TME085 - Compressible Flow

2023-06-07, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Graph drawing calculator with cleared memory

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Responsible teacher: Niklas Andersson tel.: 070-51 38 311

Solutions for the problems will be published in Canvas after the exam

Results available no later than 2023-06-28

Good luck!

Part I - Theory Questions (20 p.)

T1. (1.0 p.) How are the **compressibility** factors τ_T and τ_S defined?

T2. (2.0 p.) Gas models

(a) (0.5 p.) What do we mean by **thermally perfect gas** and **calorically perfect gas** respectively?

(b) (0.5 p.) When can air be regarded as a **calorically perfect gas**?

(c) (1.0 p.) A mixture of chemically reacting perfect gases, where the reactions are always in **equilibrium**, may be thermodynamically described as a single-species gas. How does this thermodynamic description differ from that of a calorically perfect or thermally perfect gas?

T3. (1.0 p.) What is the physical interpretation of each of the terms in the **continuity equation** on integral form

$$\frac{d}{dt}\iiint\limits_{\Omega} \rho d\mathscr{V} + \oiint\limits_{\partial\Omega} \rho \mathbf{v} \cdot \mathbf{n} dS = 0$$

T4. (1.0 p.) What is the general definition (valid for any gas) of the **total conditions** p_o, T_o, ρ_o etc at some location in the flow?

T5. (2.0 p.) Normal shocks

(a) (1.0 p.) How come that the control volume approach applied to the governing equations on **adiabatic** form gives us the normal-shock relations? *i.e.*, how do the equations "know" that there is a shock inside of the control volume?

(b) (1.0 p.) The normal shock relations actually allow two solutions, one that corresponds to a discontinuous compression (a sudden pressure increase) and one that corresponds to a discontinuous expansion (a sudden pressure decrease). However, only one of these solutions is physically valid. What thermodynamic principle guides us in the choice of the physically correct solution, and which solution is the correct one?

T6. (2.5 p.) One-dimensional flow with heat addition and/or friction

(a) (1.0 p.) Looking at the **Rayleigh** curve for one-dimensional flow with heat addition it's evident that removing heat leads to reduced entropy - how come that this is possible?

(b) (0.5 p.) In one-dimensional flow with heat addition, what is q^*?

(c) (0.5 p.) What happens in the flow when heat is added if the flow is initially supersonic and subsonic, respectively

(d) (0.5 p.) Does the total temperature T_o change due to friction?

T7. (3.0 p.) Oblique shocks and expansions

(a) (0.5 p.) How does the absolute Mach number change after a **weak/strong** stationary oblique shock?

(b) (0.5 p.) What kind of shock is obtained for a blunt body in supersonic flow?

(c) (1.0 p.) When an oblique shock is reflected at a wall (**regular reflection**), will the reflection angle be specular? Justify your answer.

(d) (1.0 p.) Describe how you can use the **Prandtl-Meyer function** to compute the change in Mach number due to a given flow deflection.

T8. (1.0 p.) Assume that we in a convergent-divergent nozzle would have a **Nozzle Pressure Ratio** (NPR) between the **normal-shock-at-exit** NPR and the NPR defining lower limit of **choked nozzle flow**, would it be possible to use the **area-Mach-number** relation throughout the nozzle? Justify and explain why or why not.

T9. (2.0 p.) Moving shocks

(a) (1.0 p.) Can a moving normal shock travel at a speed lower than the speed of sound? Explain why/why not.

(b) (1.0 p.) Describe what happens when a moving normal shock hits a solid wall.

T10. (2.0 p.) Computational Fluid Dynamics (CFD)

(a) (0.5 p.) What is meant by the term **density-based** when discussing CFD codes for compressible flow?

(b) (0.5 p.) What is meant by the term **fully-coupled** when discussing CFD codes for compressible flow?

(c) (1.0 p.) What do we mean when we say that a CFD code for compressible flow is **conservative**?

T11. (2.5 p.) Molecular energy

(a) (0.5 p.) What are the fundamental modes or forms of energy of a gas molecule?

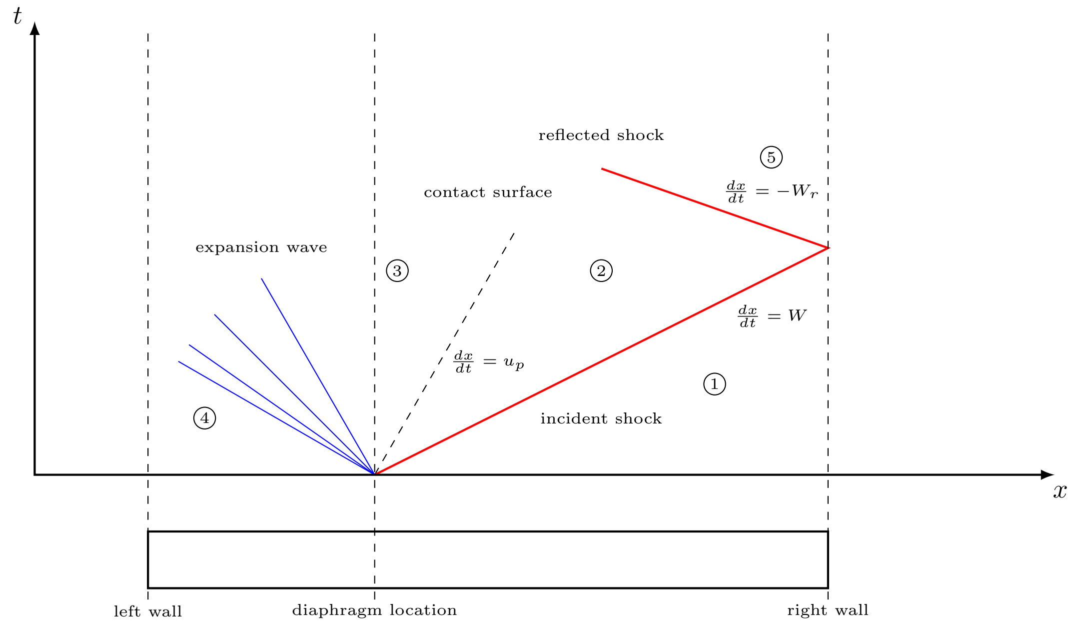
(b) (1.0 p.) How does a **mono-atomic** gas differ from a diatomic gas in terms of energy modes?

(c) (1.0 p.) Explain the concept **zero-point energy**

Part II - Problems (40 p.)

Problem 1 - SHOCK TUBE (10 p.)

A shock tube is set up such that the driver section (section 4) pressure is ten times higher than the driven section (section 1) pressure. Before the shock tube is started, the temperature in both the driver section and the driven section is 300 K. The driven section pressure is 1.0 bar. The gas used in both the driver section and the driven section is air.
Calculate the flow conditions in sections 2, 3, and 5 when the shock tube is started, i.e. find the missing values in the table below.



section	temperature [K]	pressure [Pa]	velocity [m/s]	Mach number []
1	300.	1.0e5	0.	0.
2	?	?	?	?
3	?	?	?	?
4	300.	1.0e6	0.	0.
5	?	?	?	?

Problem 2 - NOZZLE FLOW (10 p.)

Air flows through a convergent-divergent nozzle with an exit-to-throat area ratio of 3.5. The total pressure and total temperature at the nozzle inlet are 1.0 MPa and 500.0 K, respectively. Determine the pressure and temperature at the nozzle exit if the back pressure is

(a) 20.0 kPa

(b) 500.0 kPa

Problem 3 - 1D-FLOW WITH FRICTION (10 p.)

Air enters a 3.0 cm diameter pipe with a total pressure (p_o) of 100.0 kPa, a total temperature (T_o) of 300.0 K, and a velocity (u) of 100.0 m/s. The average friction factor in the pipe can be assumed to be $\bar{f} = 0.02$.

(a) For the specified conditions, calculate the air massflow through the pipe

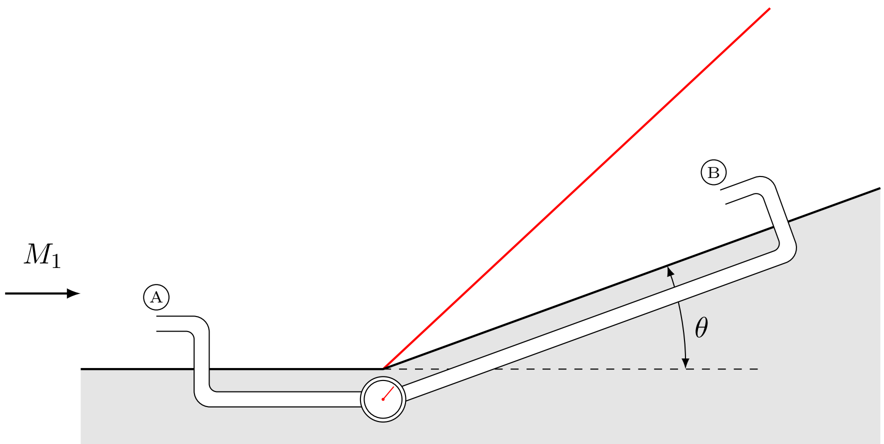
(b) Calculate the maximum pipe length possible without making changes to the flow conditions

(c) Now, let's assume that the length of the pipe is 2.5 times the length calculated in the previous task. What will happen? Calculate the air massflow through the pipe for this pipe length.

hint: any changes taking place will not effect the total conditions at the inlet

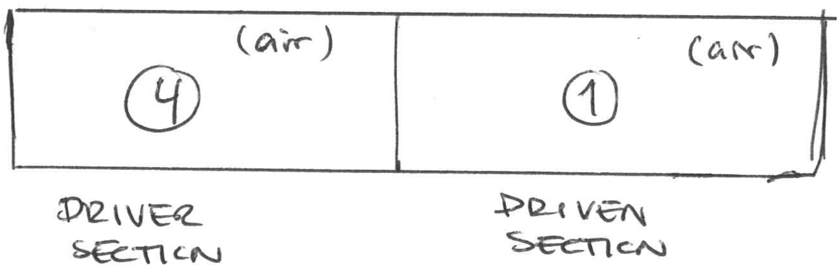
Problem 4: PITOT TUBE (10 p.)

A Mach 2.5 airflow is deflected through an oblique shock due to the presence of a compression corner (see figure below). The deflection angle θ is 20.0 degrees and the freestream pressure and temperature ahead of the compression corner are 101325.0 Pa and 300.0 K, respectively. Two pitot tubes, one placed upstream of the compression corner and one placed downstream, are connected to a pressure meter. The pressure meter measures the difference in pressure between the two pitot tubes. Both the pitot tubes are placed such that they are aligned with the flow direction. Determine the pressure difference measured by the pressure meter.



TME085 EXAM 23-06-07

P1 (SHOCK TUBE)



$T_4 = T_1 = 300K$

$p_1 = 1.0 \text{ bar } (100 \text{ kPa})$

$p_4 = 10\, p_1 = 1 \text{ MPa}$

Calculate the flow conditions in sections 2, 3, and 5.

FIRST WE WILL NEED TO FIND THE PRESSURE RATIO OVER THE INCIDEND SHOCK (p_2/p_1) USING (7.94)

(7.94)

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left(1 - \frac{(\gamma-1)((a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma(2\gamma + (\gamma+1)(p_2/p_1 - 1))}}\right)^{\frac{-2\gamma}{\gamma-1}}$$

$(\gamma_1 = \gamma_4)$

THE EQUATION CAN BE SOLVED ITERATIVELY USING NEWTON-RHAPSON OR TRIAL AND ERROR ..
(AN ESTIMATE IS ENOUGH ...)

NEWTON - RHAPSON $\Rightarrow \frac{p_2}{p_1} \approx 2.85$

WITH p_2/p_1 KNOWN, STATION 2 PROPERTIES CAN BE CALCULATED.

$$(7.10) \quad \frac{T_2}{T_1} = \frac{P_2}{P_1}\left[\frac{\frac{\gamma+1}{\gamma-1} + \frac{P_2}{P_1}}{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{P_2}{P_1}\right)}\right] \Rightarrow T_2 = 418\,K$$

$$(7.16) \quad u_2 = u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1} + \frac{\gamma-1}{\gamma+1}}\right]^{1/2} \Rightarrow u_2 = 285.1\,m/s$$

(INDUCED VELOCITY)

WHERE $a_1 = \sqrt{\gamma R T_1}$

$M_2 = u_p / a_2$ WHERE $a_2 = \sqrt{\gamma R T_2}$ $\Rightarrow M_2 = 0.70$

THE PRESSURE AND VELOCITY ARE CONSTANT OVER THE CONTACT DISCONTINUITY, TEMPERATURE CHANGES THOUGH.

$P_3 = P_2$

$u_3 = u_2 = u_p$

$$(7.85) \quad \frac{T_3}{T_4} = \left(1 - \frac{\gamma-1}{2}\left(\frac{u_3}{a_4}\right)\right)^2 \Rightarrow T_3 = 209.5\,K$$

WHERE $a_4 = \sqrt{\gamma R T_4}$

$M_3 = u_3 / a_3$ WHERE $a_3 = \sqrt{\gamma R T_3}$ $\Rightarrow M_3 = 0.98$

TO CALCULATE STATION 5 CONDITIONS, WE WILL NEED THE MACH NUMBER OF THE REFLECTED SHOCK WAVE WHICH IN TURN REQUIRES THE MACH NUMBER OF THE INCIDENT SHOCK WAVE.

$$(7.13) \quad M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right) + 1}$$

$$(7.23) \qquad \frac{M_R}{M_R^2-1} = \frac{M_s}{M_s^2-1}\sqrt{1+\frac{2(\gamma-1)}{(\gamma+1)^2}\left(M_s^2-1\right)\left(\gamma+\frac{1}{M_s^2}\right)}$$

NORMAL SHOCK RELATIONS (3.57) AND (3.59) GIVES THE PRESSURE RATIO AND TEMPERATURE RATIO OVER THE REFLECTED SHOCK..

$$(3.57) \qquad \frac{p_5}{p_2} = 1+\frac{2\gamma}{\gamma+1}\left(M_R^2-1\right)$$

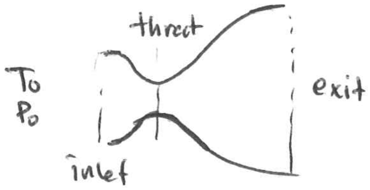
$$(3.59) \qquad \frac{T_5}{T_2} = \left(1+\frac{2\gamma}{\gamma+1}\left(M_R^2-1\right)\right)\left(\frac{2+(\gamma-1)M_R^2}{(\gamma+1)M_R^2}\right)$$

$$\Rightarrow \qquad p_5 = 701.2 \text{ kPa}$$

$$T_5 = 552 \text{ K}$$

$$M_5 = u_5/a_5 \quad (u_5 = 0) \quad = 0$$

P2 (NOZZLE FLOW)



$$\frac{A_e}{A_t} = 3.5$$

$T_0 = 500.0\ K$

$P_0 = 1.0\ MPa$

CALCULATE THE PRESSURE AND TEMPERATURE AT THE EXIT IF THE BACK PRESSURE IS

a) 20 kPa

b) 500 kPa

FROM THE AREA - MACH-NUMBER RELATION, WE CAN GET THE EXIT MACH NUMBER FOR ISENTROPIC, CHOKED SUBSONIC AND SUPERSONIC FLOW.
(CRITICAL AND SUPERCRITICAL)

(5.20) $$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

choked flow. $$\Rightarrow \left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$\Rightarrow M_{e\ subsonic} = 0.17$

$M_{e\ supersonic} = 2.8$

USING THE TOTAL PRESSURE RELATION WE GET THE CORRESPONDING EXIT PRESSURES..

(3.30) $$\frac{P_0}{P_e} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\gamma/(\gamma-1)}$$

$\Rightarrow P_{e\ subsonic,critical} = 980.4\ kPa$

$P_{e\ supercritical} = 36.9\ kPa$

CASE a IS BELOW 36.9 kPa => UNDEREXPANDED FLOW.

CASE b IS BETWEEN CRITICAL AND SUPERCRITICAL =>

WE NEED TO CALCULATE THE BACKPRESSURE CORRESPONDING TO SHOCK AT EXIT TO KNOW IF THE FLOW IS AN INTERNAL-SHOCK FLOW OR OVEREXPANDED.

USING THE NORMAL SHOCK RELATIONS, WE CAN CALCULATE P_b FOR SHOCK AT EXIT CONDITION.

$$(3.57) \quad \frac{P_b}{P_e} = 1 + \frac{2\gamma}{\gamma+1}\left(M_e^2 - 1\right) \Rightarrow P_b = 331 \text{ kPa}$$

=> CASE b IS BETWEEN CRITICAL AND SHOCK-AT-EXIT

=> THERE WILL BE A SHOCK INSIDE THE NOZZLE..

a) UNDEREXPANDED FLOW => AT THE NOZZLE EXIT THE TEMPERATURE AND PRESSURE WILL BE THE SAME AS FOR SUPERCRITICAL FLOW (DESIGN CONDITION)

$P_e = 36.9$ kPa

$M_e = 2.8$

$$(3.28) \quad \frac{T_0}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 \Rightarrow T_e = 194.7 \text{ K}$$

b)

(5.28) => EXIT MACH NUMBER FOR FLOW WITH INTERNAL SHOCK.

$$M_e = \frac{-1}{\gamma - 1} + \sqrt{\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)} \left(\frac{P_{0_1} A_t}{P_e A_e}\right)^2}$$

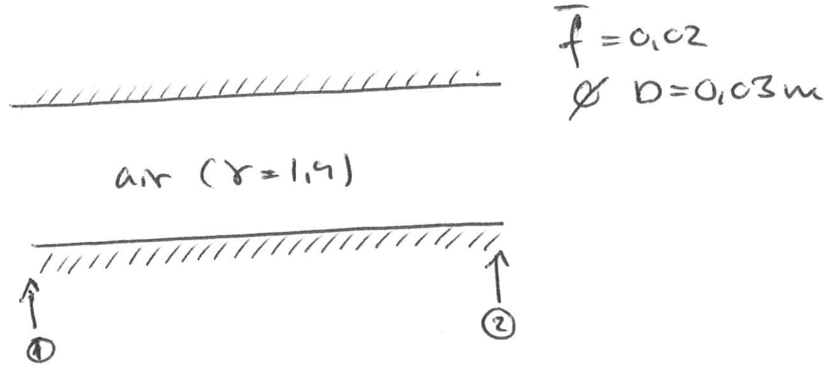
=> $M_e =$ **0.33**

subsonic flow => $P_e = P_b$

T_0 = const over shock =>

$$\frac{T_0}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 \Rightarrow T_e = \textbf{489.5} \text{ K}$$

P3 (1D-FLOW WITH FRICTION)



AT THE INLET (1) $P_0 = 100.0$ kPa, $T_0 = 300.0$ K, AND $u = 100$ m/s

a) CALCULATE THE MASSFLOW FOR THE SPECIFIED CONDITIONS.

$$\dot{m} = \rho_1 u_1 \frac{\pi D^2}{4} = \left\{ \rho = \frac{P}{RT} \right\} = \frac{P_1}{RT_1} u_1 \frac{\pi D^2}{4}$$

(3,28) $$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} \frac{u_1^2}{\gamma R T_1}$$

$$T_{01} = T_1 + \frac{1}{2} \frac{\gamma - 1}{\gamma R} u_1^2$$

$$T_{01} = T_1 + \frac{u_1^2}{2 c_p} \Rightarrow T_1 = T_{01} - \frac{u_1^2}{2 c_p} = 295 \text{ K}$$

$$M_1 = u_1 / a_1 = u_1 / \sqrt{\gamma R T_1} = 0{,}29$$

(3.30) $$\frac{P_{01}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma - 1)}$$

$$\Rightarrow P_1 = 94{,}3 \text{ kPa}$$

$$\dot{m} = \frac{P_1}{R T_1} u_1 \frac{\pi D^2}{4} = 0{,}08 \text{ kg/s}$$

b) CALCULATE THE MAXIMUM POSSIBLE PIPE LENGTH WITHOUT CHANGING THE INLET FLOW CONDITIONS

THE MAXIMUM LENGTH ~~ALLOWED~~ FOR IS $L^*(M_1)$

(3.107) $$\frac{4\bar{f}L_1^*}{D} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

$$\Rightarrow L^* = 2.17\ m$$

c) $L = 2.5L^*$, CALCULATE MASSFLOW

WITH $L > L^*$ THE INFLOW STATIC CONDITIONS WILL CHANGE SUCH THAT $L = L^*$ FOR THE NEW FLOW CONDITIONS..

(3.107) $$\frac{4\bar{f}L_1^*}{D} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

WITH $L_1^* = 2.5L_b^*$, WE GET $M_1 = 0.2$

TOTAL CONDITIONS ARE UNCHANGED (CHANGING P_0 AND T_0 REQUIRED HEAT ADDITION OR WORK)

(3.28)

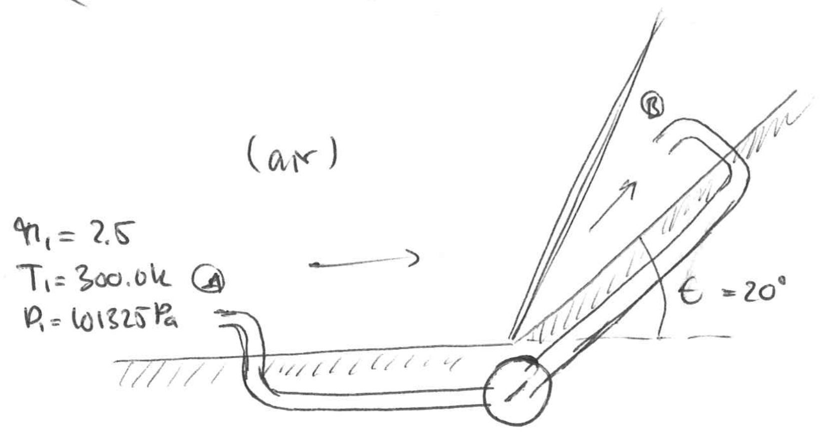
$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2 \Rightarrow T_1 = 297.6\ K$$

(3.30) $$\frac{P_{01}}{P_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_1 = 97.2\ kPa$$

$$M_1 = \frac{u_1}{a_1} = \frac{u_1}{\sqrt{\gamma R T_1}} \Rightarrow u_1 = 69.35\ m/s$$

$$\dot{m} = \frac{P_1}{RT_1}u_1\frac{\pi D^2}{4} = 0.056\ kg/s$$

P4 (PITOT TUBE)



A PRESSURE METER MEASURES THE DIFFERENCE IN PRESSURE BETWEEN PITOT-TUBE A AND PITOT-TUBE B.

CALCULATE THE MEASURED PRESSURE DIFFERENCE.

THERE WILL BE A NORMAL SHOCK IN FRONT OF EACH OF THE PITOT TUBES. BEHIND THE SHOCK THE AIR WILL SLOW DOWN TO ZERO VELOCITY ISENTROPICALLY => THE TOTAL PRESSURE WILL BE MEASURED.

PITOT-TUBE A

(NORMAL SHOCK)

(3.51) $$M_A^2 = \frac{1 + ((\gamma-1)/2)M_1^2}{\gamma M_1^2 - (\gamma-1)/2} \Rightarrow M_A = 0.51$$

(3.57) $$\frac{p_A}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1) \Rightarrow p_A = 721.9 \text{ kPa}$$

(3.30) $$\frac{p_{0A}}{A_A} = \left(1 + \frac{\gamma-1}{2}M_A^2\right)^{\gamma/(\gamma-1)} \Rightarrow p_{0A} = 863.9 \text{ kPa}$$

OBLIQUE SHOCK

$$\left.\begin{array}{l}\epsilon = 20^\circ \\ M_1 = 2.5\end{array}\right\} \Rightarrow (\epsilon\text{-}\beta\text{-}M\text{-relation}) \Rightarrow \beta = 42.9^\circ$$

$$\left.\begin{array}{ll}
(4.7) & M_{n_1} = M_1 \sin\beta \\
(4.9) & \dfrac{P_2}{P_1} = 1 + \dfrac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \\
(4.10) & M_{n_2}^2 = \dfrac{M_{n_1}^2 + (2/(\gamma-1)}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1} \\
(4.12) & M_2 = \dfrac{M_{n_2}}{\sin(\epsilon - \beta)}
\end{array}\right\} \Rightarrow \begin{array}{l} M_2 = 1.65 \\ P_2 = 325.3\,kPa \end{array}$$

PITOT-TUBE B

(NORMAL SHOCK)

$$(3.51) \quad M_B^2 = \frac{1 + ((\gamma-1)/2)M_2^2}{\gamma M_2^2 - (\gamma-1)/2} \Rightarrow M_B = 0.68$$

$$(3.57) \quad \frac{P_B}{P_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_2^2 - 1\right) \Rightarrow P_B = 973.9\,kPa$$

$$(3.30) \quad \frac{P_{0B}}{P_B} = \left(1 + \frac{\gamma-1}{2}M_B^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_{0B} = 1299\,kPa$$

PRESSURE METER READING

$$\Delta P = P_{0B} - P_{0A} = 935.3\,kPa$$