TME085 - Compressible Flow

2022-06-08, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

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Solutions for the problems will be published in Canvas after the exam

Results available no later than 2022-06-29

Good luck!

Part I - Theory Questions (20 p.)

T1. (1p.) What are the criteria for the classifications **subsonic/transsonic/supersonic/hypersonic** flow?

T2. (2p.) Gas model:

(a) When can air be regarded as a **calorically perfect gas**?

(b) What is the difference between a **calorically perfect** gas and a **thermally perfect** gas?

(c) What happens with the molecules in air at approximately 2500K, 4000K, and 9000K?

T3. (2p.) What is the physical interpretation of each of the terms in the **energy equation** on integral form

$$\frac{d}{dt}\iiint\limits_{\Omega} \rho e_o d\mathscr{V} + \oiint\limits_{\partial\Omega} \left[\rho e_o(\mathbf{v}\cdot\mathbf{n}) + p\mathbf{v}\cdot\mathbf{n}\right] dS = \iiint\limits_{\Omega} \rho\mathbf{f}\cdot\mathbf{v} d\mathscr{V}$$

T4. (1p.) For a steady-state **adiabatic** compressible flow of **calorically perfect** gas, which of the variables p_o (total pressure) and T_o (total temperature) is/are constant along streamlines? Why?

T5. (2p.) Draw a schematic **pressure-deflection diagram** and explain how it is obtained.

T6. (2p.) Describe how you can use the **Prandtl-Meyer function** to compute the change in Mach number due to a given flow deflection.

T7. (2p.) Nozzle flow:

(a) What is meant by **choked** flow in a **converging-diverging** nozzle?

(b) Explain the consequence of **free-boundary reflection** for the external flow of a nozzle operating at **overexpanded** conditions.

T8. (2p.) Moving shocks

(a) Describe what happens when a moving normal shock hits a solid wall.

(b) Can a moving normal shock travel at a speed lower than the speed of sound? Explain why/why not.

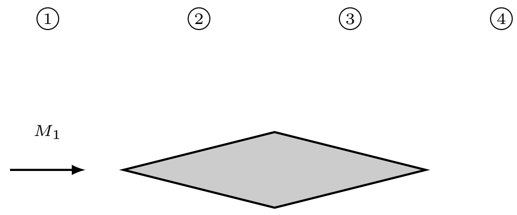
T9. (2p.) Computational Fluid Dynamics for compressible flows:

(a) When the governing equations are discretized using a **finite-volume** approach, cell face values of flow properties appears in the equations. How are these values approximated?

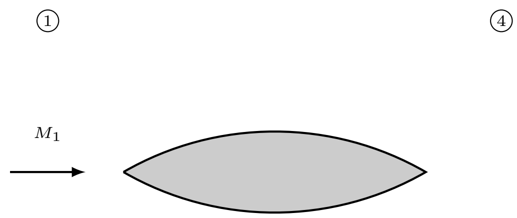
(b) What is a typical maximum **CFL number** for stable operation when applying an **explicit time stepping scheme**?

T10. (2p.) How does a **mono-atomic** gas differ from a diatomic gas in terms of energy modes?

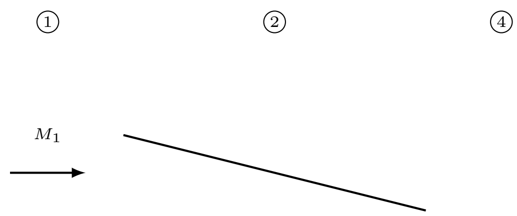
T11. (2p.) For each of the cases a-c below, make a schematic representation showing how the pressure varies from station 1 to 4 (following the flow over the upper side of each ”wing”). Any sudden changes in pressure has to be motivated.



(a)
symmetric diamond-wedge airfoil (zero angle of attack)



(b)
double circular arc airfoil (zero angle of attack)



(c)
flat plate at an angle of attack

Part II - Problems (40 p.)

Problem 1 - TANK (10 p.)

A storage tank contains air at 1000 kPa and 310K. The storage tank is evacuated and air is flowing out from the tank through a convergent nozzle with the exit area 3.0 cm^2 , to another tank. Determine the pressure in the downstream tank if the mass flow rate through the nozzle is 0.15 kg/s.

Problem 2 - PIPE FLOW WITH FRICTION (10 p.)

Atmospheric air at 101.325 kPa and 300K is drawn through a frictionless bell-mouth entrance into a 3.0 m long tube with a diameter of 0.05 m. The average friction coefficient for the tube can be assumed to be $\bar{f} = 0.005$ (as usual). The system is perfectly insulated.

(a) Calculate the maximum mass flow rate

(b) What range of back pressures (pressure downstream of the tube exit) will produce the maximum mass flow rate calculated in the previous task?

(c) Calculate the exit pressure required to produce a mass flow rate that is 90% of the maximum mass flow rate

(d) Calculate the total pressure and velocity at the exit when the mass flow rate is 90% of the maximum mass flow rate

Problem 3 - SHOCK TUBE (10 p.)

A shock tube is set up such that the pressure in the driver section is 10.35 times higher than the pressure in the driven section before the copper membrane separating the two chambers is removed and the shock tube is started. Calculate the pressure ratio over the incident shock (the shock generated as the membrane is removed, traveling into the driven section) and the Mach number of this moving shock wave (M_s) if the tail of the expansion region is standing still inside the shock tube.

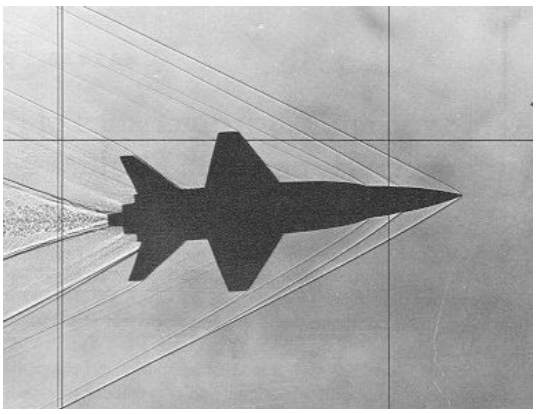
Problem 4: X-15 (10 p.)

> *"At the time the X-15 was designed, theory and empirical data (much of it from previous research airplanes) provided a good understanding of the mechanics of airflow for speeds to about Mach 3. But there were major gaps in aerodynamic knowledge above this speed. Some of these gaps were bridged by wind-tunnel tests of scale models of the X-15. However, although models of the X-15 were tested in many supersonic and hypersonic wind tunnels, they were of very small scale - 1/15 or 1/50 - and no verification had been made of the results from small-scale models for flight at hypersonic speeds. Moreover, wind tunnels approximate flow conditions rather than exactly duplicating them. Hence, a valuable part of the X-15 program would be to verify or modify the picture of hypersonic flow derived from these experimental techniques and from theoretical analyses."*
>
> https://history.nasa.gov

The Picture below is from a test of a free-flight model of an X-15 airplane that is being fired at high speed into a wind tunnel.

(a) Estimate the flight Mach number using any information about the flow that you may find in the figure

(b) If one would measure the temperature and pressure at the surface of the nose cone (behind the first oblique shock) of the X-15 model, what would the measured temperature and pressure be if the air inside the wind tunnel has the temperature 293K and the pressure 101.325 kPa



TME085 EXAM 22-06-08

P1 (TANK)

A STORAGE TANK CONTAINS AIR @ 1.0MPa AND 310.0 K
THE TANK IS EVACUATED AND THE AIR FLOWS THROUGH A CONVERGENT NOZZLE WITH THE EXIT AREA $A_e = 3.0\,cm^2$ INTO ANOTHER TANK

THE MASS FLOW IS $\dot{m} = 0{,}15\,kg/s$

DETERMINE THE PRESSURE IN THE DOWNSTREAM TANK.

CHECK IF THE NOZZLE IS CHOKED:

(5.21) $$\dot{m}_{choked} = \frac{P_o A_e}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$= 0{,}69 > 0{,}15$$

$\Rightarrow$ THE FLOW IS NOT CHOKED.

WE WILL NEED TO ITERATE TO FIND THE EXIT FLOW CONDITION..

Guess M_e.

(3.28) $$\frac{T_o}{T_e} = 1 + \frac{\gamma-1}{2} M_e^2 \Rightarrow T_e = f(T_o, M_e)$$

(3.30) $$\frac{P_o}{P_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_e = f(P_o, M_e)$$

(1.9) $$\rho_e = P_e/(RT_e) \Rightarrow \rho_e = f(T_o, P_o, M_e)$$

$$M_e = u_e/a_e = u_e/\sqrt{\gamma R T_e} \Rightarrow u_e = f(T_o, M_e)$$

$$\dot{m} = u_e A_e \rho_e$$

$\Rightarrow \dot{m}$

update M_e, iterate until converged..

Iteration gives $\dot{m} = 0.15$ for $M_e = 0.1273$.

$\Rightarrow$ $P_e \simeq 988.7$ kPa

ALTERNATIVE SOLUTION (EASIER)

$$\dot{m}_{choked} \frac{P_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$A/A^* = \dot{m}_{choked}/\dot{m}$

$\Rightarrow$ Area-Mach-relation $\Rightarrow$ $M_e = 0.127$

$P_e = f(M_e, P_0) = 988.7$ kPa

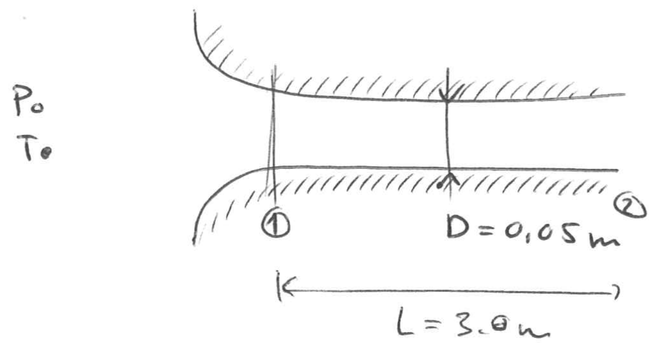
CONTINUITY $\Rightarrow$ $\rho u A = \text{const} \Rightarrow \rho u A = \rho^* a^* A^*$

$$\Rightarrow \frac{A}{A^*} = \frac{\rho^* a^*}{\rho u}$$

IN OUR CASE:

$$\left.\begin{aligned} \dot{m}_{choked} &= \rho^* a^* A_t \\ \dot{m} &= \rho u A_t \end{aligned}\right\} \Rightarrow \frac{\dot{m}_{choked}}{\dot{m}} = \frac{\rho^* a^*}{\rho u} = \frac{A}{A^*}$$

P2 (PIPE FLOW WITH FRICTION)



$P_0 = 101,325$ kPa

$T_0 = 300.0$ K

$\bar{f} = 0.005$

a) CALCULATE MAX MASS FLOW:

$\dot{m}_{max}$ WHEN THE PIPE EXIT IS CHOKED $\Rightarrow L_1^* = L$

(3.107) $$\frac{4\bar{f}L^*}{D} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

$$\Rightarrow M_1 = 0,48$$

(3.28) $$\frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2 \quad \Rightarrow T_1 = 286.5\,\text{K}$$

(3.30) $$\frac{P_0}{P_1} = \left(1 - \frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \quad \Rightarrow P_1 = 86.3\,\text{kPa}$$

$$\dot{m} = u_1 \rho_1 A_1 = M_1 a_1 \frac{P_1}{RT_1} A_1 = M_1 \sqrt{\gamma R T_1}\,\frac{P_1}{RT_1} A_1$$
$$= M_1 \frac{P_1 A_1}{\sqrt{T_1}}\sqrt{\frac{\gamma}{R}} = 0,34\ \text{kg/s}$$

b) WHAT RANGE OF BACK PRESSURES WILL PRODUCE THE MAXIMUM MASS FLOW CALCULATED IN a)

THE PRESSURE DOWNSTREAM OF THE TUBE SHOULD

(3.85) $$\frac{P_1}{P^*} = \left(\frac{1+\gamma}{2+(\gamma-1)M_1^2}\right)^{1/2}\frac{1}{M}$$

$$\Rightarrow P^* = 39,08\ \text{kPa} \quad \Rightarrow \quad P_b \le 39.08\ \text{kPa}$$

c) CALCULATE THE EXIT PRESSURE REQUIRED TO PRODUCE A MASSFLOW THAT IS 90% OF THE MAXIMUM MASSFLOW

$$\dot{m} = \rho_1 u_1 A$$

$$\left.\begin{aligned} u_1 &= M_1 a_1 = M_1\sqrt{\gamma R T_1} \\ (3.28) \quad T_1 &= T_0 \Big/ \left(1+\frac{\gamma-1}{2}M_1^2\right) \end{aligned}\right\} \Rightarrow u_1 = \frac{M_1\sqrt{\gamma R T_0}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{1/2}}$$

$$\left.\begin{aligned} \rho_1 &= \frac{P_1}{R T_1} \\ (3.30) \quad P_1 &= P_0 \Big/ \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \\ (3.28) \quad T_1 &= T_0 \Big/ \left(1+\frac{\gamma-1}{2}M_1^2\right) \end{aligned}\right\} \Rightarrow \rho_1 = \frac{P_0}{R T_0}\left(1+\frac{\gamma-1}{2}M_1^2\right)^{-1/(\gamma-1)}$$

$$\Rightarrow \dot{m} = \frac{P_0 A M_1}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(1+\frac{\gamma-1}{2}M_1^2\right)^{-(\gamma+1)/(\gamma-1)}}$$

$$\dot{m} = 0.90\,\dot{m}_{choked}$$

Iterate to find $M_1 = 0.42$

$$(3.28) \quad \frac{T_0}{T_1} = 1+\frac{\gamma-1}{2}M_1^2 \Rightarrow T_1 = 289.7\,K$$

$$(3.30) \quad \frac{P_0}{P_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_1 = 89.6\,kPa$$

$$M_1 = 0.42 \Rightarrow L_1^* = 4.85\,m$$

$$(3.107) \quad \frac{4\bar{f}L_1^*}{D} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

(3.104) $$\frac{p_1}{p^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2} \Rightarrow p^* = f(p_1, M_1)$$

& AT THE PIPE EXIT $L_2^* = L_1^* - L = 1,84\ m$

(3.107) $$\frac{4\bar{f}L_2^*}{D} = f(M_2) \Rightarrow M_2 = 0,548$$

(3.104) $$\frac{p_2}{p^*} = \frac{1}{M_2}\left(\frac{\gamma+1}{2+(\gamma-1)M_2^2}\right)^{1/2} \Rightarrow \underline{p_2 = 68,28\ kPa}$$

d) CALCULATE TOTAL PRESSURE AND FLOW VELOCITY AT THE EXIT WHEN $\dot{m} = 0.9\,\dot{m}_{choked}$

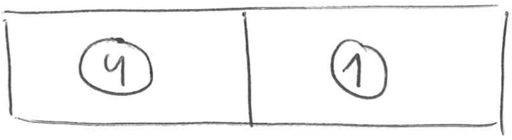
(3.103) $$\frac{T_1}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_1^2} \Rightarrow T^* = f(M_1, T_1)$$

$$\frac{T_2}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_2^2} \Rightarrow T_2 = 283,0\ K$$

$$u_2 = M_2 a_2 = M_2\sqrt{\gamma R T_2} = 184.8\ m/s$$

(3.30) $$p_{02} = p_2\left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)} \Rightarrow p_{02} = 83,75\ kPa$$

P3 (SHOCK TUBE)



$p_4 / p_1 = 10.35$

THE TAIL OF THE EXPANSION REGION STANDS STILL IN THE TUBE $\Rightarrow u_p = a_3$

THE INDUCED FLOW VELOCITY EQUALS THE SPEED OF SOUND IN THE EXPANSION TAIL REGION..

$$u_{tail} = -a_3 + u_p = 0 \Rightarrow u_p = a_3$$

(7.84)

$$\frac{a_3}{a_4} = 1 - \frac{\gamma - 1}{2}\left(\frac{u_3}{a_4}\right)$$

$$\frac{u_p}{a_4} = 1 - \frac{\gamma - 1}{2}\left(\frac{u_p}{a_4}\right)$$

$$\Rightarrow \frac{u_p}{a_4} = \frac{\gamma + 1}{2} \qquad (1)$$

(7.86)

$$\frac{p_3}{p_4} = \left(1 - \frac{\gamma - 1}{2}\left(\frac{u_3}{a_4}\right)\right)^{2\gamma/(\gamma-1)}$$

$$p_3 = p_2 \,, \quad u_p = u_3 \,, \quad \frac{u_p}{a_4} = \frac{\gamma+1}{2} \quad \Rightarrow$$

$$\frac{p_2}{p_4} = \left(1 - \left(\frac{\gamma-1}{2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{2\gamma/(\gamma-1)}$$

$$\frac{p_2}{p_1} = \frac{p_2}{p_4}\,\frac{p_4}{p_1} = \underline{2.89}$$

$$(7.13) \quad M_s = \sqrt{\frac{\gamma+1}{2}\left(\frac{p_2}{p_1} - 1\right) + 1} \quad \Rightarrow M_s = \underline{1.62}$$

P4 (X-15)

AN ESTIMATE OF THE FLOW DEFLECTION ANGLE (θ) AND SHOCK ANGLE (β) OF THE FIRST SHOCK IS OBTAINED BY MEASURING IN THE PROVIDED FIGURE (A WIDE RANGE OF RESULTS ARE ACCEPTED HERE...)

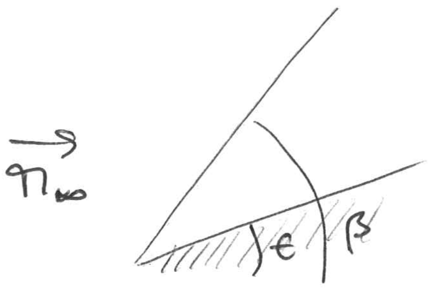
a) $\theta = 16°$

$\beta = 30°$

from the θ-β-M-relation $\Rightarrow M_\infty = 3.55$

b) $T_\infty = 293\,K$

$p_\infty = 101.325\,kPa$



(9.7) $M_{n_1} = M_1 \sin\beta$

(4.9) $$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)$$

(4.11) $$\frac{T_2}{T_1} = \frac{p_2}{p_1}\,\frac{(\gamma+1)M_{n_1}^2}{(\gamma-1)M_{n_1}^2 + 2}$$

$\Rightarrow$ $p_2 = 354.6\,kPa$, $T_2 = 442.8\,K$