TME085 - Compressible Flow

2021-03-18, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Responsible teacher: Niklas Andersson tel.: 070 - 51 38 311

Good luck!

Instructions

General Info

Due to the extraordinary situation caused by the very high risk of the covid-19 infection spreading in Sweden, Chalmers' president has decided that all written exams will be carried out from home.

Zoom will be used for identification and monitoring. You must be connected to zoom during the entire exam.

In order for the identification control process to be as smooth as possible, please connect to zoom 45 minutes before the start of the exam. You need to have a valid ID for the identification.

Exam Info

The exam consists of six problems (each problem is a separate assignment in Canvas). Each problem can give a maximum of 10 points and thus, in total, you can get 60 points on the exam. The points earned for the Compressible Flow Project is added to your exam result.

The total number of points on the exam (EP) and the bonus points earned for The Compressible Flow Project (BP) is translated into a course grade as follows:

- Fail: $(EP + BP) < 24$
- Grade 3: $24 \leq (EP + BP) < 36$
- Grade 4: $36 \leq (EP + BP) < 48$
- Grade 5: $48 \leq (EP + BP)$

Instructions

- The exam is divided into a number of separate assignments. You should submit documents (text documents of photos - pdf, jpg, png) with answers/solutions for each of these assignments. Do not wait until the last minute with the submission of files. It is better to submit files continuously as you solve the problems. You can always go back and update if you find mistakes later. You do, however, have additional 30 minutes after the exam (13:30-14:00) for scanning your solution and uploading files.
- If you use Matlab scripts, Python or any other programming languages to solve the problems you can paste your code snippets in the text document if you think that it will be helpful for the correction of the problems. Note! you will still have to explain what you have done in words, just code will not be sufficient.
- In case you have used some type of graphical representation of your solution (Matlab plots, matplotlib, gnuplot, ...), you could add these figures to your solution document if it adds value
- If you have used an iterative solution procedure using for example Matlab, you could add output from these iterations to your solution
- The exam is to be carried out individually, i.e., collaboration is not allowed.
- Due to the current circumstances, all examination aids are allowed.
- Control for plagiarism will be carried out automatically for each of the problems.

- The exam cannot be written anonymously.

Note! By uploading your exam solutions you certify that you have solved the problems on your own without receiving any help from anyone else

General Exam Guide

- Always write down and justify your assumptions
- For some problems you may have to guess values on some properties that has not been given in the problem description
- Some problem descriptions may include data that you will not need for solving the problem
- It is not uncommon that an iterative solution process in needed to be able to solve a problem
- Even if it is difficult in some situations, always try to determine whether your results are realistic or not. An unrealistic solution is worth a bit more if you make a comment about the results and why you think that it is unrealistic.
- Always write down your planned solution process in words. If you do something wrong along the way or if you run out of time and leave the problem unfinished, a description of how to solve the problem goes a long way when it comes to the number of rewarded points (if it is correct of course)
- The header of each problem indicates the total number of points and the number of subtasks.

Problem 1 - NOZZLE FLOW (10 p., 6 subtasks)

The divergent part of a minimum-length ideal nozzle contour is designed using method of characteristics (see figure below). Minimum length means that the throat section is a sharp "corner" and not as usual a smooth gradual adjustment from the convergent part to the divergent part of the nozzle. Method of characteristics is based on interaction of characteric lines (Mach waves) initiated from the throat. If operated at the correct nozzle pressure ratio, the contour will produce a shock-free flow through the nozzle and axial flow out from the nozzle. The throat area is 2.0 m^2 and the exit area is 6.6 m^2. The nozzle has a rectangular cross section area and the width in the third direction (not shown) is 1.0 m.

In your calculations, you can assume that the gas is calorically perfect air.

(a) (1p.) Using the area-Mach number relation in the divergent part a convergent-divergent nozzle, one has to be careful if the nozzle pressure ratio is within the range:

$$NPR_{choked} < NPR < NPR_{shock@nozzleexit}$$

explain why.

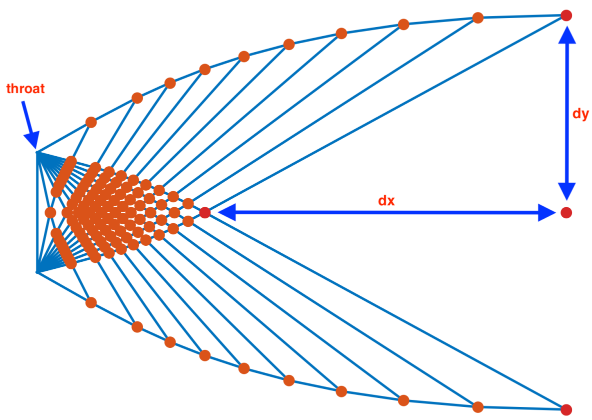
(b) (1p.) Explain the concepts subcritical, critical/choked, and supercritical nozzle operating condition

(c) (2p.) Calculate the design exit Mach number

(d) (2p.) Calculate the dx in the figure

(e) (2p.) Calculate the nozzle pressure ratio required to get supercritical conditions

(f) (2p.) Calculate the nozzle pressure ratio required to get normal-shock-@-exit conditions



Problem 2 - COMBUSTION (10 p., 5 subtasks)

Air enters a combustion chamber at 80.0 m/s, 300.0 K and 76.0 kPa. The combustion adds heat to the air corresponding to 610.0 kJ/kg. The flow can be assumed to be one-dimensional and friction is neglected, i.e., one-dimensional flow with heat addition. In your calculations, assume the air to be calorically perfect.

(a) (3p.) Calculate the heat addition that will give choked flow

(b) (3p.) Calculate the flow conditions at the exit of the combustor (Mach number, velocity, pressure, temperature)

(c) (2p.) Calculate the total pressure loss over the combustor as:

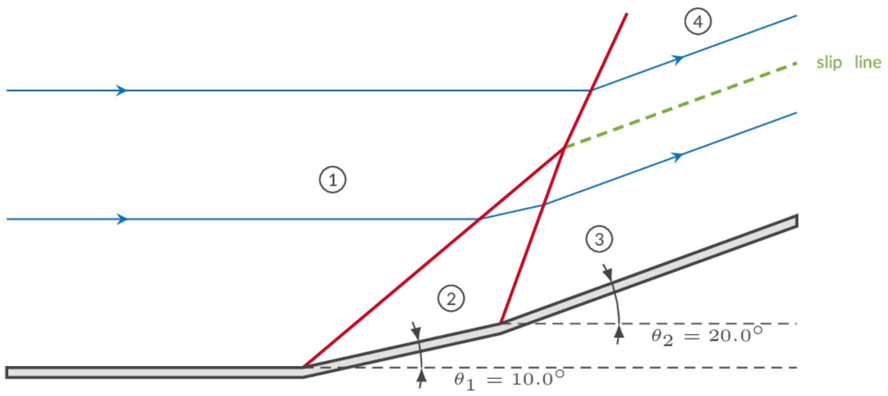
$$(p_{o_1} - p_{o_2})/p_{o_1}$$

(d) (1p.) Is the calorically perfect assumption valid? Explain why/why not.

(e) (1p.) What is the fundamental difference between calorically perfect gas and thermally perfect gas? (i.e. what molecular effects does the thermally perfect gas model include that is not included in the calorically perfect gas model)

Problem 3 - SHOCK INTERACTION (10 p., 5 subtasks)

The flow situation depicted in the figure below appears when a supersonic flow approaches a compression ramp with two consecutive discrete flow deflections. Oblique shocks are formed at each of the flow deflection locations and these shocks will eventually meet and form a single oblique shock as shown in the figure.



(a) (1p.) What is the reason for the formation of a slipline at the shock intersection point?

(b) (1p.) What flow quantities must be constant over a slipline?

(c) (5p.) With the upstream Mach number set to $M_1 = 3.0$, calculate the Mach numbers in regions 1, 2, 3, and 4 assuming that the slipline deflection is negligible, i.e., the slipline has the same direction as the wall.

(d) (2p.) Calculate the pressure ratios p_3/p_1 and p_4/p_1

(e) (1p.) Is the negligible slipline deflection a fair assumption? (justify your answer based on your previous calculations)

Problem 4 - VALVE (10 p., 3 subtasks)

Air at 300.0 K and 1.5 bar flows through a pipe at the uniform velocity 150.0 m/s. The end of the pipe is suddenly closed by a valve, which generates a shock wave propagating upstream in the pipe.

(a) (7p.) Calculate the propagation velocity of the shock wave and the pressure and temperature of the air behind the moving shock.

Hint: the density ratio over the shock can be obtained using the normal shock relations with the Mach number of the moving shock:

$$\frac{\rho_1}{\rho_2} = \frac{2 + (\gamma - 1)M_s^2}{(\gamma + 1)M_s^2}$$

This relation might be useful - at least if you solve the problem the same way as I did

(b) (2p.) Although a normal shock is adiabatic (no heat is added or removed over the shock), total enthalpy is not constant over a moving shock. Explain why.

(c) (1p.) When applying a CFD code for unsteady compressible flow, which of the following choices would you make: density-based or pressure-based, fully-coupled or segregated, conservative or non-conservative, explicit or implicit time stepping?

Problem 5 - PIPE FLOW WITH FRICTION (10 p., 2 subtasks)

A straight pipe with the diameter 50.0 mm is connected to a large air reservoir (a huge tank). The reservoir pressure and temperature are 13.8 bar and 310 K, respectively. The exit of the pipe is open to the atmosphere. The flow is adiabatic and the average friction coefficient is 0.005 (as usual ☺). The massflow through the pipe is 2.25 kg/s.

(a) (8p.) Calculate the pipe length that results in choked flow at the exit.

Hint: You will probably need to iterate to find the flow conditions at the inlet of the pipe. The following relation might be useful (depending on how you solve the problem)

$$\rho u = \frac{\dot{m}}{A} = constant$$

$$\rho u = \frac{p}{RT} a M$$

$$\frac{T_o}{T} = f(M)$$

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\gamma/(\gamma-1)}$$

(b) (2p.) What would happen if the pipe would be longer than the calculated length?

Problem 6 - ENGINE INLET (10 p., 3 subtasks)

Engine inlets designed for supersonic operation often feature inlet cones for gradual deceleration of the flow. In the schematic figure below, two engine inlets are compared. The engine inlet to the left has an inlet cone where the flow angle is changed in two discrete steps, which will produce two oblique shocks. In each of the two steps, the flow is bent 8 degrees. After passing the two oblique shocks the flow passes a normal shock when reaching the engine nacelle. In the example to the right, the flow is decelerated by a single normal shock at the engine inlet face.

(a) (5p.) Calculate the Mach number of the flow entering the engine in the two cases if the freestream Mach number is 3.0

(b) (3p.) Calculate the diffusion efficiency (defined by the relation below) for the two inlet concepts.

$$\eta = \frac{\left(\frac{p_{o_1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{p_{o_2}}{p_{o_1}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{\gamma-1}{2}\right) M_1^2}$$

Why do the diffusion efficiencies differ in the two cases?

(c) (2p.) In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

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P1 (NOZZLE FLOW)

$A_t = 2.0\,m^2$
$A_e = 6.6\,m^2$

ASSUME CALORICALLY PERFECT GAS

a) USING THE AREA-MACH NUMBER RELATION IN THE DIVERGENT PART OF A CD-NOZZLE, ONE HAS TO BE CAREFUL IF THE NOZZLE PRESSURE RATIO IS IN THE RANGE

$$NPR_{choked} < NPR < NPR_{shock@exit}$$

WHY?

THERE WILL BE A SHOCK IN THE DIVERGENT PART OF THE NOZZLE FOR THIS RANGE OF NPR:S. THE AREA-MACH RELATION IS ONLY VALID FOR ISENTROPIC FLOWS AND THUS NOT VALID OVER ~~THA~~ A SHOCK. IT IS, HOWEVER, ~~VALID OVER A SHOCK~~ BEFORE AND AFTER THE SHOCK. IT SHOULD BE NOTED THOUGH THAT A^* CHANGES OVER THE SHOCK.

b) SUBCRITICAL: SUBSONIC ISENTROPIC FLOW MACH = 1 IS NOT REACHED IN THE THROAT => THE FLOW IS NOT CHOKED => POSSIBLE TO CHANGE THE MASSFLOW BY CHANGING THE BACK PRESSURE

CRITICAL/CHOKED: MACH = 1.0 ~~AT~~ THE NOZZLE THROAT, MAX MASSFLOW FOR THE CURRENT INLET TOTAL PRESSURE. REDUCING THE BACK PRESSURE WILL NOT CHANGE THE MASSFLOW IT IS, HOWEVER, POSSIBLE TO INCREASE THE MASSFLOW ~~BACK~~ BY INCREASING THE INLET TOTAL PRESSURE.

SUPERCRITICAL: SUPERSONIC ISENTROPIC FLOW NO SHOCKS AND PERFECTLY MATCHED EXIT PRESSURE.

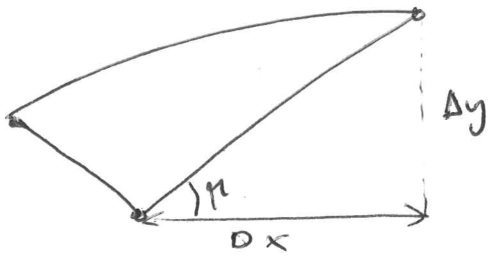
c) CALCULATE THE DESIGN EXIT MACH NUMBER

$$(5.20) \quad \left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_{esc}^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_{esc}^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

(SUPERSONIC SOLUTION)

$\Rightarrow M_{esc} = 2.74$

d) CALCULATE DX IN THE PROVIDED FIGURE



$$\sin(\mu) = \frac{1}{M_{esc}} \quad (1)$$

$$\tan(\mu) = \frac{\Delta y}{\Delta x} \quad (2)$$

$$\Delta y = \frac{A_e}{2} \quad (3)$$

$$\left.\begin{aligned} (1) &\Rightarrow \mu = f(M_{esc}) \\ (3) &\Rightarrow \Delta y = f(A_e) \end{aligned}\right\} \Rightarrow \Delta y = 3.3\ m, \ \mu = 21.4^\circ$$

$(2) \Rightarrow \Delta x = 8.4\ m$

c) CALCULATE THE NOZZLE PRESSURE RATIO REQUIRED TO GET SUPERCRITICAL FLOW

$$(3.30) \quad \left(\frac{P_o}{P_{esc}}\right) = \left(1+\frac{\gamma-1}{2}M_{esc}^2\right)^{\gamma/(\gamma-1)} = 24.68$$

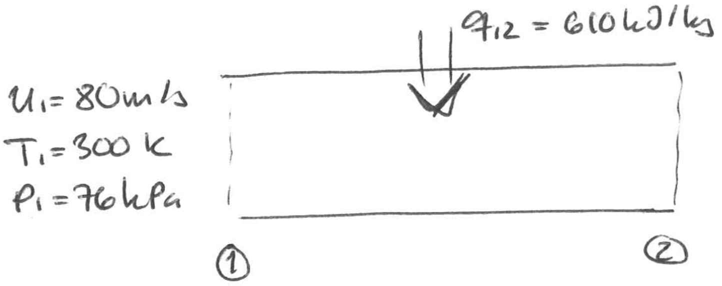
d) CALCULATE THE NOZZLE PRESSURE RATIO CORRESPONDING TO NORMAL SHOCK AT EXIT

(3.57)

$$\frac{P_{base}}{P_{esc}} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{esc}^2 - 1\right)$$

$$\frac{P_o}{P_{base}} = \frac{P_o}{P_{esc}}\,\frac{P_{esc}}{P_{base}} = 2.88$$

P2 (COMBUSTION)



ASSUME: CALORICALLY PERFECT GAS, NO FRICTION.

a) CALCULATE THE HEAT ADDITION THAT WILL LEAD TO THERMAL CHOKING.

(3.89) $$\frac{T_{01}}{T_0^*} = \frac{(\gamma+1)M_1^2}{(1+\gamma M_1^2)^2}\left(2+(\gamma-1)M_1^2\right)$$

$$M_1 = u_1/a_1 = u_1/\sqrt{\gamma R T_1} = 0.23$$

$$q_1^* = C_p\left(T_0^* - T_{01}\right) = C_p T_{01}\left(\frac{T_0^*}{T_{01}} - 1\right)$$

(3.28) $$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2 \Rightarrow T_{01} = 303.2\ K$$

$$\Rightarrow \underline{q_1^* = 1060.2\ kJ/kg}$$

b) CALCULATE THE EXIT FLOW CONDITION.

$$q_{12} = C_p\left(T_{02} - T_{01}\right) \Rightarrow T_{02} = T_{01} + q_{12}/C_p$$

$$\Rightarrow T_{02} = 910.45$$

(3.89) $$\frac{T_{01}}{T_0^*} = \frac{(\gamma+1)M_1^2}{(1+\gamma M_1^2)^2}\left(2+(\gamma-1)M_1^2\right) \Rightarrow \frac{T_{01}}{T_0^*} = 0.22$$

$$\frac{T_{02}}{T_0^*} = \frac{(\gamma+1)M_2^2}{(1+\gamma M_2^2)^2}\left(2+(\gamma-1)M_2^2\right) \Rightarrow M_2 = 0.49$$

(3.85)
$$\frac{p_1}{p^*} = \frac{1+\gamma}{1+\gamma M_1^2}$$
$$\frac{p_2}{p^*} = \frac{1+\gamma}{1+\gamma M_2^2}$$
$\Rightarrow p_2 = 61.4\ \text{kPa}$

(3.86)
$$\frac{T_1}{T^*} = M_1^2\left(\frac{1+\gamma}{1+\gamma M_1^2}\right)^2$$
$$\frac{T_2}{T^*} = M_2^2\left(\frac{1+\gamma}{1+\gamma M_2^2}\right)^2$$
$\Rightarrow T_2 = 869.4\ \text{K}$

$$u_2 = M_2 a_2 = M_2\sqrt{\gamma R T_2} = 287.1\ \text{m/s}$$

c) CALCULATE THE TOTAL PRESSURE LOSS OVER THE COMBUSTOR

$$(p_{01} - p_{02})/p_{01}$$

(3.30)
$$\frac{p_{01}}{p_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \Rightarrow p_{01} = 78.86\ \text{kPa}$$
$$\frac{p_{02}}{p_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)} \Rightarrow 72.12\ \text{kPa}$$

$$\Rightarrow (p_{01} - p_{02})/p_{01} = \underline{8.54\ \%}$$

d) IS THE CALORICALLY PERFECT GAS ASSUMPTION VALID?

THE MAXIMUM TEMPERATURE IS OUTSIDE OF THE RANGE OF TEMPERATURES FOR WHICH THE CALORICALLY PERFECT GAS ASSUMPTION IS VALID. THE TEMPERATURE IS, HOWEVER, CLOSE TO THE UPPER LIMIT $\Rightarrow$ ERRORS INTRODUCED ARE PROBABLY MINOR..

c) WHAT IS THE FUNDAMENTAL DIFFERENCE BETWEEN CALORICALLY PERFECT GAS AND THERMALLY PERFECT GAS?

THERMALLY PERFECT GAS INCLUDES MOLECULAR VIBRATONAL ENERGY ~~WHIC~~ WITH THE CONSEQUENCE THAT C_v, C_p, γ ARE FUNCTIONS OF TEMPERATURE

CALORICALLY PERFECT GAS ONLY INCLUDES TRANSLATIONAL AND ROTATIONAL ENERGY WHICH MEANS THAT C_v, C_p, γ ARE CONSTANTS.

P3 (SHOCK INTERACTION)

a) WHAT IS THE REASON FOR THE FORMATION OF THE SLIPLINE AT THE SHOCK-INTERSECTION POINT?

THE FLOW THAT PASS THROUGH THE LOWER PART OF THE SHOCK-SYSTEM (1 → 2 → 3) WILL HAVE LOWER ENTROPY THAN THE FLOW THAT PASS THROUGH THE UPPER PART (1 → 4) SINCE THE SINGLE SHOCK WILL BE STRONGER.

A SLIP LINE IS A SEPARATING LINE BETWEEN REGIONS OF DIFFERENT ENTROPY.

b) WHAT QUANTITIES MUST BE CONSTANT OVER A SLIPLINE.

PRESSURE AND FLOW DIRECTION.

c) WITH THE UPSTREAM MACH NUMBER SET TO $M_1 = 3.0$, CALCULATE M_2, M_3, M_4

$\theta_1 = 10°$, $M_1 = 3.0$

$(\theta-\beta-M) \Rightarrow \beta_1 = 27.4°$

$$\left.\begin{array}{ll} (4.7) & M_{n_1} = M_1 \sin(\beta_1) \\ (4.10) & M_{n_2}^2 = \dfrac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1} \\ (4.12) & M_2 = M_{n_2} / \sin(\beta_1 - \theta_1) \end{array}\right\} \Rightarrow M_2 = 2.5$$

$\theta_2 = 10°$, $M_2 = 2.5$

$(\theta-\beta-M) \Rightarrow \beta_2 = 31.7°$

(4.7), (4.10) & (4.12) $\Rightarrow M_3 = 2.1$

$\theta_4 = \theta_1 + \theta_2 = 20°$, $M_1 = 3.0$

$(\theta-\beta-M) \Rightarrow \beta_4 = 37.8°$

$(4.7), (4.10)$ & $(4.12) \rightarrow M_4 = 2.0$

d) CALCULATE THE PRESSURE RATIOS P_3/P_1 AND P_4/P_1

<u>1 → 2</u>

$\beta_1 = 27.4°$

$M_{n_1} = M_1 \sin(\beta_1)$

(4.9)

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) = 2.05$$

<u>2 → 3</u>

$\beta_2 = 31.7°$

$M_{n_1} = M_2 \sin(\beta_2)$

$$\frac{P_3}{P_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) = 1.87$$

<u>1 → 4</u>

$\beta_4 = 37.8°$

$M_n = M_1 \sin(\beta_4)$

$$\frac{P_4}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) = \underline{3.77}$$

$$\frac{P_3}{P_1} = \frac{P_3}{P_2}\frac{P_2}{P_1} = \underline{3.83}$$

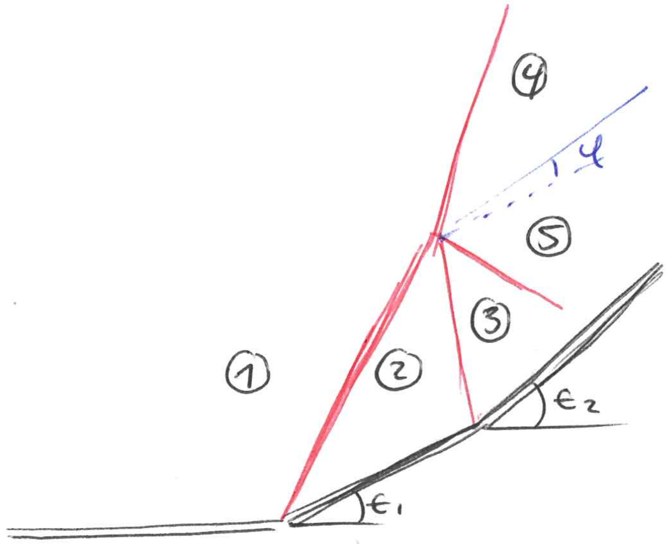
e) IS THE NEGLIGIBLE SLIPLINE DEFLECTION A FAIR ASSUMPTION ?

NO SLIPLINE DEFLECTION IMPLIES THAT

$$\frac{p_4}{p_1} = \frac{p_3}{p_1}$$

SINCE THE PRESSURE SHOULD BE THE SAME IN REGIONS 3 AND 4.

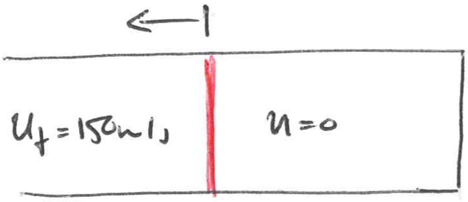
NOW, THERE IS A SLIGHT DIFFERENCE BUT IT IS STILL A FAIR ASSUMPTION. (OR MAYBE NOT AFTER ALL..)



INVESTIGATIONS SHOW THAT IN ORDER TO GET THE SAME FLOW DIRECTION AND PRESSURE, A FOURTH SHOCK WILL BE GENERATED AND THE DEFLECTION OF THE SLIPLINE WILL BE $\sim 8.45^\circ$

P4 (VALVE)

AIR AT 300.0 K AND 1.5 bar (150 kPa) FLOWS THROUGH A PIPE AT THE UNIFORM VELOCITY 150 m/s. THE END OF THE TUBE IS SUDDENLY CLOSED BY A VALVE, WHICH GENERATES A SHOCK WAVE PROPAGATING UPSTREAM.



IN RELATIVE FRAME OF REFERENCE (FOLLOWING THE SHOCK)

$$u_1 = u_f + W_s$$
$$u_2 = W_s$$

$$\rho_1 u_1 = \rho_2 u_2$$
$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$
$$h_1 + \tfrac{1}{2}u_1^2 = h_2 + \tfrac{1}{2}u_2^2$$

$$\rho_1 (u_f + W_s) = \rho_2 (W_s) \qquad (1)$$
$$P_1 + \rho_1 (u_f + W_s)^2 = P_2 + \rho_2 W_s^2 \qquad (2)$$
$$h_1 + \tfrac{1}{2}(u_f + W_s)^2 = h_2 + \tfrac{1}{2}W_s^2 \qquad (3)$$

$$(1) \Rightarrow \rho_1 (u_f + W_s) = \rho_2 W_s \Rightarrow u_f + W_s = \frac{\rho_2}{\rho_1} W_s$$

$$M_s = \frac{u_f + W_s}{a_1} \Rightarrow (u_f + W_s) = M_s a_1 \;,\; W_s = M_s a_1 - u_f$$

$$\left(\frac{\rho_1}{\rho_2}\right) = \frac{2 + (\gamma - 1) M_s^2}{(\gamma + 1) M_s^2}$$

$$M_s a_1 = \left(\frac{(\gamma+1) M_s^2}{2 + (\gamma - 1) M_s^2}\right)(M_s a_1 - u_f)$$

$$\frac{u_f}{a_1} = M_s \left(1 - \frac{2 + (\gamma + 1) M_s^2}{(\gamma+1) M_s^2}\right)$$

$$\frac{u_f}{a_1} = M_s\left(\frac{2M_s^2 - 2}{(\gamma+1)M_s^2}\right) = 2\left(\frac{M_s^2 - 1}{(\gamma+1)M_s}\right)$$

$$\frac{u_f}{2a_1} = \frac{M_s^2 - 1}{(\gamma+1)M_s}$$

$$\Rightarrow M_s = \frac{u_f}{4a_1}(\gamma+1) \overset{+}{(-)} \sqrt{\left(\frac{u_f}{4a_1}(\gamma+1)\right)^2 + 1}$$

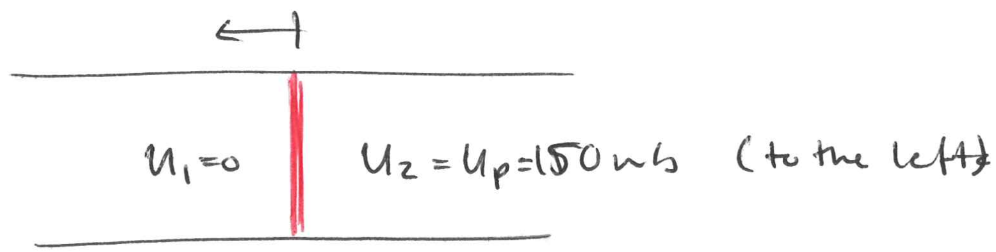
$$\Rightarrow M_s = 1.29 \quad , \quad (W_s = 298.66\ m/s)$$

(3.57)
$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_s^2 - 1) \Rightarrow P_2 = 267.2\ kPa$$

$$\frac{\rho_2}{\rho_1} = \frac{2 + (\gamma-1)M_s^2}{(\gamma+1)M_s^2} = 1.5$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} \Rightarrow T_2 = 355.8\ K$$

ALTERNATIVE SOLUTION:



FRAME OF REFERENCE FOLLOWING THE TUBE FLOW BEFORE THE VALVE IS CLOSED

(7.16)
$$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

$$\Rightarrow p_2/p_1 = 1.78 \Rightarrow P_2 = 267.2\ kPa$$

(7.13) $\mathcal{M}_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1}-1\right)+1}$

$\Rightarrow \mathcal{M}_s = 1,29$

(7.10) $$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left(\frac{\frac{\gamma+1}{\gamma-1}+\frac{p_2}{p_1}}{1+\frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}\right) \Rightarrow T_2 = 355,8\,K$$

b) ALTHOUGH A NORMAL SHOCK IS ADIABATIC, TOTAL ENTHALPY IS NOT CONSTANT OVER A MOVING SHOCK, WHY?

IT HAS TO DO WITH THE FRAME OF REFERENCE

$u_1 = 0$

$h_{01} = h_1 + \frac{1}{2}u_1^2 = h_1$

$u_2 > 0$

$h_{02} = h_2 + \frac{1}{2}u_2^2$

$h_1 < h_2 \Rightarrow h_{02} > h_{01}$

c) CFD CODE FOR COMPRESSIBLE FLOWS (UNSTEADY):

- DENSITY BASED (SOLVE FOR DENSITY IN CONTINUITY EQN)
- FULLY COUPLED (SOLVE ALL EQUATIONS COUPLED)
- CONSERVATIVE
- EXPLICIT

P5 (PIPE FLOW WITH FRICTION)

A PIPE WITH THE DIAMETER $D = 50\,mm$ IS CONNECTED TO A RESERVOIR WITH THE PRESSURE AND TEMPERATURE $p_0 = 13.8$ bar $(= 1380\,kPa)$ AND $T_0 = 310.0\,K$

THE PIPE EXIT IS OPEN TO THE ATMOSPHERE

THE AVERAGE FRICTION COEFFICIENT IS $\bar{f} = 0.005$

THE MASSFLOW IS $\dot{m} = 2.25\,kg/s$

ASSUME CALORICALLY PERFECT AIR.

$$\rho u = \frac{4\dot{m}}{\pi D^2} = \frac{p_1}{RT_1} a_1 M_1 = \frac{p_1}{RT_1}\sqrt{\gamma R T_1}\, M_1$$

$$= \frac{p_1}{\sqrt{T_1}}\sqrt{\frac{\gamma}{R}}\, M_1 \qquad (1)$$

$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \qquad (2)$$

$$\frac{p_0}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)} \qquad (3)$$

$$(1) \rightarrow (3) \Rightarrow \rho u = \frac{4\dot{m}}{\pi D^2} = \frac{p_0}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}}\, \frac{M_1}{\left(1 + \frac{\gamma-1}{2} M_1^2\right)^{(\gamma+1)/(2(\gamma-1))}}$$

$$\Rightarrow \frac{4\dot{m}\sqrt{T_0}}{\pi D^2 p_0}\sqrt{\frac{R}{\gamma}} = \frac{M_1}{\left(1 + \frac{\gamma-1}{2} M_1^2\right)^{(\gamma+1)/(2(\gamma-1))}}$$

GUESS M_1, ITERATE UNTIL CONVERGED $\Rightarrow$

$M_1 = 0.22$

(3.107)

$$\frac{4\bar{f}L_1^*}{D} = \frac{1 - M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2 + (\gamma-1)M_1^2}\right)$$

$$\Rightarrow L_1^* = 30.56\,m$$

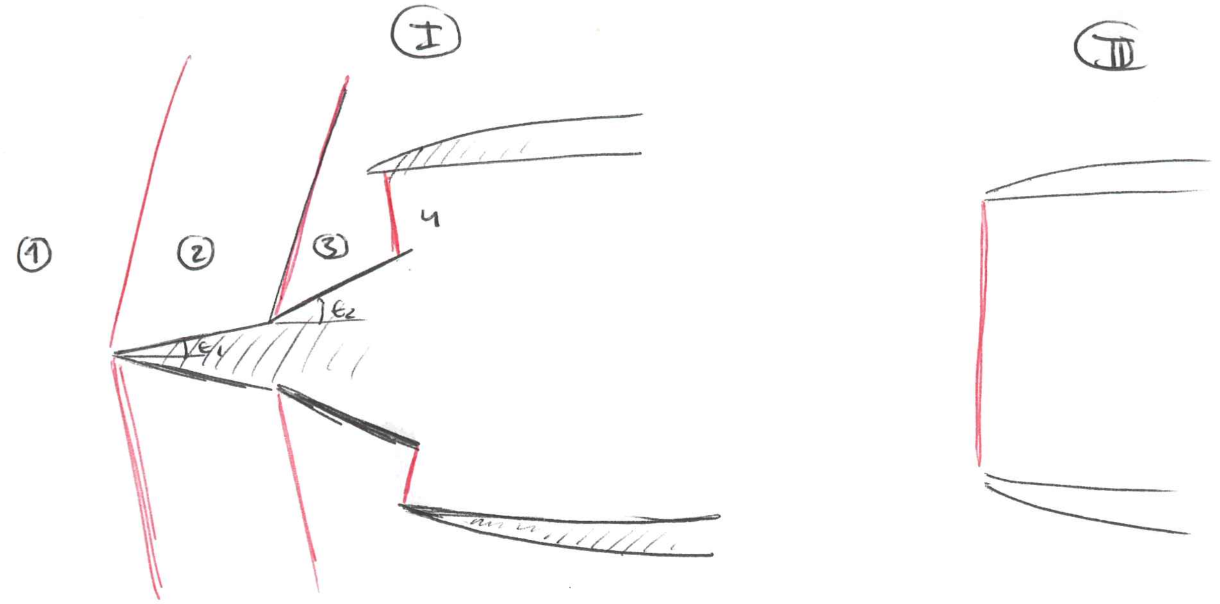
THE PIPE LENGTH THAT GIVES CHOKED EXIT IS

$$L_i^* = 30.56\,m$$

b) WHAT WOULD HAPPEN IF THE PIPE WAS LONGER THAN 30.56 m.

A LONGER PIPE WOULD RESULT IN A REDUCTION OF THE MASSFLOW SUCH THAT THE PIPE EXIT WOULD BE CHOKED. THIS IS ACCOMPLISHED BY CHANGING THE INLET STATIC FLOW PROPERTIES WITHOUT MODIFYING TOTAL FLOW PROPERTIES.

P6 (ENGINE INLET)



$\epsilon_1 = 8°$, $\epsilon_2 = 16°$ (flow deflection: 8°)

a) CALCULATE THE MACH NUMBER OF THE FLOW ENTERING THE ENGINE IN THE TWO CASES. IF THE FREESTREAM MACH NUMBER IS $M = 3.0$

(I) 1 → 2

$\epsilon_1 = 8°$, $M_1 = 3.0$

$(\epsilon - \beta - M) \Rightarrow \beta_1 = 25.6°$

(4.7) $M_{n_1} = M_1 \sin(\beta)$

(4.10) $M_{n_2}^2 = \dfrac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$

(4.12) $M_2 = M_{n_2}/\sin(\beta - \epsilon)$

$\beta_1 = 25.6°$, $\epsilon_1 = 8°$, $M_1 = 3.0$
$\Rightarrow M_2 = 2.6$

2 → 3

$\epsilon_2 = 8°$, $M_2 = 2.6$

$(\epsilon - \beta - M) \Rightarrow \beta_2 = 28.9°$

(4.7), (4.10) & (4.12) $\Rightarrow M_3 = 2.3$

3 → 4 (Normal shock)

(3.51) $M_4^2 = \dfrac{1 + ((\gamma - 1)/2)M_3^2}{\gamma M_3^2 - (\gamma-1)/2}$ $\Rightarrow M_4 = 0.54$

(II) (3.58) $$M_2^2 = \frac{1+((\gamma-1)/2)M_1^2}{\gamma M_1^2-(\gamma-1)/2} \Rightarrow M_2 = 0{,}48$$

b) CALCULATE THE DIFFUSION EFFICIENCY FOR BOTH CONCEPTS

$$\eta = \frac{\left(\frac{p_{01}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1}{\left(\frac{\gamma+1}{2}\right)M_1^2}$$

I

(3.30) $$\frac{p_{01}}{p_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)}$$

(II)

$$\frac{p_{04}}{p_{01}} = \frac{p_{04}}{p_{03}}\frac{p_{03}}{p_{02}}\frac{p_{02}}{p_{01}}$$

$$\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_2}\frac{p_2}{p_1}\frac{p_1}{p_{01}}$$

(3.30) $$\frac{p_{02}}{p_2} = \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}$$

(4.9) $$\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma+1}(M_{n1}^2-1)$$

$$\Rightarrow \frac{p_{02}}{p_{01}} = 0{,}98$$

$$\frac{p_{03}}{p_{02}} = \frac{p_{03}}{p_3}\frac{p_3}{p_2}\frac{p_2}{p_{02}}$$

(3.30) $$\frac{p_{03}}{p_3} = \left(1+\frac{\gamma-1}{2}M_3^2\right)^{\gamma/(\gamma-1)}$$

(4.9) $$\frac{p_3}{p_2} = 1+\frac{2\gamma}{\gamma+1}(M_{n1}^2-1)$$

(M_{n1} for shock 2)

$$\Rightarrow \frac{p_{03}}{p_{02}} = 0{,}99$$

$$\frac{p_{04}}{p_{03}} = \frac{p_{04}}{p_4}\frac{p_4}{p_3}\frac{p_3}{p_{03}}$$

(3.30) $$\frac{p_{04}}{p_4} = \left(1+\frac{\gamma-1}{2}M_4^2\right)^{\gamma/(\gamma-1)}$$

(3.57) $$\frac{p_4}{p_3} = 1+\frac{2\gamma}{\gamma+1}(M_3^2-1)$$

$$\frac{p_{04}}{p_{03}} = 0{,}60$$

$$\frac{p_{04}}{p_{01}} = \frac{p_{04}}{p_{03}}\frac{p_{03}}{p_{02}}\frac{p_{02}}{p_{01}} = 0{,}58$$

II

$$\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_2}\frac{p_2}{p_1}\frac{p_1}{p_{01}}$$

(3.57) $$\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma+1}(M_1^2-1)$$

(3.30) $$\frac{p_{02}}{p_2} = \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_{01}}{p_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)}$$

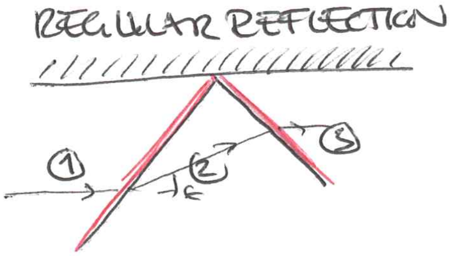
$$\frac{p_{02}}{p_{01}} = 0{,}33$$

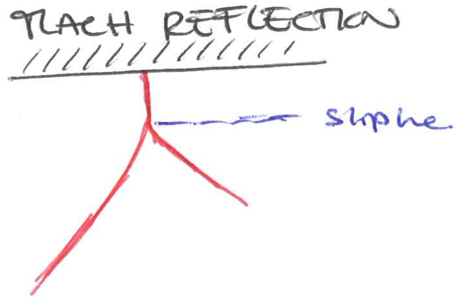
$$\eta_I = 0{,}78$$

$$\eta_{II} = 0{,}58$$

THE SHOCK SYSTEM IN CASE I LEADS TO LOWER LOSSES (~~LESS~~ ~~STRONG~~ WEAKER SHOCKS) THAN THE SINGLE NORMAL SHOCK.

c) DESCRIBE TWO TYPES OF SHOCK REFLECTIONS AT A SOLID WALL





A REGULAR REFLECTION WILL OCCUR IF POSSIBLE IN THE EXAMPEL SHOWN ABOVE THE FIRST SHOCK DEFLECTS THE FLOW AN ANGLE θ AND AT THE ~~FLOW~~ WALL THE FLOW MUST BE DEFLECTED BACK IF $\theta < \theta_{max}$ AT $M = M_2$ WE WILL GET A REGULAR REFLECTION. IF $\theta > \theta_{max}$ WE WILL INSTEAD GET A MACH REFLECTION