TME085 - Compressible Flow

2022-08-17, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Graph drawing calculator with cleared memory

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

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Solutions for the problems will be published in Canvas after the exam

Results available no later than 2022-09-09

Good luck!

Part I - Theory Questions (20 p.)

T1. (2.0 p.) Shocks:

(a) (0.5 p.) The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the **weak** type or the **strong** type. What is the main difference between these two shock types and which type is usually seen in reality?

(b) (0.5 p.) Are the normal-shock relations mathematically and physically valid for upstream Mach numbers lower than one? Justify your answer.

(c) (0.5 p.) Describe what happens when a moving normal shock hits a solid wall.

(d) (0.5 p.) In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

T2. (2.0 p.) Derive the special formula for the **speed of sound** a for a **calorically perfect** gas:

$$a = \sqrt{\gamma R T}$$

starting from the general formula:

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

T3. (1.0 p.) What is the physical interpretation of each of the terms in the **momentum equation** on integral form

$$\frac{d}{dt}\iiint_{\Omega} \rho \mathbf{v} d\mathscr{V} + \oiint_{\partial\Omega} \left[\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}\right] dS = \iiint_{\Omega} \rho \mathbf{f} d\mathscr{V}$$

T4. (3.0 p.) Gas models

(a) (0.5 p.) What do we mean by **thermally perfect gas** and **calorically perfect gas** respectively?

(b) (0.5 p.) In what temperature range (approximately) can a gas be assumed to be **calorically perfect**?

(c) (1.0 p.) A mixture of chemically reacting perfect gases, where the reactions are always in **equilibrium**, may be thermodynamically described as a single-species gas. How does this thermodynamic description differ from that of a calorically perfect or thermally perfect gas?

(d) (1.0 p.) Using **equilibrium gas** assumption in the analysis of chemically reacting nozzle flow will lead to higher exhaust temperatures than if calorically perfect gas assumption is used for the same analysis. Explain why.

T5. (1.0 p.) What does Crocco's relation say about the flow behind a curved shock?

T6. (2.0 p.) Unsteady waves:

(a) (1.0 p.) What types of waves or discontinuities are generated in a shock tube with two initially stagnant regions at different pressure (separated by a thin membrane which is removed very quickly)?

(b) (1.0 p.) What is the difference between acoustic waves and other types of waves such as shock waves and expansion waves?

T7. (2.0 p.) Compressible CFD:

(a) (0.5 p.) What is meant by the term **density-base** when discussing CFD codes for compressible flow?

(b) (0.5 p.) What is meant by the term **fully coupled** when discussing CFD codes for compressible flow?

(c) (1.0 p.) What do we mean when we say that a CFD code for compressible flow is **conservative**?

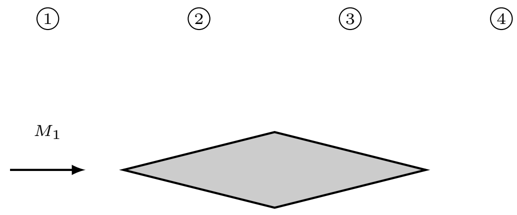
T8. (1.0 p.) What is the **Fanno** curve and what does it tell us?

T9. (2.0 p.) For a steady-state **isentropic** flow of a **calorically perfect** gas, derive the formula for T_0/T, making use of the fact that the total enthalpy h_0 is constant along the streamlines.

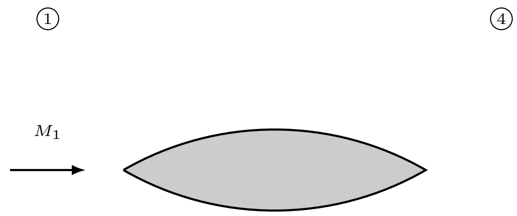
$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

T10. (2.0 p.) Derive the **isentropic relations** for **calorically perfect** gases starting from the **entropy equation**.

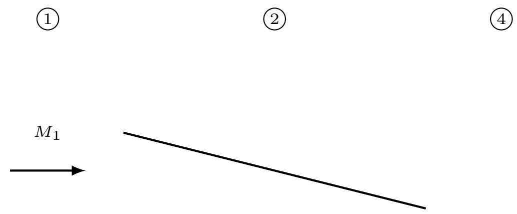
T11. (2.0 p.) For each of the cases a-c below, make a schematic representation showing how the pressure varies from station 1 to 4 (following the flow over the upper side of each "wing"). Any sudden changes in pressure has to be motivated.



(a)
symmetric diamond-wedge airfoil (zero angle of attack)



(b)
double circular arc airfoil (zero angle of attack)



(c)
flat plate at an angle of attack

Part II - Problems (40 p.)

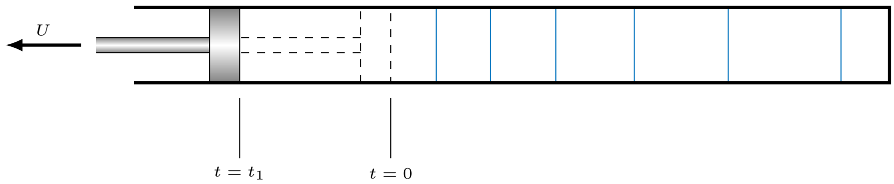
Problem 1 - PISTON (10 p.)

An expansion wave generated by the withdrawal of a piston in a tube travels into still air with the temperature $23°C$.

(a) Calculate the propagation speed of the head and tail of the expansion wave if the piston speed is 275 m/s

(b) At what piston speed will the wave start to move to the left in the tube?

discard transient start up effects, i.e., assume that the piston starts to move at the specified velocity directly from standing still.



Problem 2 - RAMJET ENGINE COMPRESSION (10 p.)

A ramjet engine is a form of jet engine that uses the forward motion of the engine itself to produce thrust. This make the operation of such an engine rather special since it cannot be started from standing still conditions. The engine does not include any moving parts such as compressors or turbines instead the pressure is raised by the shape of the engine intake. Assume that a ramjet engine is flying at Mach 2.0 and as a first compression stage the pressure of the air that enters the engine is increased by a factor of 2.3 by letting it deflect over an intake cone.

(a) Find the flow deflection angle needed to accomplish a pressure increase of 2.3 if the flight Mach number is 2.0

(b) Calculate the Mach number downstream of the oblique shock formed at the inlet of the engine, i.e., behind the oblique shock caused by the flow deflection.

(c) After the initial oblique shock, a normal shock is formed before the air enters the combustion chamber. Calculate the total pressure increase through the engine inlet

Problem 3 - RAMJET ENGINE COMBUSTION (10 p.)

A ramjet engine flies at Mach 4.0 in an atmosphere at 222 K. At the entrance to the engine combustion chamber, the Mach number of the flow has decreased through a series of shocks to 0.3. Combustion in the burner may be represented approximately as heating a perfect gas with a constant ratio of specific heats of 1.4. At the exit of the burner, the stagnation temperature of the gas is 2460 K.

(a) Assuming the burner to be a constant area duct and neglecting frictional effects, determine the Mach number of the gas leaving the burner

(b) Calculate the stagnation pressure loss due to heating

Problem 4 - NOZZLE FLOW (10 p.)

Compressed air at $40°C$ and 145 kPa from a large tank is discharged to ambient atmosphere through a convergent nozzle with the exit area 10 cm^2. Calculate the mass flow rate through the nozzle and the nozzle exit condition (Mach number, density, pressure, temperature and flow velocity) if the ambient atmosphere downstream of the nozzle exit is at

(a) 101 kPa

(b) 50 kPa

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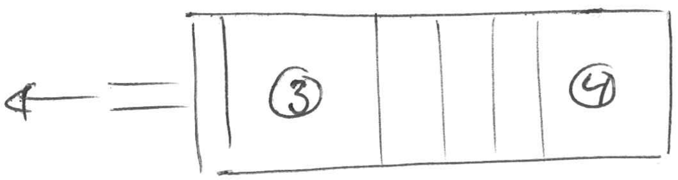
P1 (PISTON)

AN EXPANSION WAVE GENERATED BY THE WITHDRAWAL OF A PISTON IN A TUBE TRAVELS INTO STILL AIR WITH THE TEMPERATURE 23°C

a) CALCULATE THE PROPAGATION SPEED OF THE HEAD AND TAIL OF THE EXPANSION IF THE PISTON SPEED IS 275 m/s

THE AIR IN CONTACT WITH THE PISTON MUST MOVE TO THE LEFT WITH THE PISTON VELOCITY



$T_4 = 23°C$

$a_4 = \sqrt{\gamma R T_4} = 344.9$ m/s

$u_3 = u_{piston}$

(7.89) $\frac{a_3}{a_4} = 1 - \frac{\gamma - 1}{2}\left(\frac{u_3}{a_4}\right) \Rightarrow a_3 = 289.9$ m/s

$u_{head} = a_4 = 344.9$ m/s

$u_{tail} = a_3 - u_3 = 14.9$ m/s

b) AT WHAT PISTON SPEED WILL THE TAIL OF THE EXPANSION START TO MOVE TO THE LEFT?

THE LIMIT FOR RIGHT-TRAVELING TAIL IS WHEN

$u_3 = a_3 \Rightarrow u_{tail} = 0.$

(7.84)

$$\frac{a_3}{a_4} = 1 - \frac{\gamma - 1}{2}\left(\frac{u_3}{a_4}\right)$$

$$a_3 = u_3 \Rightarrow \quad \frac{u_3}{a_4} = 1 - \frac{\gamma - 1}{2}\left(\frac{u_3}{a_4}\right)$$

$$\Rightarrow \frac{u_3}{a_4}\left(1 + \frac{\gamma - 1}{2}\right) = 1 \Rightarrow$$

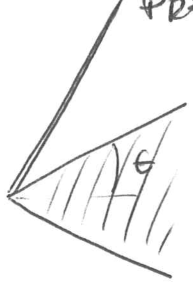
$$\frac{u_3}{a_4}\left(\frac{\gamma + 1}{2}\right) = 1 \Rightarrow u_3 = a_4\left(\frac{2}{\gamma + 1}\right)$$

$$\Rightarrow u_3 = 287.4 \text{ m/s}$$

P2 (RAMJET ENGINE COMPRESSION)

a) ~~FACTOR~~ FIND THE FLOW DEFLECTION NEEDED TO INCREASE THE PRESSURE BY A FACTOR OF 2.3

$M_1 = 2.0$



$$\frac{p_2}{p_1} = 2.3$$

(4.9) $$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)$$

(4.7) $$M_{n_1} = M_1 \sin\beta$$

$\Rightarrow \beta = 46.6°$

($\theta-\beta-M$, WITH $\beta = 46.6$ AND $M_1 = 2.0$) $\Rightarrow \theta = 15.9°$

b) CALCULATE THE MACH NUMBER BEHIND THE OBLIQUE SHOCK.

(4.10) $$M_{n2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$$

(4.12) $$M_2 = M_{n2} / \sin(\beta - \theta)$$

$\Rightarrow M_2 = 1.4$

c) AFTER THE OBLIQUE SHOCK THERE WILL BE A NORMAL SHOCK BEFORE ENTERING THE COMBUSTION CHAMBER

CALCULATE THE PRESSURE RATIO OVER THE INLET.

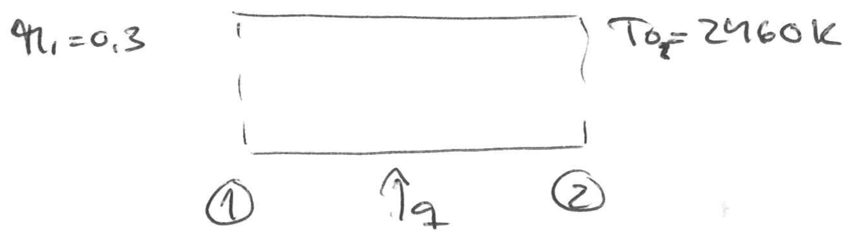
(3.57) $$\frac{p_3}{p_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_2^2 - 1\right) = 2.14$$

$$\frac{p_3}{p_1} = \frac{p_3}{p_2}\frac{p_2}{p_1} = 2.14 \times 2.3 = 4.92$$

P3 (RAMJET ENGINE COMBUSTION)

A RAMJET FLIES AT MACH = 4.0 @ $T_\infty = 222$ K
THE MACH NUMBER IS REDUCED TO M = 0.3 IN THE INLET
COMBUSTION APPROXIMATED AS HEATING OF A GAS
WITH CONSTANT $\gamma = 1.4$ (CALORICALLY PERFECT)
AT THE EXIT OF THE COMBUSTOR $T_{0_{exit}} = 2460$K

a) ASSUMING THAT THE BURNER IS A CONSTANT AREA DUCT AND NEGLECTING FRICTION, DETERMINE THE MACH NUMBER AT THE EXIT.



THE SHOCKS AT THE INLET DO NOT AFFECT THE TOTAL TEMPERATURE $\Rightarrow T_{0_1} = T_{0_\infty}$

$$T_{0_1} = T_\infty\left(1 + \frac{\gamma - 1}{2} M_\infty^2\right) \qquad (3.28)$$

$$\Rightarrow T_{01} = 932,4 \text{ K}$$

(3.89)

$$\frac{T_{01}}{T_0^*} = \frac{(\gamma+1)M_1^2}{(1+\gamma M_1^2)^2}\left(2 + (\gamma - 1)M_1^2\right)$$

$$\frac{T_{02}}{T_0^*} = \frac{(\gamma+1)M_2^2}{(1+\gamma M_2^2)^2}\left(2 + (\gamma - 1)M_2^2\right)$$

T_0^* IS CONSTANT

$\Rightarrow M_2 = 0,71$

b) CALCULATE THE TOTAL PRESSURE LOSS DUE TO HEATING

(3.88)

$$\frac{P_{01}}{P_0^*} = \left(\frac{1+\gamma}{1+\gamma M_1^2}\right)\left(\frac{2+(\gamma-1)M_1^2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$$

$$\frac{P_{02}}{P_0^*} = \left(\frac{1+\gamma}{1+\gamma M_2^2}\right)\left(\frac{2+(\gamma-1)M_2^2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$$

LOSS OF TOTAL PRESSURE $\Delta P_0 = P_{01} - P_{02}$

$$\frac{\Delta P_0}{P_{01}} = 1 - \frac{P_{02}}{P_{01}} = 1 - \frac{P_{02}}{P_0^*}\frac{P_0^*}{P_{01}} = 13.2\%$$

P4 (NOZZLE FLOW)

COMPRESSED AIR @ T=40°C AND P= 145 kPa FROM A LARGE TANK IS DISCHARGED TO AMBIENT ATMOSPHERE THROUGH A CONVERGENT NOZZLE WITH EXIT AREA 10.0 cm²

CALCULATE MASSFLOW AND EXIT CONDITIONS IF THE BACK PRESSURE IS

a) 101 kPa

b) 50 kPa

a)

CHECK CRITICAL PRESSURE (P^*)

(3,35) $$\frac{p^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$$

$$\frac{P_0}{p^*} = 1,893$$

$$\frac{P_0}{P_b} = \frac{145}{101} = 1,44 < \frac{P_0}{p^*}$$ => SUBSONIC FLOW NOT CHOKED..

(3,30) $$\frac{P_0}{P_b} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/(\gamma-1)}$$ => $M_e = 0,74$

(3,28) $$\frac{T_0}{T_e} = \left(1 + \frac{(\gamma-1)}{2} M_e^2\right)$$ => $T_e = 282.3\,K$

$$\rho_e = \frac{P_e}{RT_e} = 1,25\ kg/m^3$$

$$u_e = M_e a_e = M_e \sqrt{\gamma R T_e} = 248.4\ m/s$$

$$\dot{m} = \rho_e u_e A_e = 0,31\ kg/s$$

b)

$$\frac{p_0}{p_b} = \frac{145}{50} = 2.9 > \frac{p_0}{p^*} \Rightarrow \text{choked}$$

$P_e = p^* = 76.6\,kPa$

$T_e = T^* = 260.8\,K$ $\quad (T^*/T_0 = 2/(\gamma+1))$

$\rho_e = \frac{p_e}{RT_e} = 1.023\ kg/m^3$

$M_e = 1.0$

$u_e = M_e a_e = a_e = \sqrt{\gamma R T_e} = 323.7\ m/s$

$\dot{m} = \rho_e u_e A_e = 0.33\ kg/s$