TME085 - Compressible Flow

2023-08-16, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Responsible teacher: Niklas Andersson tel.: 070-51 38 311

Solutions for the problems will be published in Canvas after the exam

Results available no later than 2023-09-06

Good luck!

Part I - Theory Questions (20 p.)

T1. (1 p.) What are the criteria for an **isentropic** process, i.e. what conditions must be satisfied for a **steady-state** compressible flow to be **isentropic**?

T2. (2 p.) Derive the **isentropic relations** for **calorically perfect** gases starting from the **entropy equation**.

T3. (1 p.) What is the physical interpretation of each of the terms in the **momentum equation** on integral form

$$\frac{d}{dt}\iiint\limits_{\Omega} \rho\mathbf{v}d\mathscr{V} + \oint\!\!\!\oint\limits_{\partial\Omega} [\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}]\,dS = \iiint\limits_{\Omega} \rho\mathbf{f}d\mathscr{V}$$

T4. (1 p.) How do we define **total conditions** in a steady-state isentropic flow?

T5. (2 p.) The normal shock relations actually allow two solutions, one that corresponds to a discontinuous compression (a sudden pressure increase) and one that corresponds to a discontinuous expansion (a sudden pressure decrease). However, only one of these solutions is physically valid. What thermodynamic principle guides us in the choice of the physically correct solution, and which solution is the correct one?

T6. (3 p.) One-dimensional flow with heat addition

(a) (2 p.) Looking at the **Rayleigh** curve it's evident that removing heat leads to reduced entropy - how come that this is possible?

(b) (1 p.) What happens in the flow when heat is added if the flow is initially supersonic and subsonic, respectively

T7. (1 p.) How does the absolute Mach number change after a **weak/strong** stationary oblique shock?

T8. (2 p.) For a **detached shock**, indicate where you will find a normal shock, a **strong** oblique shock, a **weak** oblique shock, the **sonic line**.

T9. (2 p.) Can a moving normal shock travel at a speed lower than the speed of sound? Explain why/why not.

T10. (2 p.) Time-marching technique

(a) (1 p.) When applying a **time-marching** flow solution scheme the so-called **CFL number** is an important parameter. Define the **CFL number** and describe its significance.

(b) (1 p.) How can we use our knowledge of **characteristics** (and their speed of propagation) to guide us when determining suitable **boundary conditions** for compressible flows?

T11. (3 p.) Gas models

(a) (1 p.) A mixture of chemically reacting perfect gases, where the reactions are always in **equilibrium**, may be thermodynamically described as a single-species gas. How does this thermodynamic description differ from that of a calorically perfect or thermally perfect gas?

(b) (1 p.) In what temperature range (approximately) can a gas be assumed to be **calorically perfect**?

(c) (1 p.) Explain the concept **energy state**.

Part II - Problems (40 p.)

Problem 1 - MOVING SHOCK WAVE (10 p.)

A shock moves at a velocity of 400.0 m/s (relative to a stationary observer) into stagnant air (i.e. the fluid ahead of the moving shock is standing still in relation to a stationary observer). The temperature and pressure in the air ahead of the moving shock is 101.325 kPa and 300.0 K, respectively. If a Prandtl tube (a device that measures the difference between total and static pressure ($p_o - p$)) is placed in the fluid behind the moving shock, what will the resulting measured pressure difference be?

Problem 2 - FREE-BOUNDARY REFLECTION (10 p.)

An oblique shock formed in a Mach 2.5 air flow deflects the flow an angle of 10.0°. The shock interacts with e free boundary[1] which leads to a "reflection" in the form of an expansion region.

(a) Why is the oblique shock "reflected" as an expansion region when interacting with the free boundary? (What physical constraint is the reason for this behavior?)

(b) Calculate the flow Mach number and flow angle downstream of the expansion region (the flow can be assumed to be axial upstream of the oblique shock)

[1] An example of a free boundary is the shear layer that builds in the jet flow downstream of a nozzle as the region where the high-speed jet flow and the slower surrounding fluid meets

Problem 3 - ONE-DIMENSIONAL FLOW WITH FRICTION (10 p.)

Air flows steadily from a large reservoir through a convergent-divergent nozzle into a 0.3 m diameter pipe with a length of 3.5 m. The conditions in the reservoir are such that the Mach number and pressure at the inlet of the pipe are 2.0 and 101.3 kPa, respectively. The average friction factor ($\bar{f}$) for the flow in the pipe is estimated to be 0.005.

(a) If no shocks occur, calculate the mach number and pressure at the pipe exit

(b) Calculate the pressure downstream of the pipe (the back pressure) if there is a normal shock at the pipe exit

(c) Calculate the Mach number and pressure at the pipe exit if there is a shock halfway down the pipe

Problem 4 - NOZZLE FLOW (10 p.)

Air is expanded through a convergent-divergent nozzle from a large reservoir in which the pressure and temperature are 600.0 kPa and 40.0°C, respectively. The design back pressure for the nozzle is 100.0 kPa.

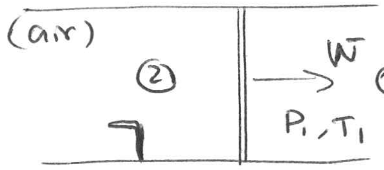
(a) Calculate A_e/A^* (the exit to throat area ratio)

(b) Calculate the exit velocity at design conditions

(c) For what range of back pressures will the nozzle be overexpanded?

THE085 EXAM 23-08-16

P1 (MOVING SHOCK WAVE)



$W = 400$ m/s
$P_1 = 101.325$ kPa
$T_1 = 300.0$ K

THE ~~PITOT~~ Prandl-TUBE IS PLACED IN THE INDUCED FLOW BEHIND THE MOVING SHOCK

$$M_s = \frac{W}{a_1} \quad \text{WHERE } a_1 = \sqrt{\gamma R T_1} \quad \Rightarrow M_s = 1.15$$

(7.13) $$M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \quad \Rightarrow \quad \frac{p_2}{p_1} = 1.38$$

(7.16) $$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2} \Rightarrow u_p = 82.2 \text{ m/s}$$

$$M_2 = u_2/a_2 = \frac{u_p}{\sqrt{\gamma R T_2}}$$

(7.10) $$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left[\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}\right]$$

$\Rightarrow M_2 = 0.23$

$M_2 < 1.0 \Rightarrow$ no shock in front of ~~pitot~~ prandtl tube.

(3.30) $$\frac{p_{02}}{p_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)} \Rightarrow p_{02} = 105.1 \text{ kPa}$$

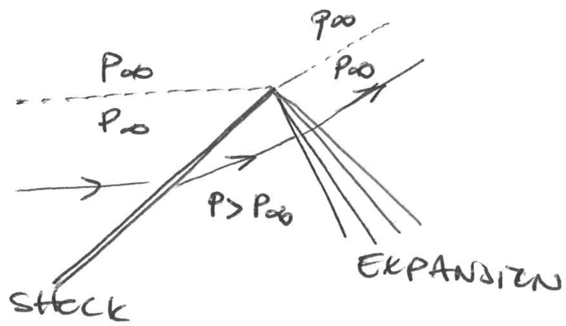
$$p_2 = p_1 \cdot 1.38 \quad \Rightarrow \quad p_{02} - p_2 = 5.07 \text{ kPa}$$

P2 (FREE BOUNDARY REFLECTION)

AN OBLIQUE SHOCK FORMED IN A MACH 2.5 FLOW DEFLECTS THE FLOW AN ANGLE OF 10°
THE SHOCK INTERACTS WITH A FREE BOUNDARY, WHICH LEADS TO A REFLECTION IN THE FORM OF AN EXPANSION REGION.

a) WHY IS THE SHOCK REFLECTED AS AN EXPANSION?

- AFTER THE OBLIQUE SHOCK, THE PRESSURE HAS BEEN INCREASED BY THE DISCONTINUOUS COMPRESSION PROCESS. AT THE FREE BOUNDARY THE PRESSURE MUST BE CONSTANT, WHICH MEANS THAT THE PRESSURE MUST BE DECREASED. THE PRESSURE REDUCTION IS ACCOMPLISHED BY AN EXPANSION AND THUS THE SHOCK WILL BE "REFLECTED" AS AN EXPANSION AT THE FREE BOUNDARY. THIS WILL LEAD TO A NET TURNING OF THE FLOW AS THE FLOW DEFLECTION CAUSED BY THE EXPANSION WILL BE IN THE SAME DIRECTION AS THE FLOW DEFLECTION RELATED TO THE OBLIQUE SHOCK.



b) CALCULATE THE FLOW MACH NUMBER AND FLOW ANGLE AFTER THE EXPANSION
(THE FLOW CAN BE ASSUMED TO BE AXIAL UPSTREAM OF THE SHOCK)

$$\left.\begin{array}{l} M_\infty = 2.5 \\ \theta = 10^\circ \end{array}\right\} \Rightarrow (\theta-\beta-M) \Rightarrow \beta = 31.85^\circ$$

$$\left.\begin{array}{ll} (4.7) & M_{n_1} = M_1 \sin\beta \\ (4.9) & \dfrac{P_2}{P_1} = 1 + \dfrac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \\ (4.10) & M_{n_2}^2 = \dfrac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1} \\ (4.12) & M_2 = \dfrac{M_{n2}}{\sin(\beta-\theta)} \end{array}\right\} \Rightarrow \begin{array}{l} \dfrac{P_2}{P_1} = 1.86 \\ M_2 = 2.08 \end{array}$$

TOTAL PRESSURE DOWNSTREAM OF THE SHOCK (AND THROUGH THE EXPANSION)

$$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{P_{03}}{P_3} = \left(1 + \frac{\gamma-1}{2} M_3^2\right)^{\gamma/(\gamma-1)}$$

$P_{03} = P_{02}$, $P_3 = P_1$ (THAT IS WHY THE EXPANSION IS FORMED)

$$\Rightarrow \frac{P_{03}}{P_3} = \frac{P_{02}}{P_1} = \frac{P_{02}}{P_2}\frac{P_2}{P_1}$$

$$\frac{p_{02}}{p_2}\frac{p_2}{p_1} = \left(1+\frac{\gamma-1}{2}M_3^2\right)^{\gamma/(\gamma-1)} \Rightarrow \underline{M_3 = 2{,}48}$$

THE FLOW DEFLECTION OVER THE EXPANSION IS OBTAINED USING THE PRANDTL-MEYER FUNCTION

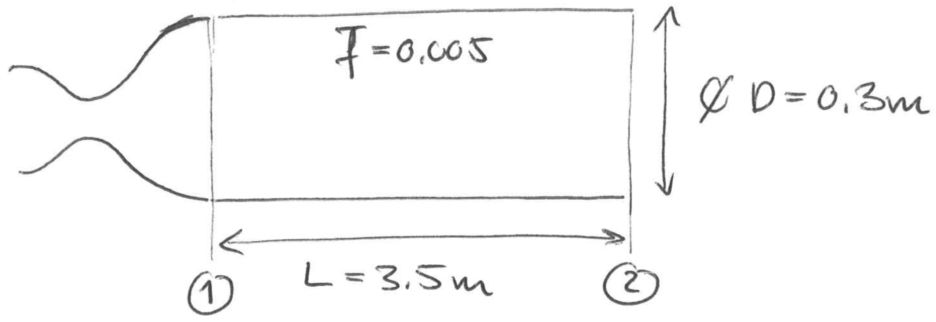
(4.44)

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}$$

$$\Delta\theta = \nu(M_3) - \nu(M_2) = 10{,}03°$$

THE FLOW DIRECTION HAS ALREADY BEEN CHANGED 10° BY THE SHOCK $\Rightarrow$ NET FLOW DEFLECTION : $\underline{20°}$

P3 (ONE - DIMENSIONAL FLOW WITH FRICTION)



$M_1 = 2.0$
$P_1 = 101.3 kPa$

a) CALCULATE EXIT MACH NUMBER AND PRESSURE IF THERE ARE NO SHOCKS INSIDE THE TUBE

CHECK MAX LENGTH: (L^*)

$$(3.107) \quad \frac{4\bar{f}L^*}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma} \ln\left(\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right)$$

$$\Rightarrow L_1^* = 4{,}57\, m \ (> 3{,}5 m)$$

$$\Rightarrow L_2^* = L_1^* - L = 1{,}07\, m$$

$$(3.107) \Rightarrow \underline{M_2 = 1.32}$$

$$(3.104) \quad \frac{P_1}{P^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2}$$

$$\frac{P_2}{P^*} = \frac{1}{M_2}\left(\frac{\gamma+1}{2+(\gamma-1)M_2^2}\right)^{1/2}$$

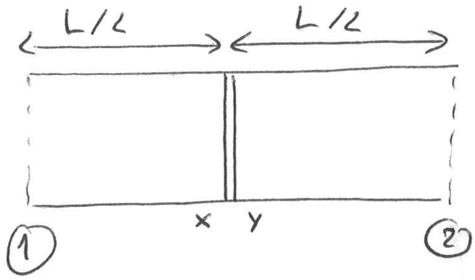
$$P^* = \text{const}$$

$$\Rightarrow \underline{P_2 = 177.3\, kPa}$$

b) CALCULATE THE PRESSURE DOWNSTREAM OF THE PIPE IF THERE IS A SHOCK AT THE PIPE EXIT.

(3.57) $$\frac{P_b}{P_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_2^2 - 1\right) \Rightarrow P_b = 331.0 \text{ kPa}$$

c) CALCULATE THE MACH NUMBER AND PRESSURE AT THE PIPE EXIT IF THERE IS A SHOCK HALFWAY DOWN THE PIPE.



$L_1^* = 4.57$

$L_x^* = L_1^* - \frac{L}{2} = 2.82$

(3.107) $\Rightarrow M_x = 1.64$

(3.104)
$$\left.\begin{array}{l} \dfrac{P_1}{p^*} = \dfrac{1}{M_1}\left(\dfrac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2} \\[2ex] \dfrac{P_x}{p^*} = \dfrac{1}{M_x}\left(\dfrac{\gamma+1}{2+(\gamma-1)M_x^2}\right)^{1/2} \\[2ex] p^* = \text{const} \end{array}\right\} P_x = 133.1 \text{ kPa}$$

NORMAL SHOCK:

(3.51) $$M_y^2 = \frac{1 + ((\gamma-1)/2)M_x^2}{\gamma M_x^2 - (\gamma-1)/2} \Rightarrow M_y = 0.66$$

(3.57) $$\frac{P_y}{P_x} = 1 + \frac{2\gamma}{\gamma+1}\left(M_x^2 - 1\right) \Rightarrow P_y = 398 \text{ kPa}$$

(3.107) $$\frac{4\bar{f}L_y^*}{D} = \frac{1-M_y^2}{\gamma M_y^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_y^2}{2+(\gamma-1)M_y^2}\right)$$

$$\Rightarrow L_y^* = 4.65$$

$$L_2^* = L_y^* - \frac{L}{2} = 2.9$$

(3.107) $$\frac{4\bar{f}L_2^*}{D} = \frac{1-M_2^2}{\gamma M_2^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2}\right)$$

$$\Rightarrow \underline{M_2 = 0{,}71}$$

(3.104) $$\left.\begin{aligned} \frac{p_y}{p^*} &= \frac{1}{M_y}\left(\frac{\gamma+1}{2+(\gamma-1)M_y^2}\right)^{1/2} \\ \frac{p_2}{p^*} &= \frac{1}{M_2}\left(\frac{\gamma+1}{2+(\gamma-1)M_2^2}\right)^{1/2} \\ p^* &= \text{const} \end{aligned}\right\} \Rightarrow \underline{p_2 = 366.0\ \text{kPa}}$$

P4 (NOZZLE FLOW)

AIR IS EXPANDED THROUGH A C-D NOZZLE

UPSTREAM STAGNATION CONDITIONS:

$T_0 = 40°C$

$P_0 = 600.0\ kPa$

DESIGN BACK PRESSURE: $P_b = 100.0\ kPa$

a) CALCULATE A_e/A^*

$$(3.30)\quad \frac{P_0}{P_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\gamma/(\gamma-1)} \Rightarrow (P_e = P_b) \Rightarrow$$

$$\Rightarrow M_e = 1.83$$

AREA-MACH RELATION

$$(5.20)\quad \left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow \underline{A_e/A^* = 1.47}$$

b) CALCULATE THE EXIT VELOCITY AT DESIGN CONDITIONS.

$$(3.28)\quad \frac{T_0}{T_e} = 1 + \frac{\gamma+1}{2} M_e^2 \Rightarrow T_e = 187.6\ K$$

$$u_e = M_e \cdot a_e = M_e\sqrt{\gamma R T_e} = \underline{501.9\ m/s}$$

c) FOR WHAT RANGE OF BACK PRESSURES WILL THE FLOW BE OVEREXPANDED.

THE RANGE OF BACK PRESSURES ASKED FOR IS FROM NORMAL SHOCK AT EXIT TO DESIGN BACK PRESSURE ⇒ NEED TO GET THE BACK PRESSURE THAT GIVES A NORMAL SHOCK AT THE EXIT

NORMAL SHOCK AT EXIT:

$$(3.57) \qquad \frac{P_{b,n}}{P_e} = 1 + \frac{2\gamma}{\gamma+1}\left(M_e^2 - 1\right) \Rightarrow P_{b,n} = 373.3\,kPa$$

OVEREXPANDED NOZZLE FLOW

$$\underline{100.0\,kPa < P_b < 373.3\,kPa}$$