TME085 - Compressible Flow

2022-03-17, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

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Solutions for the problems will be published in Canvas after the exam

Results available no later than 2022-04-08

Good luck!

Part I - Theory Questions (20 p.)

T1. (2 p.)

(a) For a given flow, how can we determine if compressibility effects are important?

(b) Use the Bernoulli equation (even though it's not valid for compressible flows) to obtain an estimate of the Mach number for which compressible effects becomes significant and must be considered.

(c) What are the criteria for the classifications subsonic/transsonic/supersonic/hypersonic flow?

T2. (1 p.)

(a) What defines a reversible process?

(b) What defines an adiabatic process?

T3. (1p.) What is the physical interpretation of each of the terms in the momentum equation on integral form

$$\frac{d}{dt}\iiint_{\Omega} \rho \mathbf{v} d\mathscr{V} + \oiint_{\partial\Omega} [\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p\mathbf{n}]\, dS = \iiint_{\Omega} \rho \mathbf{f} d\mathscr{V}$$

T4. (2p.)

(a) What is the general definition (valid for any gas) of the total conditions p_o, T_o, ρ_o,... etc at some location in the flow?

(b) What is the general definition (valid for any gas) of the critical/sonic conditions p^*, T^*, ρ^*,... etc at some location in the flow?

T5. (2p.) Are the normal-shock relations mathematically and physically valid for upstream Mach numbers lower than one? Justify your answer.

T6. (2 p.)

(a) In one-dimensional flow with heat addition, what is q^*?

(b) What happens in the flow when heat is added if the flow is initially supersonic and subsonic, respectively

(c) Describe how adding and/or removing heat from a one-dimensional flow in theory could be used to resemble the flow in a convergent-divergent nozzle

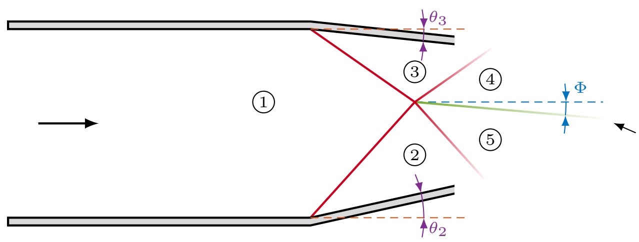
T7. (2 p.)

(a) How does the absolute Mach number change after a weak/strong stationary oblique shock?

(b) What happens if there is no possible solution? What is the reason for the no-solution situation?

(c) What kind of shock is obtained for a blunt body in supersonic flow?

T8. (2p.) What are the constrains that leads to the generation of the separating line between regions 4 and 5 (represented by a green line in the figure below)? What is the reason for the need for this separating line?



T9. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T10. (2 p.)
Derive the area-velocity relation in quasi-1D flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T11. (2 p.)

(a) When applying a time-marching flow solution scheme the so-called CFL number is an important parameter. Define the CFL number and describe its significance. What is a typical maximum CFL number for an explicit time stepping scheme.

(b) What is meant by the terms "*density-based*" and "*fully coupled*" when discussing CFD codes for compressible flow?

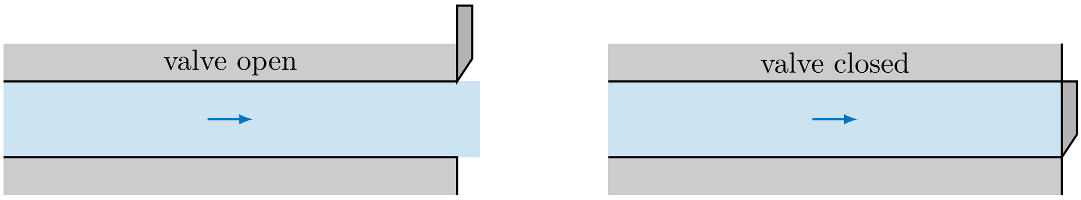
(c) How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - VALVE (10 p.)

Air flows out of a duct at a velocity of 250 m/s with a temperature of 0°C and a pressure of 70 kPa. A valve at the end of the duct is suddenly closed. Find the pressure acting on the valve immediately after it is closed.

Hint: a shock will be generated when the valve is closed



Problem 2 - NOZZLE FLOW (10 p.)

A convergent-divergent nozzle is designed to expand air form a chamber in which the pressure is 800 kPa and the temperature is 40°C to give a Mach number of 2.7 at the nozzle exit. The nozzle throat area is 0.08 m^2.
Calculate:

(a) The nozzle exit area

(b) The design back pressure

(c) The lowest back pressure for which the flow through the entire nozzle is subsonic

(d) The range of back pressures for which the nozzle flow is overexpanded

(e) The range of back pressures for which the nozzle flow is underexpanded

(f) The massflow at design conditions if the flow is pulled through the nozzle (the back pressure is adjusted to control the nozzle flow and the inlet total pressure is fixed at 800 kPa)

(g) The massflow at design conditions if the flow is pushed through the nozzle (the inlet total pressure is adjusted to control the nozzle flow and the back pressure is fixed at 101 kPa)

Problem 3 - WEDGE FLOW (10 p.)

Air at a pressure of 60 kPa and a temperature of -20°C flows over a symmetric wedge. The freestream Mach number is 2.5 and the wedge leading edge angle θ is 8.0 degrees. The need for flow deflection as the flow passes the wedge will lead to generation of shocks. The shock generated on the lower side of the wedge will impinge the wall at the axial coordinate A (measured from the leading edge of the wedge). By deflecting the wall at A, as indicated in Figure 1 below, the shock may be terminated at A. Without the deflection of the wall at A (Figure 2), the shock will be reflected at the wall. The vertical distance between the wall and the leading edge of the wedge (h) is 1.0 m.

(a) Calculate the axial distance from the leading edge of the wedge to A, the point on the wall where the shock impinges (see Figure 1 below)

(b) What wall deflection angle α will lead to termination of the shock at A (see Figure 1 below)

(c) Plot the wall pressure ($y = 0.0$) as a function of axial coordinate from the leading edge of the wedge ($x = 0.0$) to $x = 1.5A$

(d) Plot the pressure along a line $y = h/2$ as a function of axial coordinate from the leading edge of the wedge ($x = 0.0$) to $x = 1.5A$

Note: the figures are just schematic representations

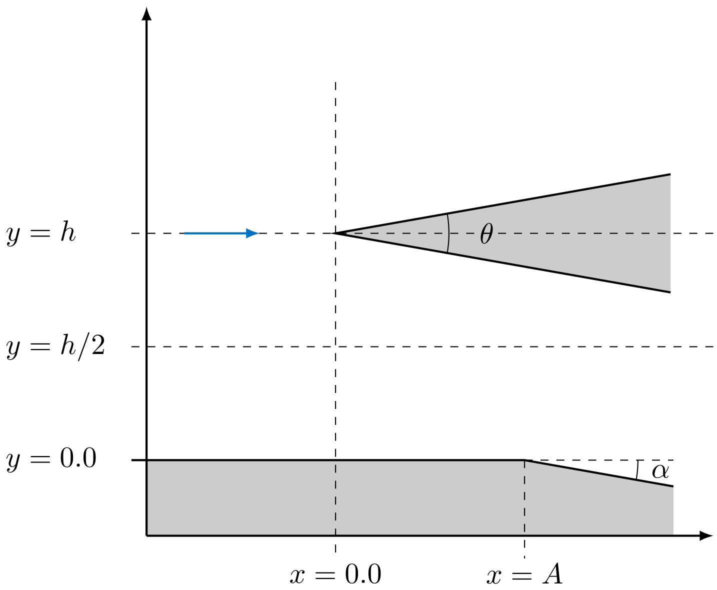


Figure 1

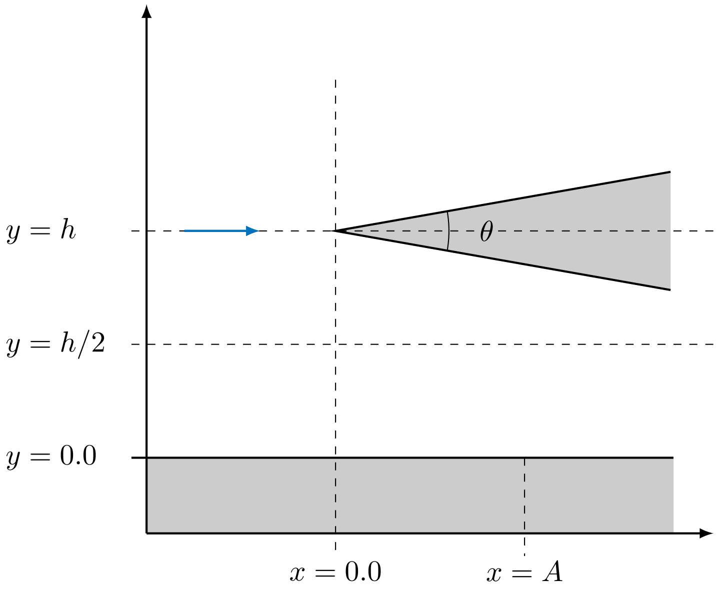


Figure 2

Problem 4 - FLOW WITH FRICTION (10 p.)

Air enters a 10.0 m long stainless-steel pipe at a temperature of 20°C and a pressure of 250 kPa. The massflow through the pipe is 0.12 kg/s. The average friction factor for the pipe is $f = 0.005$. Find the pipe diameter D such that the pressure at the exit of the pipe is 126 kPa.

Hint: you will most likely need to solve this problem iteratively.

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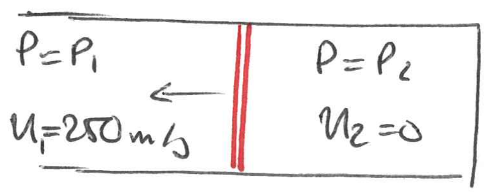
P1 (VALVE)

AIR FLOWS OUT FROM A DUCT AT A VELOCITY OF 250 m/s WITH A TEMPERATURE OF 0°C AND A PRESSURE OF 70 kPa. A VALVE AT THE END OF THE DUCT IS SUDDENLY CLOSED.

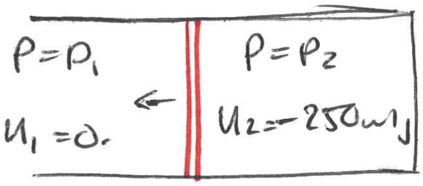
FIND THE PRESSURE ACTING ON THE VALVE IMMEDIATELY AFTER IT IS CLOSED.

CLOSING THE VALVE MEANS THAT THERE WILL BE A SOLID WALL AT THE RIGHT END OF THE TUBE AND THUS THE VELOCITY WILL BE ZERO AT THE RIGHT END

A SHOCK WILL BE FORMED TO FULFIL THE CONTRAINT



IF WE LOOK AT THIS PROBLEM WITH A FRAME OF REFERENCE FOLLOWING THE PIPE FLOW WE GET.



THUS WE CAN USE THE STANDARD RELATION BETWEEN INDUCED FLOW VELOCITY AND SHOCK PRESSURE RATIO

(7.16) $$u_2 = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

$$\Rightarrow \frac{p_2}{p_1} = 2.64 \qquad \Rightarrow \underline{p_2 = 184.7}$$

THE MACH NUMBER OF A SHOCK GENERATING A PRESSURE RATIO p_2/p_1 IS GIVEN BY

(7.13) $$M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \qquad \Rightarrow \underline{M_s = 1.55}$$

P2 (NOZZLE FLOW)

A C-D NOZZLE IS DESIGNED TO EXPAND AIR FROM A CHAMBER IN WHICH THE PRESSURE IS 800 kPa AND THE TEMPERATURE IS 40°C TO GIVE A MACH NUMBER AT THE NOZZLE EXIT OF 2.7

THE NOZZLE THROAT AREA IS 0,08 m²

a) CALCULATE THE EXIT AREA

$M_{e_{sc}} = 2{,}7 \Rightarrow$

$$(5.20) \quad \left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_{e_{sc}}^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_{e_{sc}}^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow \frac{A_e}{A^*} = 3{,}18$$

CHOKED FLOW $\Rightarrow A^* = A_t \Rightarrow A_e = 0{,}25\ m^2$

b) CALCULATE THE DESIGN BACK PRESSURE

ISENTROPIC FLOW $\Rightarrow$

$$(3.30) \quad \frac{p_0}{p_e} = \left(1+\frac{\gamma-1}{2}M_{e_{sc}}^2\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow p_{e_{sc}} = 34{,}36\ kPa \quad (= p_b)$$

c) CALCULATE THE LOWEST BACK PRESSURE FOR WHICH THE FLOW THROUGH THE NOZZLE IS SUBSONIC.

CHOKED, SUBSONIC FLOW:

$$(5.20) \quad \left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_{e_c}}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_{e_c}^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUBSONIC SOLUTION:

$$M_{e_c} = 0{,}19$$

(3.30) $$\frac{P_0}{P_{e_c}} = \left(1 + \frac{\gamma - 1}{2} M_{e_c}^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_{e_c} = P_b = 781.0 \text{ kPa}$$

d) CALCULATE THE RANGE OF BACK PRESSURES FOR WHICH THE FLOW IS OVEREXPANDED.

OVEREXPANDED FLOW : FROM SHOCK-AT-EXIT TO DESIGN (SUPERCRITICAL) FLOW

NORMAL SHOCK AT EXIT :

(3.57) $$\frac{P_b}{P_{e_{sc}}} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{e_{sc}}^2 - 1\right) \Rightarrow P_b = 286.5 \text{ kPa}$$

$\Rightarrow$ OVEREXPANDED :

$$34.36 < P_b < 286.5 \quad (\text{kPa})$$

e) FIND THE RANGE OF BACK PRESSURES FOR WHICH THE FLOW IS UNDEREXPANDED.

UNDEREXPANDED $\Rightarrow$ BACK PRESSURE LOWER THAN THE DESIGN BACK PRESSURE.

$$P_b < 34.36 \text{ kPa}$$

f) DESIGN MASSFLOW IF THE FLOW IS PULLED THROUGH THE NOZZLE.

CHOKED MASS FLOW:

(5.21) $$\dot{m} = \frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} = 146.2 \text{ kg/s}$$

g) CALCULATE THE DESIGN MASSFLOW IF THE FLOW IS PUSHED THROUGH THE NOZZLE WITH A EXIT PRESSURE OF 101 kPa

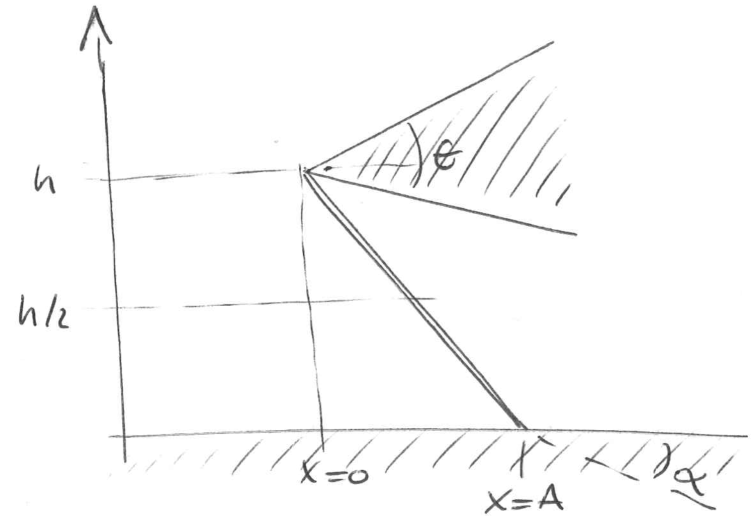
(3.30) $$\frac{P_0}{P_e} = \left(1 + \frac{\gamma - 1}{2} M_{esc}^2\right)^{\gamma/(\gamma-1)}$$

$P_e = P_b = 101\ kPa$ $\Rightarrow P_0 = 2.36\ MPa$

(5.21) $$\dot{m} = \frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$\Rightarrow \dot{m} = 431.2\ kg/s$

P3 (WEDGE FLOW)



$P_1 = 60.\,kPa$, $T_1 = -20°C$, $M_1 = 2.5$, $\epsilon = 8.0°$, $h = 1.0m$

a) CALCULATE THE DISTANCE FROM THE LEADING EDGE OF THE WEDGE TO THE LOCATION WHERE THE SHOCK IMPINGES THE WALL. (A)

FLOW DEFLECTION $= \epsilon/2 = 4.0°$

UPSTREAM MACH NUMBER $= 2.5$

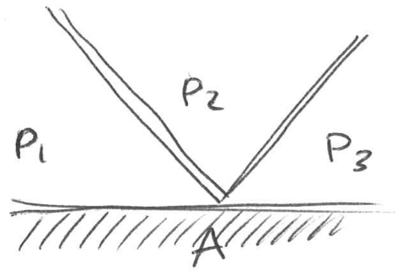
$(\epsilon - \beta - M) \Rightarrow \beta = 26.6°$

$A = h / \tan(\beta) = 2.0\ m$

b) THE SHOCK WILL BE TERMINATED AT (A) IF THERE IS NO NEED FOR FLOW DEFLECTION

$\Rightarrow \alpha = \frac{\epsilon}{2} = 4°$

d) PLOT THE WALL PRESSURE AS A FUNCTION OF AXIAL COORDINATE FROM $x=0.0$ TO $x=1.5A$



WE NEED THE PRESSURE DOWNSTREAM OF THE FIRST SHOCK AND ALSO DOWNSTREAM OF THE SECOND SHOCK FOR THE CASE WHERE THE SHOCK IS REFLECTED.

FIRST SHOCK:

$$\left.\begin{aligned}
&(4.7) && M_{n_1} = M_1 \sin\beta \\
&(4.9) && \frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \\
&(4.10) && M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1} \\
&(4.12) && M_2 = M_{n_2} / \sin(\beta-\theta)
\end{aligned}\right\} \Rightarrow \begin{aligned} &M_2 = 2.33 \\ &P_2 = 77.8\,kPa \\ &P_2/P_1 = 1.3 \end{aligned}$$

SECOND SHOCK:

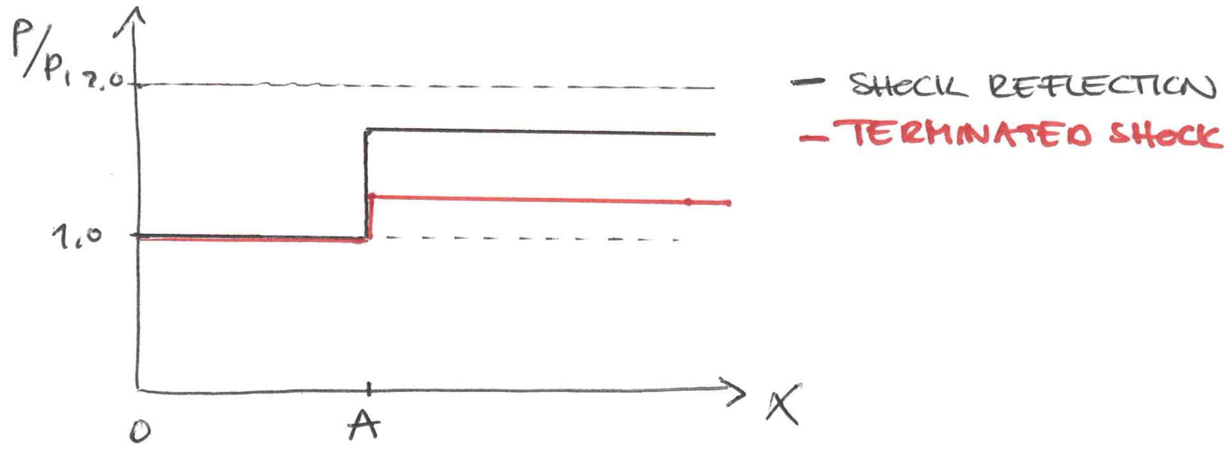
THE FLOW IS DEFLECTED BACK 4° ($\theta = 4°$)

($\theta-\beta-M$ WITH $M = M_2$ AND $\theta = 4°$) $\Rightarrow$ $\beta = 28.5°$

(4.7), (4.9), (4.10), AND (4.12) $\Rightarrow$

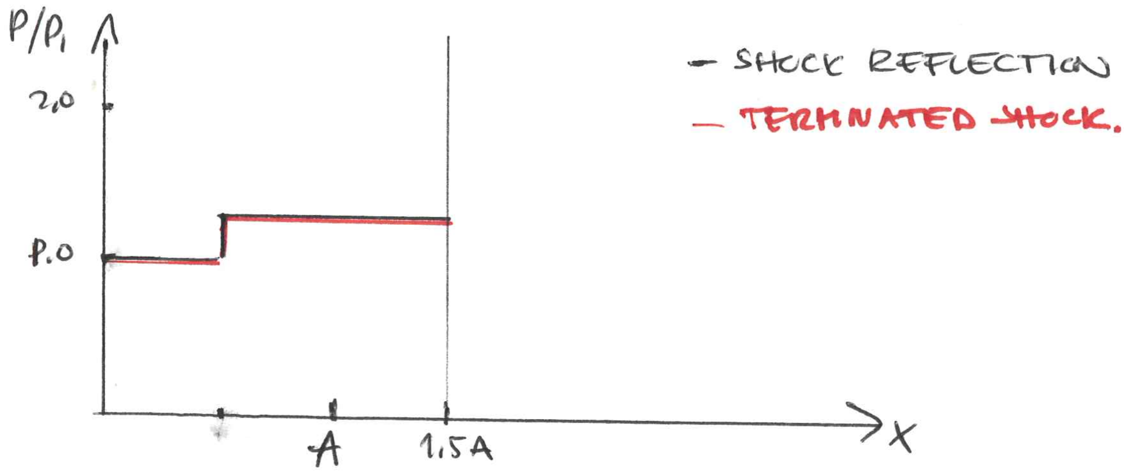
$M_3 = 2.17$, $P_3 = 99.4\,kPa$, $P_3/P_1 = 1.66$

WALL PRESSURE :

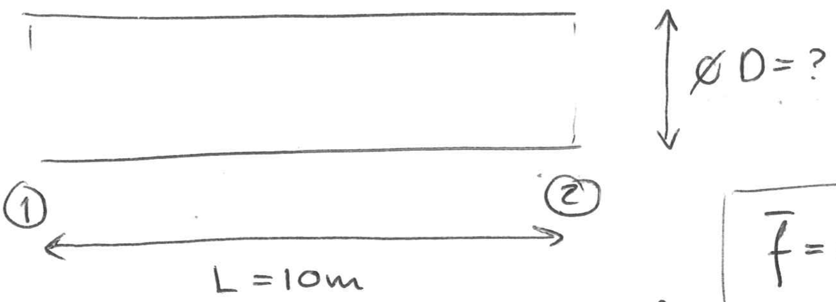


PRESSURE ALONG $y = h/2$:

(GEOMETRIC CHECK SHOWS THAT THE LINE $y = h/2$ WILL NOT INTERSECT WITH THE SECOND SHOCK IN THE INTERVAL $x = 0 \rightarrow A \cdot 1.5$)



P4 (FLOW WITH FRICTION)



$\bar{f} = 0{,}005$

$\dot{m} = 0{,}12$ kg/s

$T_1 = 20°C$

$P_1 = 200kPa$

$P_2 = 126kPa$

$T_2 = ?$

FIND THE PIPE DIAMETER (D) THAT GIVES THE EXIT PRESSURE $P_2 = 126kPa$ AND MASS FLOW $\dot{m} = 0{,}12$ kg/s

THIS PROBLEM IS SOLVED USING AN ITERATIVE PROCESS.

$$T_1 = 20°C \Rightarrow a_1 = \sqrt{\gamma R T_1} = 343{,}1 \text{ m/s}$$

$$\rho_1 = \frac{P_1}{RT_1} = 2{,}97 \text{ kg/m}^3$$

1. GUESS M_1

2. CALCULATE D

$$\dot{m} = \rho_1 u_1 \frac{\pi D}{4} = \rho_1 M_1 a_1 \frac{\pi D}{4} \Rightarrow D =$$

3. CALCULATE L_1^*

$$(3.107) \quad \frac{4\bar{f}L_1^*}{D} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

$$\Rightarrow L_1^* =$$

4. CALCULATE L_2^*

$$L_2^* = L_1^* - L$$

5. if $L_2^* > 0$, CALCULATE EXIT PRESSURE USING P^*

$$(3.107) \quad \frac{4\bar{f}L_2^*}{D} = \frac{1-M_2^2}{\gamma M_2^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2}\right)$$

$$\Rightarrow M_2 =$$

(3.104)

$$\left.\begin{aligned} \frac{p_1}{p^*} &= \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2} \\ \frac{p_2}{p^*} &= \frac{1}{M_2}\left(\frac{\gamma+1}{2+(\gamma+1)M_2^2}\right)^{1/2} \end{aligned}\right\} \Rightarrow P_2 =$$

IF $P_2 \simeq P_{2\,target}$ YOU'R DONE OTHERWISE UPDATE M_1 AND ITERATE..

ITERATION GIVES:-

$M_1 \approx 0{,}24$

$D \approx 0{,}025\,m$