TME085 - Compressible Flow

2023-03-16, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Graph drawing calculator with cleared memory

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

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Solutions for the problems will be published in Canvas after the exam

Results available no later than 2023-04-06

Good luck!

Part I - Theory Questions (20 p.)

T1. (3.0 p.) Shocks:

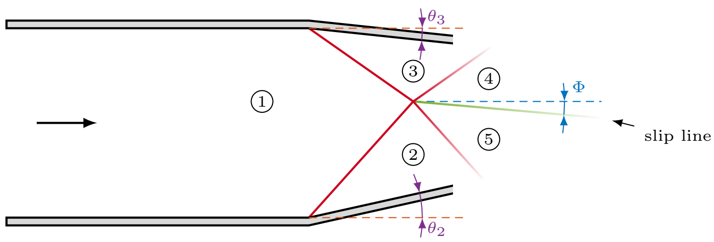
(a) (0.5 p.) The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the **weak** type or the **strong** type. What is the main difference between these two shock types and which type is usually seen in reality?

(b) (0.5 p.) Are the normal-shock relations mathematically and physically valid for upstream Mach numbers lower than one? Justify your answer.

(c) (0.5 p.) Describe what happens when a moving normal shock hits a solid wall.

(d) (0.5 p.) In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

(e) (1.0 p.) What are the constraints that leads to the generation of the separating line between regions 4 and 5 (represented by a green line in the figure below)? What is the reason for the need for this separating line?



T2. (1.0 p.) What are the implications of the **area-velocity relation** for quasi-one-dimensional flow?

T3. (2.0 p.) Governing equations:

(a) (1.0 p.) What is the physical interpretation of each of the terms in the **momentum equation** on integral form

(b) (1.0 p.) How can the **substantial derivative** operator be interpreted physically?

$$\frac{d}{dt}\iiint_{\Omega} \rho \mathbf{v} d\mathscr{V} + \oiint_{\partial\Omega} [\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}]\, dS = \iiint_{\Omega} \rho \mathbf{f} d\mathscr{V}$$

T4. (3.0 p.) Gas models

(a) (0.5 p.) What do we mean by **thermally perfect gas** and **calorically perfect gas** respectively?

(b) (0.5 p.) In what temperature range (approximately) can a gas be assumed to be **calorically perfect**?

(c) (1.0 p.) Explain what the **Boltzmann distribution** describes and what sparsely populated implies.

(d) (1.0 p.) Using **equilibrium gas** assumption in the analysis of chemically reacting nozzle flow will lead to higher exhaust temperatures than if calorically perfect gas assumption is used for the same analysis. Explain why.

T5. (2.0 p.) Unsteady waves:

(a) (1.0 p.) What types of waves or discontinuities are generated in a shock tube with two initially stagnant regions at different pressure (separated by a thin membrane which is removed very quickly)?

(b) (1.0 p.) How are the two **characteristic** curves C^+ and C^- defined?

T6. (2.0 p.) Compressible CFD:

(a) (0.5 p.) What is meant by the term **density-base** when discussing CFD codes for compressible flow?

(b) (0.5 p.) When the governing equations are discretized using a **finite-volume** approach, cell face values of flow properties appears in the equations. How are these values approximated?

(c) (1.0 p.) What do we mean when we say that a CFD code for compressible flow is **conservative**?

T7. (1.0 p.) Heat addition:

(a) (0.5 p.) What is the **Rayleigh** curve and what does it tell us?

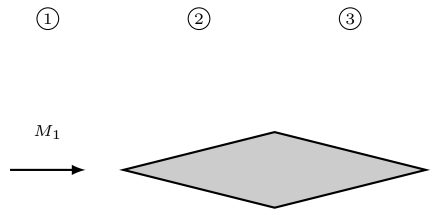
(b) (0.5 p.) What happens in the flow when heat is added if the flow is initially supersonic and subsonic, respectively

T8. (2.0 p.) For a steady-state **isentropic** flow of a **calorically perfect** gas, derive the formula for T_0/T, making use of the fact that the total enthalpy h_0 is constant along the streamlines.

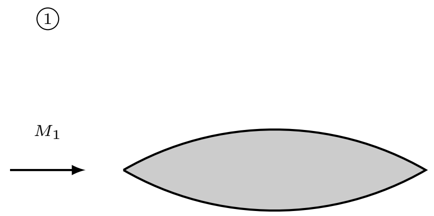
$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

T9. (2.0 p.) Derive the **isentropic relations** for **calorically perfect** gases starting from the **entropy equation**.

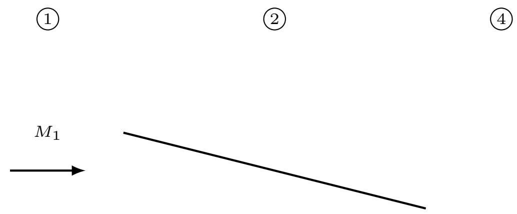
T10. (2.0 p.) For each of the cases a-c below, make a schematic representation showing how the pressure varies from station 1 to 4 (following the flow over the upper side of each "wing"). Any sudden changes in pressure has to be motivated.



(a)
symmetric diamond-wedge airfoil (zero angle of attack)



(b)
double circular arc airfoil (zero angle of attack)



(c)
flat plate at an angle of attack

Part II - Problems (40 p.)

Problem 1 - PIPE FLOW WITH FRICTION (10 p.)

Air flows through a 20-cm-diameter pipe with an average Darcy friction factor of 0.005. At the pipe entrance the Mach number is 2.0, the static pressure is 101325 Pa, and the temperature is 288 K.

(a) What is the maximum possible pipe length without generation of shocks in the pipe? Calculate the pressure and temperature at the exit for this pipe length.

(b) What is the maximum possible pipe length without changing the mass flow through the pipe? Calculate the pressure and temperature at the exit for this pipe length.

(c) What is the difference in total pressure at the pipe exit for the two cases above?

Problem 2 - NOZZLE FLOW (10 p.)

Air flows through a convergent-divergent nozzle with exit-to-throat area ratio of 4.0. The total pressure upstream of the nozzle is 3.0 bar and the total temperature is 300 K.

Calculate:

(a) The exit Mach numbers corresponding to:

1. choked flow (subsonic flow in divergent part of the nozzle)
2. supercritical nozzle flow (perfectly matched supersonic flow)

(b) The nozzle pressure ratios (NPR) ranges corresponding to

1. subsonic nozzle flow
2. nozzle flow with internal shock in the divergent section
3. overexpanded nozzle flow
4. underexpanded nozzle flow

(c) The flow deflection angle downstream of the oblique shock generated at the nozzle exit for a nozzle pressure ratio (NPR) of 15.0

Problem 3 - MOVING SHOCK WAVE (10 p.)

An explosion generates a moving shock wave behind which the induced flow reaches a velocity of 180.0 m/s. The shock wave moves into stagnant air at 101325 Pa and 288 K.

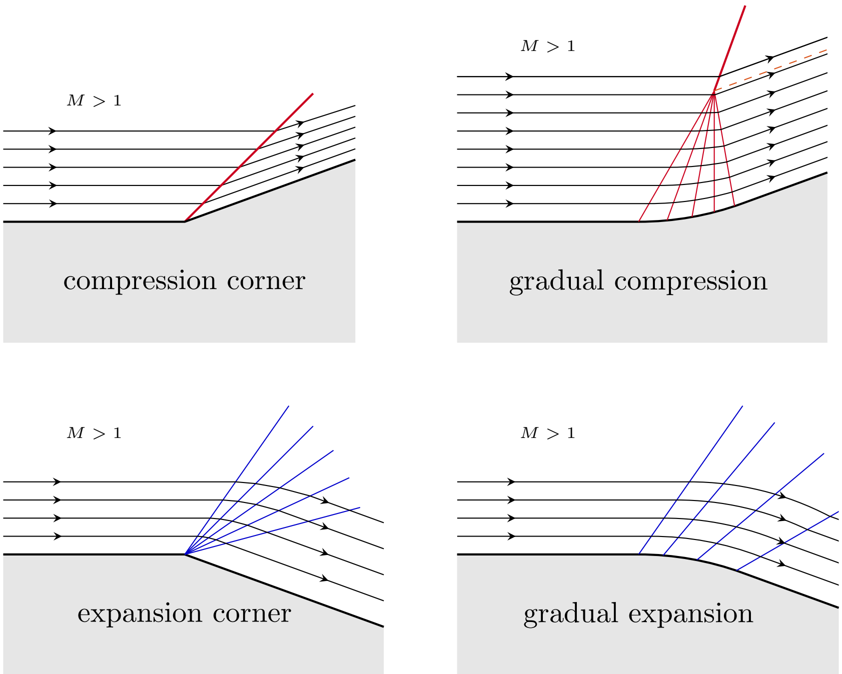
Calculate:

(a) The velocity with which the shock wave moves into the stagnant gas

(b) The Mach number of the induced flow behind the shock wave

(c) The total temperature behind the shock wave

Problem 4 - ISENTROPIC FLOW COMPRESSION PROCESSES (10 p.)



A gradual compression (top right figure above) may be analyzed using theory based on isentropic flow processes (in the same way as flow expansion is analyzed). Close to the wall where compression is built up from a large number of isolated Mach waves this approach is correct. In contrast to an expansion where the Mach waves becomes more and more separated with increasing wall distance (see figure above), the distance of the isolated Mach waves in a gradual compression will be shorter and shorter with increasing wall distance and eventually they will coalesce into a single oblique shock (top right figure). In the outer region of the gradual compression region, the interaction between Mach waves is significant and it becomes less and less accurate to use theory based on isentropic processes.

(a) Calculate the Mach number, temperature, and pressure downstream of a gradual compression that gives a total change in flow direction of 20° assuming that the compression can be considered to be isentropic. The Mach number upstream of the compression is 3.0 and the pressure and temperature in the upstream flow are 1.0 bar and 300 K, respectively.

(b) For oblique shocks (top left figure above) associated with very small flow deflection angles, one almost gets the correct results if analyzing the shock using isentropic theory instead of the oblique shock relations since such shocks are rather weak and consequently the deviation from constant total pressure is close to insignificant. In literature on gas dynamics, one may find that oblique shocks with a relative total pressure loss of less than 0.5% may be approximated to be isentropic. Surprisingly, the criteria of maximum 0.5% relative total pressure loss is fulfilled for flow deflection angles up to almost 5.0 degrees for the upstream flow conditions used in the previous task. Verify by calculating the relative total pressure loss for a 5.0-degree flow deflection as follows:

$$\frac{p_{o_1} - p_{o_2}}{p_{o_1}}$$

where indices 1 and 2 denotes flow stations upstream of the shock and downstream of the shock, respectively.

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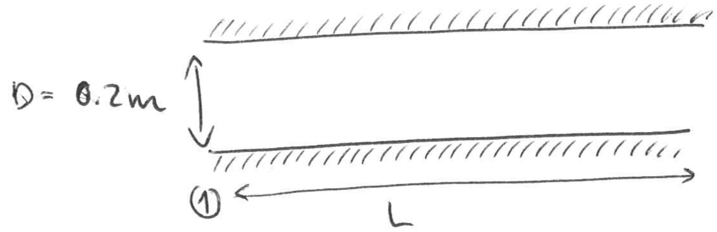
P1 (PIPE FLOW WITH FRICTION)

$\bar{f} = 0.005$



$M_1 = 2.0$

$p_1 = 101325$ Pa

$T_1 = 288$ K

a) WHAT IS THE MAXIMUM POSSIBLE LENGTH WITHOUT GENERATION OF SHOCKS IN THE PIPE?
(CALCULATE THE EXIT TEMPERATURE AND PRESSURE FOR THIS PIPE LENGTH)

THE MAXIMUM LENGTH IS L^* CALCULATED FROM THE GIVEN INLET CONDITIONS, THE TEMPERATURE AND PRESSURE AT THE EXIT WILL BE T^* AND p^*, RESPECTIVELY.

(3.107) $$\frac{4\bar{f}L_1^*}{D} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

$$\Rightarrow L_1^* = 3.05\,m$$

(3.103) $$\frac{T_1}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_1^2} \Rightarrow T^* = 432\,K$$

(3.104) $$\frac{p_1}{p^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2} \Rightarrow p^* = 248.2\,kPa$$

b) WHAT IS THE MAXIMUM POSSIBLE PIPE LENGTH WITHOUT CHANGING THE MASSFLOW THROUGH THE PIPE? (CALCULATE THE PRESSURE AND TEMPERATURE AT THE EXIT FOR THIS PIPE LENGTH)

IF THE PIPE LENGTH EXCEEDS THE LENGTH CALCULATED IN THE PREVIOUS TASK, A SHOCK WILL FORM INSIDE THE PIPE. THE LOCATION OF THE SHOCK DEPENDS ON THE LENGTH OF THE PIPE. THE LONGEST POSSIBLE PIPE WITHOUT CHANGING THE INLET CONDITIONS WILL BE WHEN THE SHOCK STANDS AT THE PIPE ENTRANCE.

NORMAL SHOCK AT STATION 1 =>

(3.51) $$M_2^2 = \frac{1+((\gamma-1)/2)M_1^2}{\gamma M_1^2-(\gamma-1)/2} \Rightarrow M_2 = 0.58$$

(3.57) $$\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma+1}(M_1^2-1) \Rightarrow p_2 = 248.2\,kPa$$

(3.59) $$\frac{T_2}{T_1} = \left(1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right)\left(\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right)$$

$$\Rightarrow T_2 = 486\,K$$

NOW, CALCULATE $L^*(M2)$ USING (3.107) =>

$L^* = 5.88\,m$

THE TEMPERATURE AND PRESSURE WILL BE THE SAME AT THE EXIT (SAME AS IN P1a)

THE SHOCK DOES NOT CHANGE p^* AND T^*

c) WHAT IS THE DIFFERENCE IN TOTAL PRESSURE AT THE PIPE EXIT FOR THE TWO CASES.

THE PRESSURE AT THE EXIT IS THE SAME FOR BOTH CASES (p^*) AND THE MACH NUMBER IS SONIC $M=1.0 \Rightarrow p_0$ IS THE SAME! (THE MORE RAPID DECREASE FOR THE SUPERSONIC CASE IS BALANCED BY THE NORMAL SHOCK AND A

P2 (NOZZLE FLOW)

AIR FLOWS THROUGH A C-D NOZZLE WITH $A_e/A_t = 4.0$

UPSTREAM CONDITIONS: $P_0 = 3.0$ bar ($= 300.0$ kPa), $T_0 = 300.0$ K

a) CALCULATE THE EXIT MACH NUMBER FOR:

1. CHOKED, ISENTROPIC, SUBSONIC FLOW (CRITICAL FLOW)
2. CHOKED, ISENTROPIC, SUPERSONIC FLOW (SUPERCRITICAL FLOW)

1) SUBSONIC SOLUTION OF THE AREA-MACH RELATION

$$(5.20) \quad \left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$\Rightarrow \underline{M_e = 0.15}$

2) SUPERSONIC SOLUTION OF THE AREA MACH RELATION

(5.20) $\Rightarrow \underline{M_e = 2.94}$

b) FIND THE NPR-RANGES CORRESPONDING TO

1. SUBSONIC FLOW
2. NOZZLE FLOW WITH INTERNAL SHOCK
3. OVEREXPANDED NOZZLE FLOW
4. UNDEREXPANDED NOZZLE FLOW

THE FOLLOWING PRESSURES ARE NEEDED TO GET THE NPR-RANGES ASKED FOR:

BACKPRESSURE FOR SUBSONIC CHOKED FLOW

BACK PRESSURE FOR SUPERSONIC, ISENTROPIC, CHOKED FLOW

BACK PRESSURE FOR NORMAL-SHOCK-AT-EXIT CONDITION

CRITICAL FLOW

(3.30)

$$NPR_c = \frac{P_o}{P_{b_c}} = \left(1 + \frac{\gamma - 1}{2} M_{e_c}^2\right) = 1.015$$

SUPERCRITICAL FLOW

(3.30)

$$NPR_{sc} = \frac{P_o}{P_{b_{sc}}} = \left(1 + \frac{\gamma - 1}{2} M_{e_{sc}}^2\right) = 33.57$$

NORMAL SHOCK AT EXIT

(3.57)

$$\frac{P_{bnse}}{P_{b\,sc}} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{e_{sc}}^2 - 1\right)$$

$$NPR_{nse} = \frac{P_o}{P_{bnse}} = \frac{P_o}{P_{b\,sc}} \frac{P_{b\,sc}}{P_{bnse}} = 3.38$$

1. SUBSONIC FLOW

$$NPR < NPR_c = 1.015$$

2. INTERNAL SHOCKS

$$NPR_c < NPR < NPR_{nse}$$

$$(1.015 < NPR < 3.38)$$

3. OVEREXPANDED

$$NPR_{nse} < NPR < NPR_{sc}$$

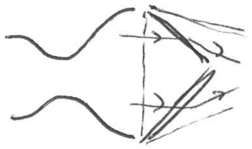
$$(3.38 < NPR < 33.57)$$

4. UNDER EXPANDED

$$NPR > NPR_{sc} = 33.57$$

c) CALCULATE THE FLOW DEFLECTION ANGLE DOWNSTREAM OF THE NOZZLE FOR $NPR = 15.0$

$NPR = 15.0 \Rightarrow$ OVEREXPANDED FLOW



SHOCK PRESSURE RATIO: $PR = NPR_{sc} / NPR$

$$\left.\begin{array}{ll} (4.9) & PR = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \\ (4.7) & M_{n_1} = M_{e_{sc}} \sin\beta \end{array}\right\} \Rightarrow \beta = 29.2^\circ$$

$(\theta - \beta - M) \Rightarrow$

$(M = M_{e_{sc}}, \beta) \Rightarrow \theta = 11.5^\circ$

FLOW DEFLECTION $\theta = 11.5^\circ$ (TOWARDS THE CENTERLINE.)

P3 (MOVING SHOCK WAVE)

AN EXPLOSION GENERATES A MOVING SHOCK WAVE BEHIND WHICH THE INDUCED FLOW REACHES A VELOCITY OF 180 m/s
THE AIR AHEAD OF THE SHOCK: ($u=0$, $P=101325$ Pa, $T=288$ K)

a) CALCULATE THE VELOCITY OF THE MOVING SHOCK

(7.16)
$$u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1}-1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1}+\frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$
(WHERE $a_1=\sqrt{\gamma R T_1}$, $T_1=288$)

ITERATE TO FIND (P_2/P_1) $\Rightarrow$

$$\frac{P_2}{P_1} = 2.01$$

(7.13)
$$M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1}-1\right)+1} = 1.37$$

$$W = M_s\, a_1 = M_s\sqrt{\gamma R T_1} = \{T_1=288\} = \underline{464.9 \text{ m/s}}$$

b) CALCULATE THE MACH NUMBER OF THE INDUCED FLOW

(7.10)
$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\left(\frac{\frac{\gamma+1}{\gamma-1}+\frac{P_2}{P_1}}{1+\frac{\gamma+1}{\gamma-1}\left(\frac{P_2}{P_1}\right)}\right) \Rightarrow T_2 = 355.2\text{K}$$

$$M_2 = u_2/a_2 = u_p/\sqrt{\gamma R T_2} = \underline{0.48}$$

C) CALCULATE THE TOTAL TEMPERATURE BEHIND THE SHOCK WAVE

(3.28) $\frac{T_{02}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2 \Rightarrow \underline{T_{02} = 371{,}3\ K}$

P4 (ISENTROPIC FLOW COMPRESSION PROCESS)

a) A GRADUAL (ISENTROPIC) COMPRESSION TURNS THE FLOW 20°.

UPSTREAM FLOW CONDITION:

$M_1 = 3.0$
$P_1 = 1.0$ bar $(= 100.0$ kPa$)$
$T_1 = 300$ K

CALCULATE THE DOWNSTREAM MACH NUMBER, TEMPERATURE, AND PRESSURE.

$$(4.44) \quad \nu(M_1) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_1^2-1)} - \tan^{-1}\sqrt{M_1^2-1}$$

$$\Rightarrow \nu(M_1) = 49.76°$$

THE COMPRESSION TURNS THE FLOW 20° $\Rightarrow$

$$\nu(M_2) = 29.76° \quad (\nu(M_2) = \nu(M_1) - 20°)$$

$$(4.44) \Rightarrow \underline{M_2 = 2.12}$$

THE COMPRESSION IS ISENTROPIC $\Rightarrow$ T_0, P_0 ARE NOT AFFECTED BY THE COMPRESSION

$$(3.28) \quad \frac{T_1}{T_2} = \frac{1 + \frac{(\gamma-1)}{2} M_2^2}{1 + \frac{(\gamma-1)}{2} M_1^2} \Rightarrow \underline{T_2 = 441.4 \text{ K}}$$

$$(3.30) \quad \frac{P_1}{P_2} = \left(\frac{1 + \frac{(\gamma-1)}{2} M_2^2}{1 + \frac{(\gamma-1)}{2} M_1^2}\right)^{\gamma/(\gamma-1)} \Rightarrow \underline{P_2 = 386.4 \text{ kPa}}$$

b) EVALUATE THE RELATIVE TOTAL PRESSURE LOSS FOR AN OBLIQUE SHOCK THAT GIVES A 5° FLOW DEFLECTION

$$\left.\begin{matrix} M_1 = 3.0 \\ \epsilon = 5.0° \end{matrix}\right\} \Rightarrow (\epsilon - \beta - M) \Rightarrow \beta = 23.1°$$

$$\left.\begin{array}{ll} (4.7) & M_{n1} = M_1 \sin\beta \\ (4.9) & \dfrac{P_2}{P_1} = 1 + \dfrac{2\gamma}{\gamma+1}(M_{n1}^2 - 1) \\ (4.10) & M_{n2}^2 = \dfrac{M_{n1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n1}^2 - 1} \\ (4.12) & M_2 = M_{n2}/\sin(\beta-\epsilon) \end{array}\right\} \Rightarrow \begin{array}{l} M_2 = 2.75 \\ P_2 = 145.4 \text{ kPa} \end{array}$$

$$(3.30) \qquad \frac{P_{01}}{P_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}$$

RELATIVE TOTAL PRESSURE LOSS

$$\frac{P_{01} - P_{02}}{P_{01}} = 0.53\,\%$$ (VERY CLOSE TO THE SUGGESTED 0.5%)