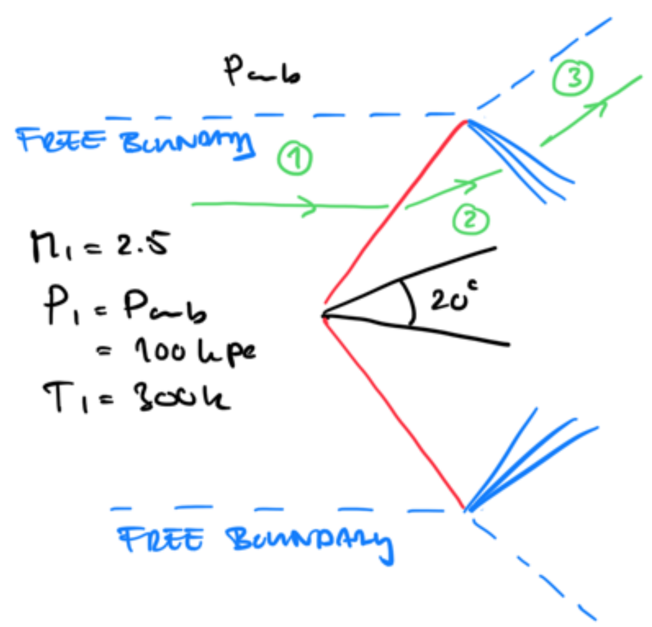
$\underline{373.3\ kPa > P_b > 100.0\ kPa}$

c) FIND THE RANGE OF BACK PRESSURES FOR WHICH NORMAL SHOCKS WILL APPEAR INSIDE THE NOZZLE.

$P_{b\,choked} > P_b > P_{b\,nse}$

$\underline{524.81\ kPa > P_b > 373.3\ kPa}$

P4 (WEDGE FLOW)



A WEDGE IS PLACED INSIDE A SUPERSONIC PLANAR JET. AN OBLIQUE SHOCK IS FORMED AT THE LEADING EDGE OF THE WEDGE TO TURN THE FLOW SUCH THAT IT FOLLOWS THE WEDGE SURFACE.

WHEN THE SHOCK REACHES THE FREE BOUNDARY (THE INTERFACE BETWEEN THE SUPERSONIC JET AND THE SURROUNDING AIR), IT WILL BE REFLECTED AS AN EXPANSION THAT REDUCES THE PRESSURE SUCH THAT $P_3 = P_{amb}$.

CALCULATE: M_3, T_3, P_3

1 → 2 (OBLIQUE SHOCK)

($\theta - \beta - M$: $\theta = 10°$, $M_1 = 2.5$) $\Rightarrow \beta = 31.9°$

(4.7) $M_{n_1} = M_1 \sin\beta$

(4.9) $$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)$$

(4.10) $$M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$$

(4.11) $$\frac{T_2}{T_1} = \frac{P_2}{P_1}\left(\frac{(\gamma-1)M_{n_1}^2 + 2}{(\gamma+1)M_{n_1}^2}\right)$$

(4.12) $$M_2 = \frac{M_{n_2}}{\sin(\beta-\theta)}$$

$\Rightarrow M_2 = 2.09$

$P_2 = 186.4$ kPa

$T_2 = 360.9$ K

FOR THE NEXT STEP, WE WILL NEED TOTAL TEMPERATURE AND TOTAL PRESSURE

(3.30) $$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}$$

$\Rightarrow P_{02} = 1.67$ MPa

(3.28) $$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2}M_2^2$$

$\Rightarrow T_{02} = 675.0$ K

2 → 3 (EXPANSION)

$P_3 = P_1$ (PRESSURE SHOULD MATCH THE AMBIENT PRESSURE AFTER EXPANSION)

$$\frac{P_2}{P_3} = \left(\frac{1 + \frac{\gamma-1}{2}M_3^2}{1 + \frac{\gamma-1}{2}M_2^2}\right)^{\gamma/(\gamma-1)}$$

$\Rightarrow M_3 = 2.48$

$$\frac{T_2}{T_3} = \left(\frac{1 + \frac{(\gamma-1)}{2}M_3^2}{1 + \frac{(\gamma-1)}{2}M_2^2}\right)$$

$\Rightarrow T_3 = 302.1$ K

$M_2 = 2.1$	$M_3 = 2.5$
$T_2 = 360.9$ K	$T_3 = 302.1$ K
$P_2 = 186.4$ kPa	$P_3 = 100.0$ kPa