TME085 - Compressible Flow

2021-06-08, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Responsible teacher: Niklas Andersson tel.: 070 - 51 38 311

Good luck!

Instructions

General Info

Due to the extraordinary situation caused by the very high risk of the covid-19 infection spreading in Sweden, Chalmers' president has decided that all written exams will be carried out from home.

Zoom will be used for identification and monitoring. You must be connected to zoom during the entire exam.

In order for the identification control process to be as smooth as possible, please connect to zoom 45 minutes before the start of the exam. You need to have a valid ID for the identification.

Exam Info

The exam consists of six problems (each problem is a separate assignment in Canvas). Each problem can give a maximum of 10 points and thus, in total, you can get 60 points on the exam. The points earned for the Compressible Flow Project 2021 is added to your exam result.

The total number of points on the exam (EP) and the bonus points earned for The Compressible Flow Project (BP) is translated into a course grade as follows:

- Fail: $(EP + BP) < 24$
- Grade 3: $24 \leq (EP + BP) < 36$
- Grade 4: $36 \leq (EP + BP) < 48$
- Grade 5: $48 \leq (EP + BP)$

Instructions

- The exam is divided into a number of separate assignments. You should submit documents (text documents of photos - pdf, jpg, png) with answers/solutions for each of these assignments. Do not wait until the last minute with the submission of files. It is better to submit files continuously as you solve the problems. You can always go back and update if you find mistakes later. You do, however, have additional 30 minutes after the exam (13:30-14:00) for scanning your solution and uploading files.
- If you use Matlab scripts, Python or any other programming languages to solve the problems you can paste your code snippets in the text document if you think that it will be helpful for the correction of the problems. Note! you will still have to explain what you have done in words, just code will not be sufficient.
- In case you have used some type of graphical representation of your solution (Matlab plots, matplotlib, gnuplot, ...), you could add these figures to your solution document if it adds value
- If you have used an iterative solution procedure using for example Matlab, you could add output from these iterations to your solution
- The exam is to be carried out individually, i.e., collaboration is not allowed.
- Due to the current circumstances, all examination aids are allowed.
- Control for plagiarism will be carried out automatically for each of the problems.

- The exam cannot be written anonymously.

Note! By uploading your exam solutions you certify that you have solved the problems on your own without receiving any help from anyone else

General Exam Guide

- Always write down and justify your assumptions
- For some problems you may have to guess values on some properties that has not been given in the problem description
- Some problem descriptions may include data that you will not need for solving the problem
- It is not uncommon that an iterative solution process in needed to be able to solve a problem
- Even if it is difficult in some situations, always try to determine whether your results are realistic or not. An unrealistic solution is worth a bit more if you make a comment about the results and why you think that it is unrealistic.
- Always write down your planned solution process in words. If you do something wrong along the way or if you run out of time and leave the problem unfinished, a description of how to solve the problem goes a long way when it comes to the number of rewarded points (if it is correct of course)
- The header of each problem indicates the total number of points and the number of subtasks.

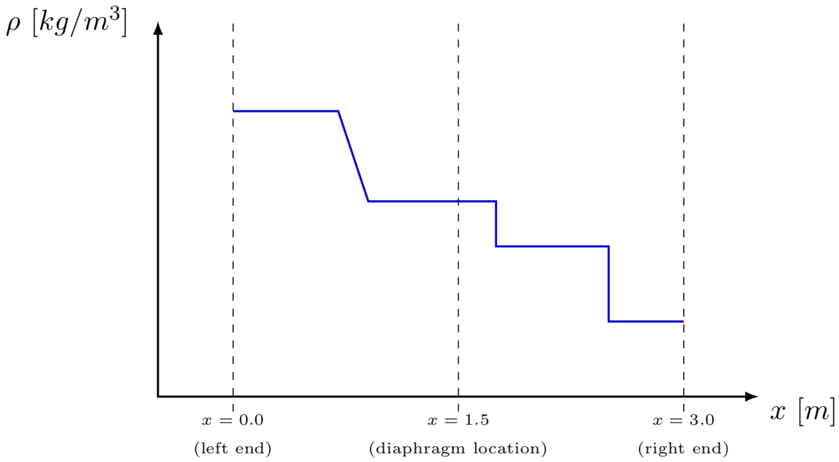
Problem 1 - SHOCK TUBE (10 p., 3 subtasks)

The figure below shows the distribution of density in a shock tube at a time t = 0.0025 seconds after the diaphragm was broken in a shock tube test. Unfortunately the original test data was lost and the density values are not to scale due to a bug in the data processing program. In order to be able to rerun the test, the temperature in the driven section is needed, i.e. the temperature to the right of the diaphragm before it was broken.

(a) (8.0p.) Help the lab engineers to calculate the temperature in the driven section using the information available in the figure and your profound knowledge on shock tube physics. *The gas in the shock tube is air and we can assume calorically perfect gas.*

(b) (1.0p.) What types of waves or discontinuities are generated in a shock tube

(c) (1.0p.) How does the absolute Mach number change after a weak/strong stationary oblique shock?



Problem 2 - NOZZLE FLOW (10 p., 5 subtasks)

A convergent divergent nozzle with the exit-to-throat area ratio of 4.0 ($A_t = 0.5\ m^2$, $A_e = 2.0\ m^2$) is operated such that a normal shock is standing at the nozzle exit plane (see figure below). At this operating condition the pressure and temperature just downstream of the nozzle exit are $p_b = 1.0\ bar$ and $T_b = 300.0\ K$, respectively.

The gas that flows through the nozzle is air that can be assumed to be calorically perfect.

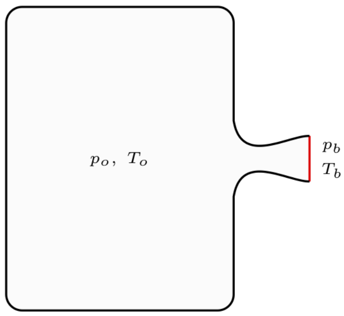
(a) (4.0p.) calculate flow Mach number and static pressure just upstream of the nozzle exit plane

(b) (2.0p.) calculate total pressure p_o and total temperature T_o upstream of the nozzle (plenum chamber)

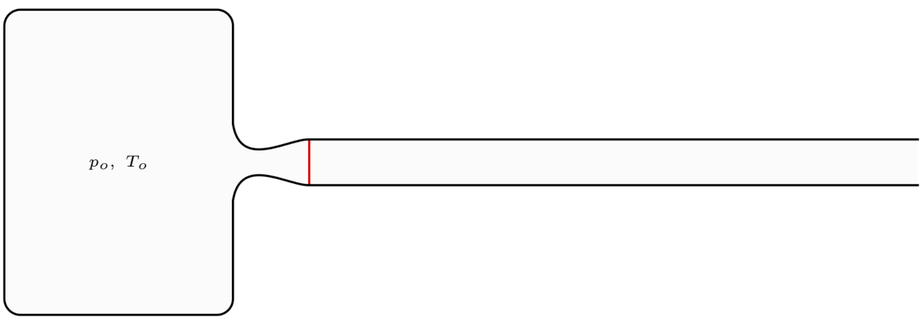
(c) (1.0p.) what is the maximum mass flow through the nozzle at the given conditions

(d) (2.0p.) a pipe with an average friction factor of $\bar{f} = 0.005$ is attached to the nozzle as shown in the lower figure below. If we assume that the conditions downstream of the pipe exit can be changed as needed, what is the maximum possible length of this extension pipe without altering the nozzle flow conditions?

(e) (1.0p.) calculate the pressure and temperature at the pipe exit



converegent-diveregent nozzle with a shock at the nozzle exit plane



converegent-diveregent nozzle with an attached pipe

Problem 3 - WEDGE FLOW (10 p., 4 subtasks)

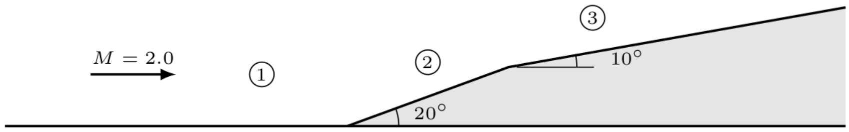
A 20° wedge with a 10° shoulder (depicted the figure below) is situated in a flow with a free stream Mach number of $M = 2.0$

(a) (2.0p) draw a schematic sketch of the important flow features in the flow over the wedge

(b) (4.0p) calculate the Mach numbers in regions 2 and 3

(c) (2.0p) assume that the flow would pass a simple 10° wedge (without the shoulder), the resulting flow direction would be the same. Would the total pressure in the directed flow be greater or lower than in the case with the shoulder? What is the reason for this?

(d) (2.0p.) How does the total pressure change between region 2 and 3. Explain?



Problem 4 - NOZZLE EXPANSION (10 p., 3 subtasks)

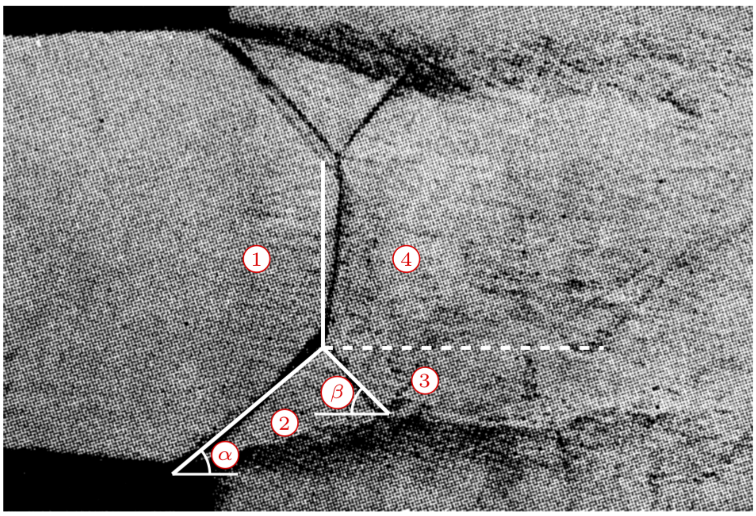
The figure below shows a Schlieren photograph of an overexpanded jet. The Schlieren image shows a snap shot the instantaneous flow field. In order to be able to analyze the flow, a schematic representation of the shock system has been added to the figure. The angles of the oblique shocks are $\alpha = 40°$ and $\beta = 45°$, respectively. Between region 1 and 4 there is a normal shock. The dashed line between region 3 and 4 is a slip line.

The gas is air and we can assume calorically perfect gas.

(a) (8.0p.) Calculate the exit Mach number, i.e. the Mach number in region 1

(b) (1.0p.) Explain the concepts: critical, perfectly expanded (supercritical), overexpanded, underexpanded

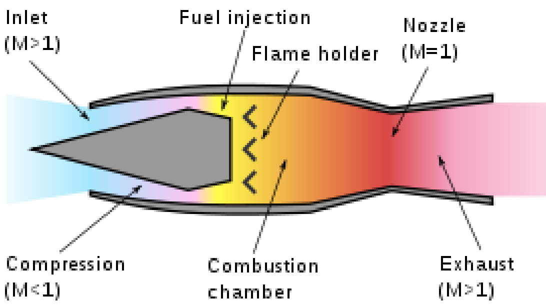
(c) (1.0p.) What are the implications of the area-velocity relation for quasi-one-dimensional flow



Problem 5 - RAM JET ENGINE (10 p., 5 subtasks)

The ramjet engine is an air-breathing engine concept with very few moving parts. The absence of moving parts makes this concept much less complex than a corresponding gas turbine construction and therefore it was an appealing option in the early days of supersonic aviation. For a ramjet engine, the speed of the aircraft itself is sufficient to compress the air in the engine intake, which eliminates the need for compressors. As the air enters the engine intake, a system of shocks decelerates the flow to subsonic speeds and raises the pressure. After the engine intake, the air passes through a subsonic combustor in which fuel is sprayed into the airstream and burned. After the combustor the gas passes through a nozzle where the flow is again accelerated to supersonic speed generating a propelling jet flow.

Suppose that it is possible to represent the ramjet engine by a pipe where first supersonic air flow is decelerated by a normal shock (i.e. the system of oblique shocks in the engine inlet diffusor is replaced by a single normal shock). After the normal shock, heat is added to the flow in the combustor section after which the flow leaves the engine through passing the exit section. Assume that the aircraft, for which the ramjet engine is the propelling unit, operates at an altitude of 6000 m and moves through the air at a speed which gives a Mach number of 2.5 (thermodynamic data as a function of altitude is given in the figure below)



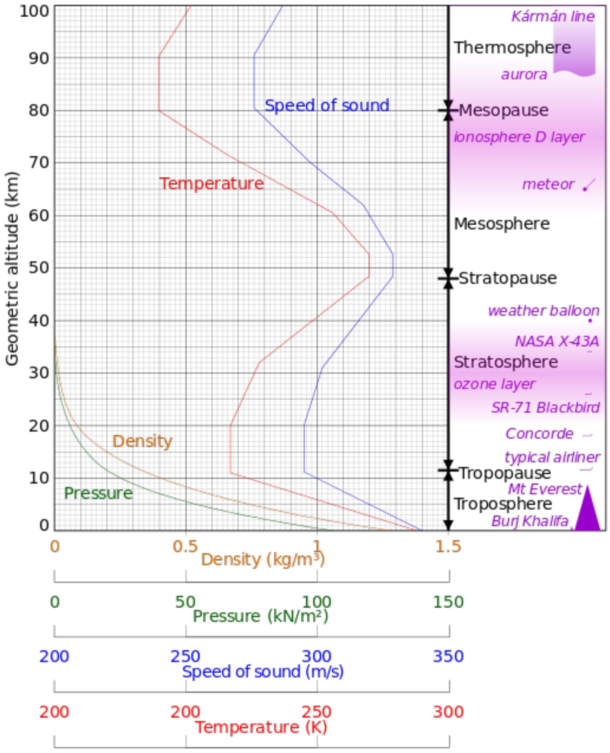
(a) (4.0p.) The normal shock at in the engine intake slows down the flow and increases the pressure. Calculate the pressure and temperature in the burner inlet section.

(b) (2.0p.) Calculate the highest possible temperature that can be obtained in the combustor section of the engine. The mass of the added fuel and changes in the gas composition can be neglected.

(c) (2.0p.) Calculate the amount of heat added per unit mass when the maximum temperature is reached

(d) (1.0p.) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

(e) (1.0p.) What is the difference between a calorically perfect gas and a thermally perfect gas?



Geometric altitude (km)
100
90
80
70
60
50
40
30
20
10
0
Kármán line
Thermosphere
aurora
Speed of sound
Mesopause
ionosphere D layer
Temperature
meteor
Mesosphere
Stratopause
weather balloon
NASA X-43A
Stratosphere
ozone layer
SR-71 Blackbird
Concorde
Density
typical airliner
Tropopause
Pressure
Mt Everest
Troposphere
Burj Khalifa
0
0.5
1
1.5
Density (kg/m³)
0
50
100
150
Pressure (kN/m²)
200
250
300
350
Speed of sound (m/s)
200
200
250
300
Temperature (K)

Problem 6 - MOVING SHOCK (10 p., 4 subtasks)

A shock moves in an open-ended tube with the velocity (relative to the stagnant air into which it propagates) of 415 m/s. The pressure and temperature in the air in front of the shock is 101 kPa and 288 K, respectively. When the incident shock reaches the open end, a reflected expansion fan is generated in order to maintain a constant pressure at the boundary.

Calculate:

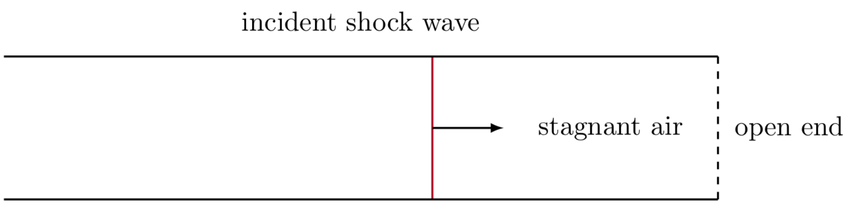
(a) (2.0p.) The pressure increase over the incident shock wave

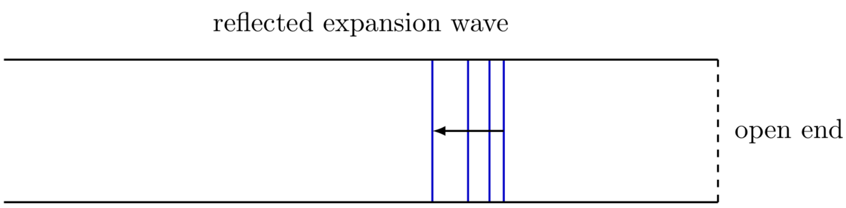
(b) (2.0p.) The induced velocity behind the incident shock wave

(c) (3.0p.) The propagation velocity of the leading part of the expansion wave (the left-most part)

(d) (3.0p.) The propagation velocity of the trailing part of the expansion wave (the right-most part)

hint: The expansion waves are left-running characteristics (C^-) with $dx = u - a$ and the Riemann invariant J^+ is constant over the expansion region.





TME085 EXAM 2021-06-08

P1 (SHOCK TUBE)

ASSUME CALORICALLY PERFECT AIR.

a) $\Delta t = 0.0025$

$$\Delta X_{shock} = 2.0 \times \left(\frac{1.5}{3.0}\right) = 1.0$$

$$\Delta X_{-cd} = 0.5 \times \left(\frac{1.5}{3.0}\right) = 0.25$$

ESTIMATES FROM THE PROVIDED FIGURE.

$$u_p = \Delta X_{-cd} / \Delta t = 100.0 \text{ m/s}$$

$$W_s = \Delta X_{-shock} / \Delta t = 400.0 \text{ m/s}$$

(7.16)

$$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

$$M_s = W_s / a_1 = W_s / \sqrt{\gamma R T_1}$$

(7.13)

$$M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \Rightarrow \left(M_s^2 - 1\right)\left(\frac{2\gamma}{\gamma+1}\right) + 1 = \frac{p_2}{p_1}$$

GUESS T_1

CALCULATE $M_s \Rightarrow p_2/p_1 \Rightarrow u_p$

UPDATE T_1

ITERATE UNTIL CONVERGED $\Rightarrow$ 278.75 K

b)

\# SHOCK WAVE

\# CONTACT DISCONTINUITY (SEPARATING THE GASES ORIGINALLY SEPARATED BY THE SHOCK-TUBE DIAPHRAGM. DISCONTINUITY IN DENSITY, TEMPERATURE, ENTROPY.. TRAVELS IN THE DIRECTION OF THE INCIDENT SHOCK WITH THE INDUCED FLOW VELOCITY.

PRESSURE AND FLOW VELOCITY IS CONTINUOUS OVER THE CONTACT SURFACE (PRESSURE AND VELOCITY ARE THE SAME IN REGION 2 AND 3.

EXPANSION

c) HOW DOES THE ABSOLUTE MACH NUMBER CHANGE AFTER A WEAK / STRONG OBLIQUE SHOCK

AFTER A WEAK SHOCK, THE ABSOLUTE MACH NUMBER IS ALMOST ALWAYS SUPERSONIC (THE EXCEPTION IS FOR FLOW DEFLECTIONS CLOSE TO THE MAXIMUM POSSIBLE FLOW DEFLECTION)

AFTER A STRONG SHOCK, THE ~~[illegible]~~ ABSOLUTE MACH NUMBER IS ALWAYS SUBSONIC.

P2 (NOZZLE FLOW)

$$\frac{A_e}{A_t} = 4.0 \quad (A_t = 0.5\,m^2,\ A_e = 2.0\,m^2)$$

NORMAL SHOCK AT EXIT FOR $P_b = 1.0$ bar (100 kPa), $T_b = 300$ K

ASSUME CALORICALLY PERFECT AIR.

a) CALCULATE THE STATIC PRESSURE JUST UPSTREAM OF THE NOZZLE EXIT PLANE.

(5.20)
$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_{esc}^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_{esc}^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUPERSONIC SOLUTION : $M_{esc}^{*} = 2.94$

(3.57) NORMAL SHOCK:

$$\frac{P_b}{P_{esc}} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{esc}^2 - 1\right) \Rightarrow P_{esc} = 10.1\ \text{kPa}$$

b)

(3.59)
$$\frac{T_b}{T_{esc}} = \frac{P_b}{P_{esc}}\left(\frac{2+(\gamma-1)M_{esc}^2}{(\gamma+1)M_{esc}^2}\right) \Rightarrow T_{esc} = 119.9\ \text{K}$$

(ASSUMING THAT T_b IS THE JET TEMPERATURE DOWNSTREAM OF THE SHOCK)

(3.28)
$$\frac{T_0}{T_{esc}} = 1 + \frac{\gamma-1}{2}M_{esc}^2 \Rightarrow T_0 = 323.8\ \text{K}$$

(3.30)
$$\frac{P_0}{P_{esc}} = \left(1+\frac{\gamma-1}{2}M_{esc}^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_0 = 338.5\ \text{kPa}$$

c) WHAT IS THE MAXIMUM MASSFLOW THROUGH THE NOZZLE ~~FOR~~ AT THE GIVEN CONDITIONS?

(5.21)
$$\dot{m}_{choked} = \frac{P_0 A_t}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} = 386.2\ \text{kg/s}$$

d) A PIPE WITH $\bar{f} = 0.005$ IS ADDED AT THE NOZZLE EXIT. WHAT IS THE MAXIMUM POSSIBLE PIPE LENGTH WITHOUT ALTERING THE NOZZLE FLOW (ASSUMING THAT THE PIPE EXIT PRESSURE CAN BE ADJUSTED AS NEEDED)

PIPE INLET MACH NUMBER:

(3.51) $$M_{in}^2 = \frac{1 + ((\gamma-1)/2) M_{esc}^2}{\gamma M_{esc}^2 - (\gamma-1)/2} \Rightarrow M_{in} = 0.48$$

(3.107)

$$\frac{4\bar{f}L^*}{D} = \frac{1-M_{in}^2}{\gamma M_{in}^2} + \frac{\gamma+1}{2\gamma} \ln\left(\frac{(\gamma+1)M_{in}^2}{2+(\gamma-1)M_{in}^2}\right)$$

$$\Rightarrow L^* = 100.26\,m$$

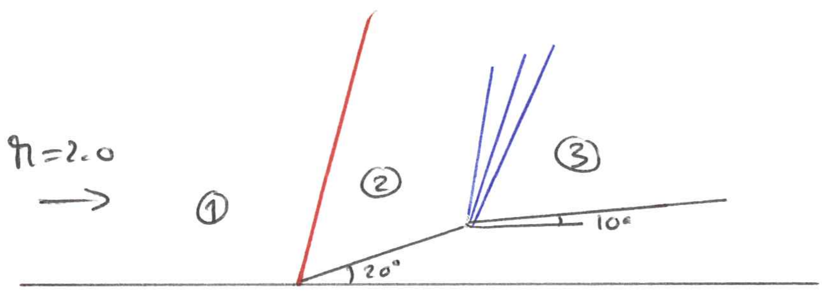
e) CALCULATE THE PRESSURE AND TEMPERATURE AT THE EXIT OF THE PIPE.

$P = P^*$, $T = T^*$

(3.103) $$\frac{T_{in}}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_{in}^2} \Rightarrow T^* = 261.46\,K$$

(3.104) $$\frac{P_{in}}{P^*} = \frac{1}{M_{in}}\left(\frac{\gamma+1}{2+(\gamma-1)M_{in}^2}\right)^{1/2} \Rightarrow P^* = 44.7\,kPa$$

P3 (WEDGE FLOW)



b) CALCULATE THE MACH NUMBER IN REGIONS 2 AND 3

$1 \to 2$ (OBLIQUE SHOCK)

($\theta - \beta - M$, WITH $M = 2.0$ AND $\theta = 20°$) $\Rightarrow \beta = 53.4°$

$$(4.7) \quad M_{n_1} = M_1 \sin\beta$$

$$(4.10) \quad M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$$

$$(4.12) \quad M_2 = \frac{M_{n_2}}{\sin(\beta - \theta_1)}$$

$\Rightarrow M_2 = 1.2$

$2 \to 3$ (EXPANSION)

$$(4.44) \quad \nu(M_2) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2 - 1)} - \tan^{-1}\sqrt{M_2^2 - 1}$$

$$= 3.8°$$

$$\nu(M_3) = \nu(M_2) + \theta_2 = 13.8°$$

$(4.44) \Rightarrow M_3 = 1.56$

c)

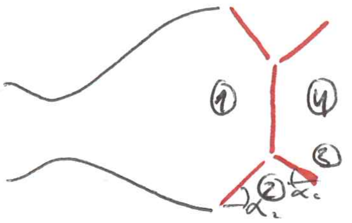
THE REDUCTION OF TOTAL PRESSURE IS HIGHER IN THE SHOULDER CASE THAN IN THE SIMPLE WEDGE CASE

THE REASON IS THAT A 20° DEFLECTION RESULTS IN A STRONGER SHOCK THAN A 10° DEFLECTION WITH GREATER REDUCTION OF TOTAL PRESSURE AS A CONSEQUENCE

THE EXPANSION IS ISENTROPIC AND THUS THE SHOULDER EXPANSION PROCESS DOES NOT AFFECT THE TOTAL PRESSURE.

P4 (NOZZLE EXPANSION)

OVEREXPANDED JET



$\alpha_1 = 40°$
$\alpha_2 = 45°$

THE FIRST OBLIQUE SHOCK WILL TURN THE FLOW INWARDS AN ANGLE θ AND THE REFLECTED SHOCK WILL TURN THE FLOW BACK AGAIN (NOT NECESSARILY THE SAME ANGLE)

THE PRESSURE AFTER THE SECOND OBLIQUE SHOCK SHOULD MATCH THE PRESSURE IN REGION 4 (AFTER THE NORMAL SHOCK)

1 → 4 NORMAL SHOCK

$$(3.57) \quad \frac{p_4}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

1 → 2 OBLIQUE SHOCK $(\theta\text{-}\beta\text{-}M,\ \beta_1 = \alpha_1,\ M = M_1) \Rightarrow \theta_1 = ?$

$$(4.7) \quad M_{n_1} = M_1 \sin(\beta_1)$$

$$(4.9) \quad \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$$

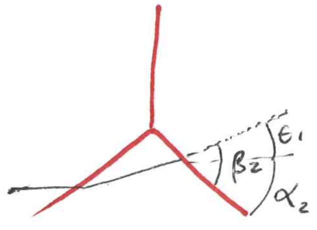
$$(4.10) \quad M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$$

$$(4.12) \quad M_2 = \frac{M_{n2}}{\sin(\beta_1 - \theta_1)}$$

2 → 3 OBLIQUE SHOCK $(\theta\text{-}\beta\text{-}M,\ \beta = \alpha_2 + \theta_1,\ M = M_2) \Rightarrow \theta_2 = ?$

$$(4.7) \quad M_{n_1} = M_2 \sin(\beta_2)$$

$$(4.9) \quad \frac{p_3}{p_2} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$$



$\beta_2 = \alpha_2 + \theta_1$

$$(4.10) \quad M_{n2}^2 = \frac{M_{n1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n1}^2 - 1}$$

$$(4.12) \quad M_3 = \frac{M_{n2}}{\sin(\beta_2 - \theta_2)}$$

<u>1 → 3</u> (TWO OBLIQUE SHOCKS)

$$\frac{P_3}{P_1} = \frac{P_3}{P_2}\frac{P_2}{P_1}$$

ITERATE UNTIL $P_3 \simeq P_4$ ⇒

$$M_1 = 2.4$$

$$\theta_1 = 16.5$$

$$\theta_2 = 17.2$$

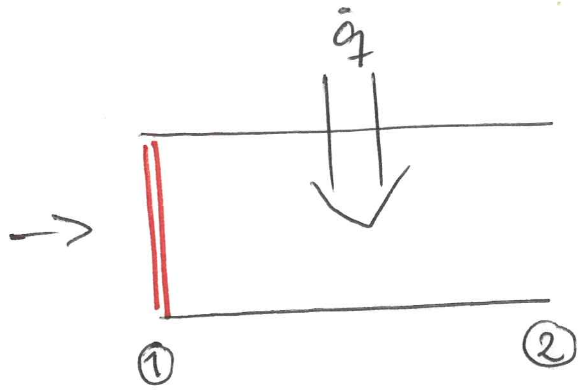
$$M_2 = 1.7$$

$$M_3 \simeq 1.0$$

P2 (RAMJET ENGINE)

A RAMJET ENGINE OPERATES AT $M = 2.5$ AT AN ALTITUDE OF 6000 m

$$H = 6000\,\text{m} \Rightarrow \begin{cases} T_\infty = 250\,\text{K} \\ P_\infty = 50\,\text{kPa} \end{cases}$$



(NORMAL SHOCK AT INLET)

$$(3.51) \quad M_1^2 = \frac{1 + ((\gamma-1)/2) M_\infty^2}{\gamma M_\infty^2 - (\gamma-1)/2} \Rightarrow M_1 = 0.51$$

a) CALCULATE PRESSURE AND TEMPERATURE AT THE INLET OF THE BURNER (1)

NORMAL SHOCK:

$$\frac{P_1}{P_\infty} = 1 + \frac{2\gamma}{\gamma+1}(M_\infty^2 - 1) \qquad (3.57)$$

$$\frac{T_1}{T_\infty} = \left(1 + \frac{2\gamma}{\gamma+1}(M_\infty^2 - 1)\right)\left(\frac{2 + (\gamma-1)M_\infty^2}{(\gamma+1)M_\infty^2}\right) \qquad (3.59)$$

$\Rightarrow 356.3\,\text{kPa} \quad (P_1)$

$537.4\,\text{K} \quad (T_1)$

b) THE HIGHEST TEMPERATURE POSSIBLE IS T^* (THERMAL CHOKING)

$$(3.86) \quad \frac{T_1}{T^*} = M_1^2 \left(\frac{1+\gamma}{1+\gamma M_1^2}\right)^2 \Rightarrow T^* = 660{,}2\,K$$

c) CALCULATE THE AMOUNT OF HEAT ADDED PER UNIT MASS TO GET THE MAXIMUM TEMPERATURE.

$$(3{,}28) \quad \frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \Rightarrow T_{01} = 562{,}5\,K$$

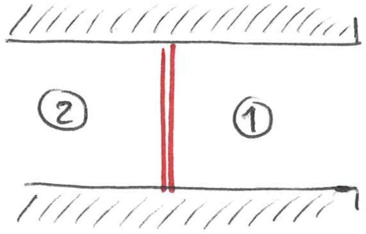
$$(3{,}89) \quad \frac{T_{01}}{T_0^*} = \frac{(\gamma+1)M_1^2}{(1+\gamma M_1^2)^2}\left(2 + (\gamma-1)M_1^2\right) \Rightarrow T_0^* = 792.2\,K$$

$$q^* = c_p\,(T_0^* - T_{01}) = 230.7\,kJ/kg$$

P3 (MOVING SHOCK)

A SHOCK MOVES AT A VELOCITY OF 415 m/s THROUGH AN OPEN-ENDED TUBE. THE PRESSURE AND TEMPERATURE AHEAD OF THE SHOCK IS 101 kPa AND 288 K, RESPECTIVELY.

a) CALCULATE THE PRESSURE INCREASE OVER THE MOVING SHOCK.



THE SHOCK MACH NUMBER $M_s = \frac{W}{a_1} = \frac{W}{\sqrt{\gamma R T_1}} = 1{,}22$

$$(7.13) \qquad M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1}-1\right)+1} \Rightarrow \underline{\frac{p_2}{p_1} = 1{,}57}$$

b) CALCULATE THE INDUCED FLOW VELOCITY BEHIND THE MOVING SHOCK.

$$(7.16) \qquad u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1}-1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

WHERE $a_1 = \sqrt{\gamma R T_1}$

$$\Rightarrow \quad \underline{u_p = 113{,}5 \text{ m/s}}$$

CHECK INDUCED FLOW MACH NUMBER:

$$\left.\begin{aligned} M_2 &= \frac{u_p}{a_2} = \frac{u_p}{\sqrt{\gamma R T_2}} \\ \frac{T_2}{T_1} &= \frac{p_2}{p_1}\left(\frac{\frac{\gamma+1}{\gamma-1}+\frac{p_2}{p_1}}{1+\frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}\right) \end{aligned}\right\} \Rightarrow M_2 = 0{,}31 \text{ (SUBSONIC)}.$$

$(T_2 = 328{,}5\text{ K})$

c) AT THE PIPE OPENING, THE SHOCK WILL BE TERMINATED AND AN EXPANSION MOVING BACK INTO THE TUBE WILL BE FORMED.

CALCULATE THE PROPAGATION VELOCITY OF THE LEADING PART OF THE EXPANSION

THE EXPANSION PROPAGATES INTO A REGION WITH THE FLOW VELOCITY $u_2 = u_p$ AND THE SPEED OF SOUND $a_2 = \sqrt{\gamma R T_2}$

THUS THE EXPANSION HEAD MOVES TO THE LEFT AT THE VELOCITY

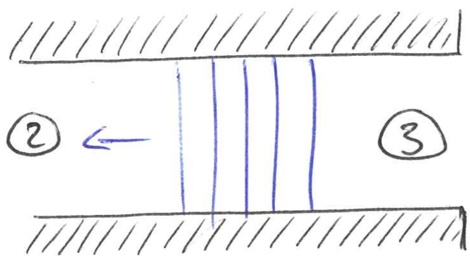
$$u_{HEAD} = u_2 - a_2 = \underline{-249.8 \text{ m/s}}$$

d) CALCULATE THE PROPAGATION VELOCITY OF THE TAIL OF THE EXPANSION.

THE J_+-INVARIANT WILL BE CONSTANT OVER THE EXPANSION

$$J_+ = u_2 + \frac{2a_2}{\gamma - 1} = u_3 + \frac{2a_3}{\gamma - 1}$$

$$a_3 = \sqrt{\gamma R T_3}$$



THE EXPANSION IS ISENTROPIC

$$\left(\frac{P_2}{P_3}\right) = \left(\frac{T_2}{T_3}\right)^{\gamma/(\gamma-1)}$$

$P_3 = P_1$ (THAT IS WHY THE EXPANSION IS FORMED..)

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{T_2}{T_3}\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow T_3 = T_2 \Big/ \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

NOW, LET'S GO BACK TO THE $J+$ INVARIANT

$$u_2 + \frac{2a_2}{\gamma-1} = u_3 + \frac{2a_3}{\gamma-1}$$

$$u_2 + \frac{2}{\gamma-1}(a_2 - a_3) = u_3 \Rightarrow u_3 = 226.8 \text{ m/s}$$

$$u_{TAIL} = u_3 - a_3 = \underline{-113.8 \text{ m/s}}$$