TME085 - Compressible Flow

2020-08-19, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Responsible teacher: Niklas Andersson tel.: 070 - 51 38 311

Good luck!

Instructions

General Info

Due to the extraordinary situation caused by the very high risk of the covid-19 infection spreading in Sweden, Chalmers' president has decided that all written exams will be carried out from home.

Exam Info

The exam consists of six problems (each problem is a separate assignment in Canvas) The problems can give a maximum of 10 points each. In total you can get 60 points on the exam. The points earned for the Compressible Flow Project is added to your exam result if applicable.

The total number of points on the exam (EP) and the bonus points earned from The Compressible Flow Project (BP) is translated into a course grade as follows:

- Fail: $EP < 24$ (i.e. the bonus points can not be used to pass the course)
- Grade 3: $24 \leq EP < (36 - BP)$
- Grade 4: $(36 - BP) \leq EP < (48 - BP)$
- Grade 5: $(48 - BP) \leq EP$

Niklas Andersson will be available for questions related to the exam from 8:30 until 13:30 the day of the exam (2020-08-19)

If you would like to get in contact with Niklas during the exam, you can send a Canvas message, call or send a mail

- mobile: 070-5138311
- mail: niklas.andersson@chalmers.se

Instructions

The written exam should be handed in through Canvas at the latest 15 minutes past the end of the exam time. If it is not possible to hand-in the exam through Canvas, it should be sent to the niklas.andersson@chalmers.se as soon as possible.

- The exam is divided into a number of separate assignments. You should submit a document with answers/solutions for each of these assignments. Do not wait until the last minute with the submission of files. It is better to submit files continuously as you solve the problems. You can always go back and update if you find mistakes later.
- Answers should be written in a text document (one document for each assignment). Calculations etc. may be written on paper and subsequently be photographed or scanned and included as images in the text document.
- If you use Matlab scripts, Python or any other programming languages to solve the problems you can paste your code snippets in the text document if you think that it will be helpful for the correction of the problems. Note! you will still have to explain what you have done in words, just code will not be sufficient.
- In case you have used some type of graphical representation of your solution (Matlab plots, matplotlib, gnu plot, ...), you could add these figures to your solution document if it adds value

- If you have used an iterative solution procedure using for example Matlab, you could add output from these iterations to your solution
- The exam is to be carried out individually, ie., collaboration is not allowed.
- Due to the current circumstances, all examination aids are allowed.
- Control for plagiarism will be carried out automatically for each of the problems.
- The exam cannot be written anonymously.

Note! By uploading your exam solutions you certify that you have solved the problems on your own without receiving any help from anyone else

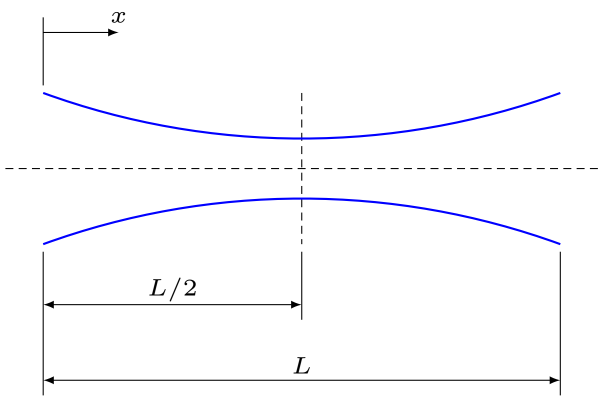
General Exam Guide

- Always write down and justify your assumptions
- For some problems you may have to guess values on some properties that has not been given in the problem description
- Some problem descriptions may include data that you will not need for solving the problem
- It is not uncommon that an iterative solution process in needed to be able to solve a problem
- Even if it is difficult in some situations, always try to determine whether your results are realistic or not. An unrealistic solution is worth a bit more if you make a comment about the results and why you think that it is unrealistic.
- Always write down your planned solution process in words. If you do something wrong along the way or if you run out of time and leave the problem unfinished, a description of how to solve the problem goes a long way when it comes to the number of rewarded points (if it is correct of course)
- The header of each problem indicates the total number of points and the number of subtasks.

Problem 1 - NOZZLE FLOW (10 p., 3 subtasks)

A converging-diverging nozzle with an exit to throat area ratio, A_e/A_t, of 1.633, is designed to operate with atmospheric pressure at the exit plane, $p_e = p_{atm}$. The converging-diverging nozzle area, A, varies with position, x, as:

$$\frac{A(x)}{A_t} = \left(\frac{A_e}{A_t} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1$$



(a) At what pressure ratio or ranges of pressure ratios will the nozzle flow be (a) perfectly expanded, (b) overexpanded, (c) underexpanded?

(b) Calculate the maximum mass flow through the nozzle if the reservoir temperature is 293.0 K and the exit area is 0.33 m^2

(c) Will there be a normal shock in the nozzle if nozzle pressure ratio is $p_o/p_{atm} = 1.5$? If so, at what position (x/L) will the normal shock occur?

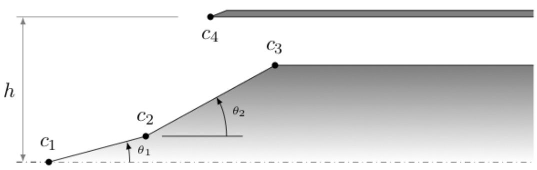
Problem 2 - ENGINE INTAKE DESIGN (10 p., 3 subtasks)

An axisymmetric engine intake for supersonic flight is represented in two dimensions in the figure below. The intake geometry can be modified in flight such that the engine intake performance is optimized. Let's say that the aircraft flies at a speed corresponding to Mach number 3.0 and that the surrounding pressure and temperature are -20.0 degrees Celsius and 0.47 bar, respectively. The intake geometry is designed such that, at design conditions, three oblique shocks should be formed with endpoints in the coordinates $c_1 - c_4$. After these three initial oblique shocks, a system of shocks will be generated in the engine intake duct that we will not consider here. The angles θ_2 and θ_3 are 4.0 and 12.0 degrees, respectively.

(a) Let's say that one would like to measure the pressure in the freestream downstream of the three first oblique shocks using a pitot tube, what would the measured pressure be? (6p.)

(b) What would happen if the flight Mach number were to be increased without modifying the geometry (no calculations needed) (2p.)

(c) What would happen if the flight Mach number were to be decreased without modifying the geometry (no calculations needed) (2p.)



coordinate	x [m]	y [m]
c_1	0.00	0.00
c_2		
c_3		
c_4		0.4

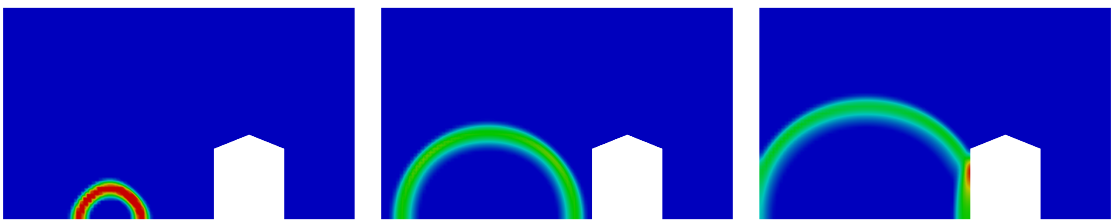
Problem 3 - GROUND EXPLOSION (10 p., 3 subtasks)

The pictures below give a schematic representation of a shock wave generated by a ground explosion.

(a) Calculate the pressure ratio over the propagating shock wave if the induced velocity behind the shock wave corresponds to a Mach number of 0.9 (6p.)

(b) Under the circumstances calculated in subtask 1, what will the propagation velocity of the shock wave be (2p.)

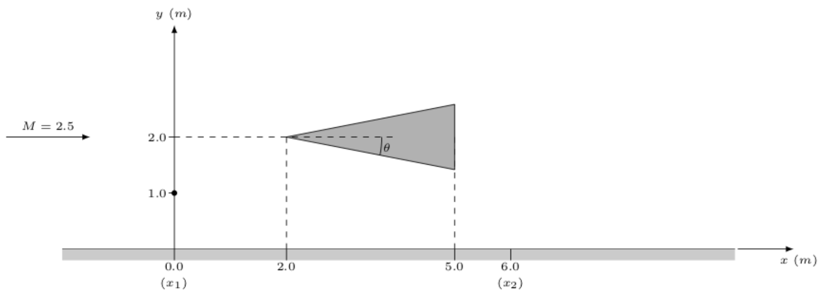
(c) Let's assume that the shock wave reaches a solid wall (as indicated in the figure), at the ground, where the shock propagates in the wall-normal direction, calculate the pressure and temperature at the wall as the shock reflects (2p.)



Problem 4 - WEDGE FLOW (10 p.)

A wedge-shaped object is placed over a flat surface according to the figure below. Air at 1.0 bar and 300.0 K is flowing passed the wedge. The freestream Mach number ahead of the object is 2.5. The wedge half angle (θ) is 11.0 degrees and the axial extent of the wedge is 3.0 m.

Calculate and plot the pressure difference between locations along a streamline starting at (0.0,1.0), the black dot on the y-axis in the figure, and locations on the wall with the same axial coordinate, i.e. pressure difference as a function of axial coordinate, for $x_1 \leq x \leq x_2$. Locations of discrete changes in pressure difference should be justified by calculations.



Problem 5 - TUBE FLOW WITH FRICTION (10 p.)

An experimental setup for estimation of the friction coefficient for supersonic air flow in a tube comprises a convergent divergent nozzle attached to a round tube. The nozzle inlet conditions are $p_o = 6.73\ MPa$ and $T_o = 312.0\ K$. The nozzle throat diameter (d_{th}) and the nozzle exit diameter (D_e) are 0.0061 m and 0.0127 m, respectively. The tube attached to the nozzle has the same inner diameter as the nozzle at the nozzle exit plane, i.e. it is an axial extension of the nozzle. The nozzle flow is isentropic throughout and the flow in the convergent part of the nozzle is supersonic. The entire system can be assumed to be adiabatic.

Pressure is measured at two axial locations in the tube, see table below. Based on the given data, calculate the average friction coefficient for the specific tube and flow.

station	x/D_e	p (kPa)
1	1.75	238.0
2	29.6	485.0

Problem 6 - FLOW OVER FLAT PLATE (10 p., 3 subtasks)

Two researchers usually working with low-speed flows are testing a flat plate at an angle of attack ($\alpha = 20°$) in a high-speed freestream ($U_\infty = 860\ m/s$) in a supersonic wind tunnel. The wind tunnel freestream temperature and pressure are 20 degrees and 1.0 atm, respectively. The plate has a length in the flow direction $L = 0.4\ m$ and is significantly longer in the spanwise direction thus the flow can be assumed to be fairly well represented using a 2D flow model. The researchers finds it rather strange that although the flow is attached to the surface of the flat plate, the force balance, the devise used for measuring forces on the object placed in the wind tunnel, indicates a large drag component in the total force exerted on the flat plate even for zero angle of attack. Doing some quick hand calculations, they figure out that the force is way greater than forces associated with surface friction and thus they ask the operators of the wind tunnel to recalibrate the force balance. The operators simply laugh at them and asks them to go back and do their homework.

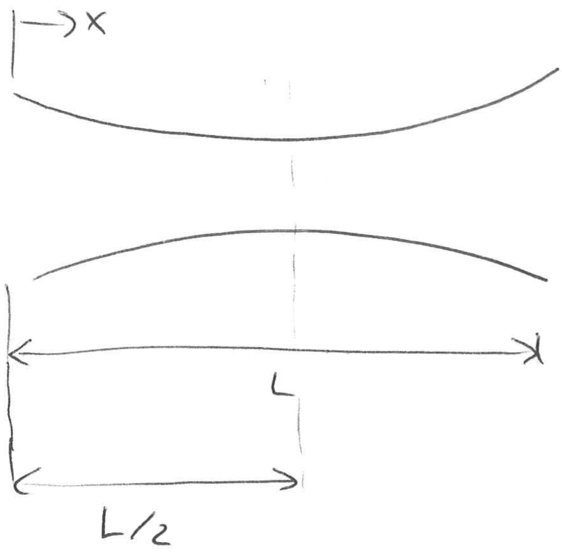
(a) What physical phenomena caused the drag component in the measured force? (1p.)

(b) Calculate the lift and drag components of the force exerted on the flat plate as a function of span length (3p.)

(c) Locally, near the trailing edge, there will be a net turning of the flow by the presence of the flat plate. calculate the flow angle just downstream of the trailing edge of the plate. (6p.)

TME085 EXAM 2020-08-19

P1 (NOZZLE FLOW)



$$\frac{A(x)}{A_t} = \left(\frac{A_e}{A_t} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1$$

$$\frac{A_e}{A_t} = 1.633$$

a) FOR WHAT PRESSURE RATIO OR RANGES OF PRESSURE RATIOS WILL THE NOZZLE FLOW BE

i) PERFECTLY EXPANDED

ii) OVEREXPANDED

iii) UNDEREXPANDED

TO FIND THE RANGES OF PRESSURE RATIOS, WE NEED THE PRESSURE RATIOS CORRESPONDING TO SHOCK AT EXIT AND SUPERCRITICAL FLOW (LATER WE WILL ALSO NEED THE CRITICAL PRESSURE RATIO)

(5.20) $$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$M_{ec} = 0.39$ (SUBSONIC SOLUTION)

$M_{ex} = 1.96$ (SUPERSONIC SOLUTION)

(3.30)
$$NPR_c = \frac{p_o}{p_{ec}} = \left(1 + \frac{\gamma-1}{2}M_{ec}^2\right)^{\gamma/(\gamma-1)} = 1.11$$

$$NPR_{sc} = \frac{p_o}{p_{esc}} = \left(1 + \frac{\gamma-1}{2}M_{ex}^2\right)^{\gamma/(\gamma-1)} = 7.36$$

NORMAL SHOCK AT EXIT

(3.57)

$$\frac{P_{bnse}}{P_{esc}} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{esc}^2 - 1\right)$$

$$NPR_{nse} = \frac{P_0}{P_{bnse}} = \frac{P_0}{P_{esc}}\,\frac{P_{esc}}{P_{bnse}} = 1.7$$

PERFECTLY EXPANDED NOZZLE FLOW:

$$NPR = NPR_{sc} = 7.36$$

OVEREXPANDED NOZZLE FLOW:

$$NPR_{nse} < NPR < NPR_{sc}$$

$$1.7 < NPR < 7.36$$

UNDEREXPANDED NOZZLE FLOW:

$$NPR > NPR_{sc} = 7.36$$

b) CALCULATE THE MAXIMUM MASS FLOW THROUGH THE NOZZLE IF THE RESERVOIR ~~PRESSURE~~ TEMPERATURE IS 293.0K AND THE EXIT AREA IS $0.33\,m^2$

WE WILL ALSO NEED THE TOTAL PRESSURE

ASSUME THAT THE AMBIENT PRESSURE IS $P_{AMB} = 101325$ Pa

$\Rightarrow P_0 = NPR_c \cdot P_{amb}$ (CHOKED FLOW)

(5.21) $$\dot{m} = \frac{P_0 A_t}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} = 53.6\ kg/s$$

c) WILL THERE BE A SHOCK INSIDE THE NOZZLE IF THE PRESSURE RATIO IS $P_0/P_{amb} = 1.5$? IF SO, AT WHAT x/L WILL THE SHOCK BE LOCATED?

WE WILL GET A SHOCK IN THE NOZZLE FOR $1.11 < NPR < 1.7$

$\Rightarrow$ THERE WILL BE A SHOCK FOR $NPR = 1.5$

(5.28) $$M_e^2 = \frac{-1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}\left(\frac{P_{01}A_t}{P_e A_e}\right)^2}$$

$(P_{01}/P_e = 1.5) \Rightarrow M_e = 0.52$

(3.30)
$$\frac{P_{02}}{P_e} = \left(1 + \frac{\gamma-1}{2}M_e^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}}{P_e}\frac{P_e}{P_{01}} = 0.80$$

(5.21)
$$\dot{m} = \frac{P_{01}A_1^*}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} = \frac{P_{02}A_2^*}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

(T_0 IS NOT AFFECTED BY THE SHOCK)

$\Rightarrow P_{01}A_1^* = P_{02}A_2^*$

(5.20)
$$\left(\frac{A}{A_1^*}\right)^2 = \frac{1}{M_1^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_1^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\left(\frac{A}{A_2^*}\right) = \frac{1}{M_2^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_2^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

(3.51) $$M_2^2 = \frac{1 + ((\gamma-1)/2)M_1^2}{\gamma M_1^2 - (\gamma-1)/2}$$

ITERATE $\Rightarrow M_1 = 1.83$ (MACH NUMBER AHEAD OF SHOCK)

$A_1^* = A_t$

(5.20)
$$\left(\frac{A}{A_t}\right)^2 = \frac{1}{M_1^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_1^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

GIVEN AREA FUNCTION

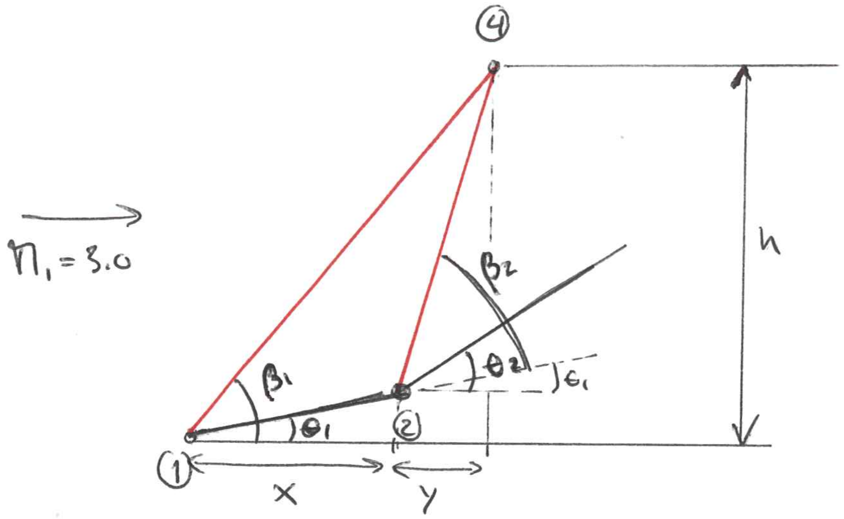
$$\left(\frac{A}{A_t}\right) = \left(\frac{A_e}{A_t} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1 \Rightarrow$$

$$\left(\frac{A}{A_t} - 1\right) \Big/ \left(\frac{A_e}{A_t} - 1\right) = \left(2\frac{x}{L} - 1\right)^2 \Rightarrow$$

$$\frac{x}{L} = \frac{1}{2}\left(\sqrt{\left(\frac{A}{A_t} - 1\right) \Big/ \left(\frac{A_e}{A_t} - 1\right)} + 1\right) = \underline{0.93}$$

P4 (ENGINE INTAKE)

OBTAIN ENGINE INTAKE COORDINATES FOR A THREE-SHOCK DESIGN.



$$(x+y)\tan\beta_1 = h \qquad (1)$$

$$x\tan\epsilon_1 + y\tan(\beta_2+\theta_1) = h \qquad (2)$$

$$(1) \Rightarrow \quad x\tan(\beta_1) = h - y\tan(\beta_1)$$

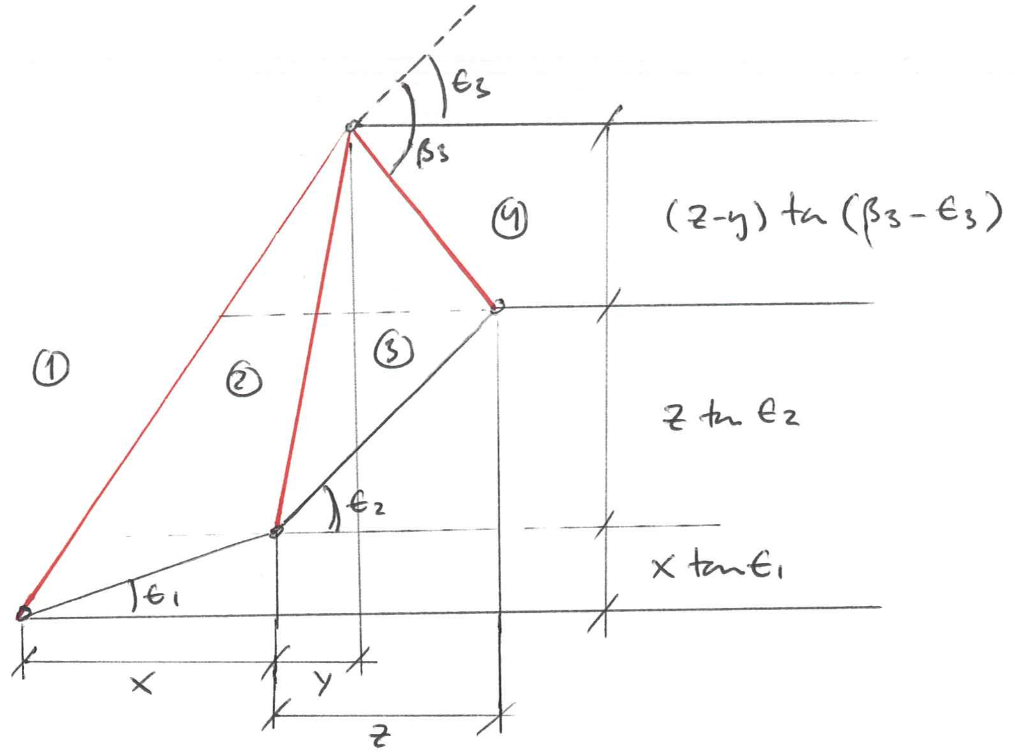
$$x = \frac{h - y\tan(\beta_1)}{\tan\beta_1} = \frac{h}{\tan\beta_1} - y \qquad (3)$$

(3) in (2) $\Rightarrow$

$$\left(\frac{h}{\tan\beta_1} - y\right)\tan\theta_1 + y\tan(\beta_2+\epsilon_1) = h$$

$$y\left(\tan(\beta_2+\theta_1) - \tan(\epsilon_1)\right) = h\left(1 - \frac{\tan\epsilon_1}{\tan\beta_1}\right)$$

$$\Rightarrow y = h\left(1 - \frac{\tan\epsilon_1}{\tan\beta_1}\right) \Big/ \left(\tan(\beta_2+\epsilon_1) - \tan(\epsilon_1)\right) \qquad (4)$$



$$x \tan \epsilon_1 + z \tan \epsilon_2 + (z-y) \tan(\beta_3 - \epsilon_3) = h$$

$$x \tan \epsilon_1 + z(\tan \epsilon_2 + \tan(\beta_3 - \epsilon_3)) - y \tan(\beta_3 - \epsilon_3) = h$$

$$z(\tan \epsilon_2 + \tan(\beta_3 - \epsilon_3)) = h + y \tan(\beta_3 - \epsilon_3) - x \tan \epsilon_1 \quad (5)$$

TO BE ABLE TO CALCULATE THE COORDINATES WE NEED THE SHOCK ANGLES β_1, β_2 AND β_3

(4.7) $M_{n_1} = M_1 \sin \beta_1$, WITH β_1 FROM $(\epsilon - \beta - M_1)$

(4.10) $M_{n_2}^2 = \dfrac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$

(4.12) $M_2 = M_{n_2} / \sin(\beta_1 - \epsilon_1)$

$M_1 = 3.0$, $\epsilon_1 = 4.0°$ => $\beta_1 = 22.35°$

=> $M_2 = 2.8$

$M_2 = 2.8$, $\epsilon_2 = (12° - 4°)$ => $\beta_2 = 27.16°$

=> $M_3 = 2.4$

$M_3 = 2.4$, $\epsilon_3 = 12°$ => $\beta_3 = 34.6°$

=> $M_4 = 1.9$

COORDINATE 1:

$$x_1 = 0.0$$
$$y_1 = 0.0$$

COORDINATE 2:

$$x_2 = X$$
$$y_2 = X \tan(\epsilon_1)$$

COORDINATE 3:

$$x_3 = X + Z$$
$$y_3 = X \tan(\epsilon_1) + Z \cancel{\tan}(\epsilon_2)$$

COORDINATE 4:

$$x_4 = h / \tan \beta_1$$
$$y_4 = h$$

COORD.	X	Y
1	0.0	0.0
2	0.35	0.08
3	1.36	0.24
4	0.97	0.40

b) CALCULATE THE FLOW CONDITION DOWNSTREAM OF THE THREE OBLIQUE SHOCKS.

$$(4.7) \quad M_{n_1} = M_1 \sin\beta$$

$$(4.9) \quad \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)$$

OBLIQUE SHOCK RELATIONS

$$(4.8) \quad \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_{n_1}^2}{(\gamma-1)M_{n_1}^2 + 2}$$

$$(4.11) \quad \frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$$

$$(4.10) \quad M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$$

$$(4.12) \quad M_2 = M_{n_2} / \sin(\beta - \epsilon)$$

USE (4.7) − (4.12) TO CALCULATE

$$\frac{p_2}{p_1}, \frac{p_3}{p_2}, \frac{p_4}{p_3}$$

$$\frac{T_2}{T_1}, \frac{T_3}{T_2}, \frac{T_4}{T_3}$$

$$p_4 = \frac{p_4}{p_3}\frac{p_3}{p_2}\frac{p_2}{p_1}p_1 = 227{,}3 \text{ kPa}$$

$$T_4 = \frac{T_4}{T_3}\frac{T_3}{T_2}\frac{T_2}{T_1}T_1 = 403.5 \text{ K}$$

$$\rho_4 = \frac{p_4}{RT_4} = 1{,}96 \text{ kg/m}^3$$

$$M_4 = 1{,}94$$

$$u_4 = M_4 a_4 = M_4\sqrt{\gamma R T_4} = 782.6 \text{ m/s}$$

C) WHAT WOULD HAPPEN IF THE FLIGHT MACH NUMBER WAS INCREASED WITHOUT MODIFYING THE GEOMETRY?

THE ANGLE OF THE FIRST SHOCK WOULD DECREASE AND MOVE INTO THE INTAKE

d) FLIGHT MACH NUMBER DECRESES

THE ANGLE OF THE FIRST SHOCK WOULD DECREASE AND MISS THE INTAKE LIP

PRESSURE MEASUREMENT USING PITOT TUBE

=> NORMAL SHOCK IN FRONT OF PITOTUBE

PITOT TUBE MEASURES THE TOTAL PRESSURE

FIRST WE NEED TO CALCULATE THE PRESSURE DOWNSTREAM OF THE SHOCK SYSTEM.

$$p_4 = \frac{p_4}{p_3}\frac{p_3}{p_2}\frac{p_2}{p_1}p_1 = 227.3\ kPa$$

$$M_4 = 1.9$$

NORMAL SHOCK AT $M = 1.9$ =>

(3.57)

$$\frac{p_5}{p_4} = 1 + \frac{2\gamma}{\gamma+1}\left(M_4^2 - 1\right)$$

(3.51)

$$M_5^2 = \frac{1 + ((\gamma-1)/2)M_4^2}{\gamma M_4^2 - (\gamma-1)/2}$$

=> $p_5 = 963.9\ kPa$

$M_5 = 0.59$

(3.30)

$$\frac{p_{05}}{p_5} = \left(1 + \frac{\gamma-1}{2}M_5^2\right)^{\gamma/(\gamma-1)} \Rightarrow p_{05} = 1217.5\ kPa$$

P3 (GROUND EXPLOSION)

a) CALCULATE THE PRESSURE RATIO OVER THE SHOCK IF THE INDUCED FLOW MACH NUMBER IS 0.9

$$(7.16) \qquad u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1}-1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1}+\frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

$$M_p = \frac{u_p}{a_2} = \frac{1}{\gamma}\sqrt{\frac{T_1}{T_2}}\left(\frac{P_2}{P_1}-1\right)\left(\left(\frac{2\gamma}{\gamma+1}\right)\Big/\left(\frac{P_2}{P_1}+\frac{\gamma-1}{\gamma+1}\right)\right)^{1/2}$$

$$(7.10) \qquad \frac{T_2}{T_1} = \frac{P_2}{P_1}\left(\frac{\frac{\gamma+1}{\gamma-1}+\frac{P_2}{P_1}}{1+\frac{\gamma+1}{\gamma-1}\left(\frac{P_2}{P_1}\right)}\right)$$

AMBIENT PRESSURE AND TEMPERATURE NOT GIVEN:

GUESS: $P_{amb} = P_1 = 101.325\ kPa$

$T_{amb} = T_1 = 293\ K$

ITERATE EQUATIONS ABOVE $\rightarrow$ $\frac{P_2}{P_1} = 2.69$

b) CALCULATE THE PROPAGATION VELOCITY OF THE SHOCK WAVE.

$$(7.13) \qquad M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1}-1\right)+1} = 1.56$$

$$W_s = M_s a_1 = M_s\sqrt{\gamma R T_1} = 536.7\ m/s$$

c) CALCULATE TEMPERATURE AND ~~PRESSURE~~ PRESSURE AFTER REFLECTION

$$(7.23) \qquad \frac{M_R}{M_R^2-1} = \frac{M_s}{M_s^2-1}\sqrt{1+\frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2-1)\left(\gamma+\frac{1}{M_s^2}\right)}$$

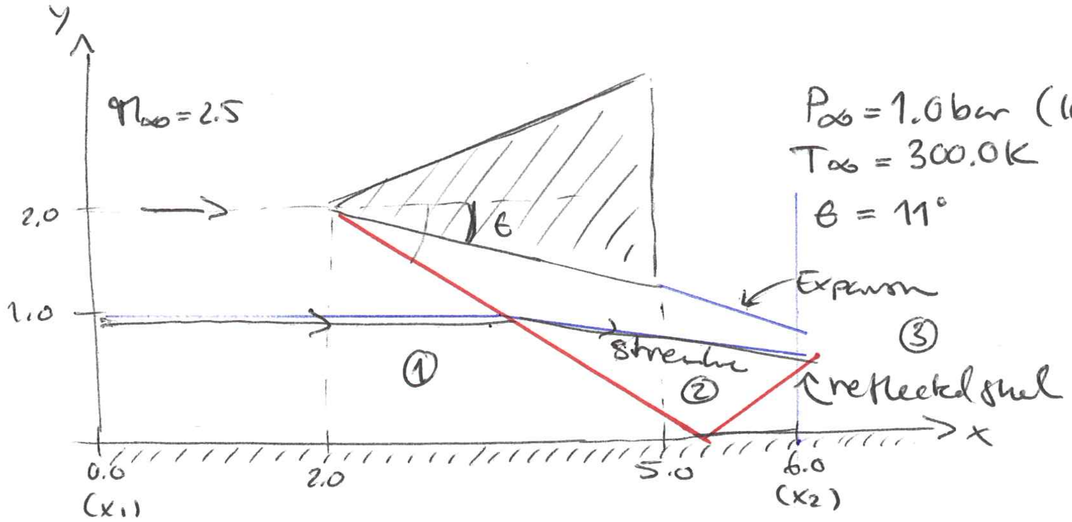
$$(3.57) \qquad \frac{P_5}{P_2} = 1+\frac{2\gamma}{\gamma+1}(M_R^2-1)$$

$$(3.59) \qquad \frac{T_5}{T_2} = \frac{P_5}{P_2}\left(\frac{2+(\gamma-1)M_R^2}{(\gamma+1)M_R^2}\right)$$

$$\left.\begin{aligned}\frac{P_5}{P_1} &= \frac{P_5}{P_2}\frac{P_2}{P_1}\\ \frac{T_5}{T_1} &= \frac{T_5}{T_2}\frac{T_2}{T_1}\end{aligned}\right\} \Rightarrow \begin{aligned}P_5 &= 672.7\ kPa\\ T_5 &= 519.9\ K\end{aligned}$$

TMEO85 EXAM 19-06-10

P1 (WEDGE FLOW)



$P_\infty = 1.0$ bar (100.0 kPa)
$T_\infty = 300.0$ K
$\theta = 11°$

CALCULATE AND PLOT THE PRESSURE DIFFERENCE BETWEEN LOCATIONS ALONG A STREAMLINE STARTING AT (0.0; 1.0) AND LOCATIONS ON THE WALL (y=0) WITH THE SAME AXIAL COORDINATES.

$(\theta = 11°, M_\infty = 2.5)$ $(\theta-\beta-M) \Rightarrow \beta_1 = 32.8°$

(4.7) $M_{n_1} = M_1 \sin\beta_1$

(4.9) $\dfrac{p_2}{p_1} = 1 + \dfrac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$

(4.10) $M_{n_2}^2 = \dfrac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$

(4.12) $M_2 = M_{n_2} / \sin(\beta_1 - \theta)$

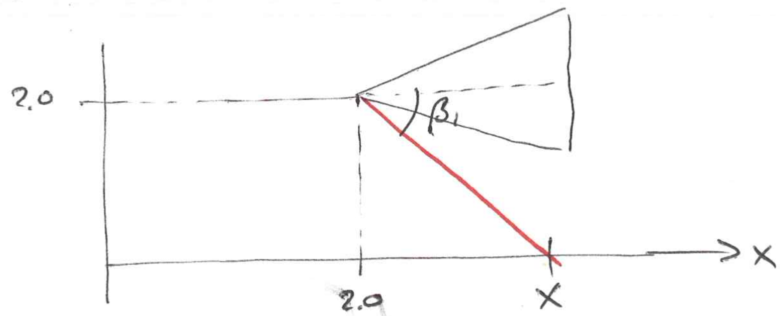
$\Rightarrow$ $M_2 = 2.04$, $\dfrac{p_2}{p_1} = 1.97$

SHOCK REFLECTION AT THE WALL:

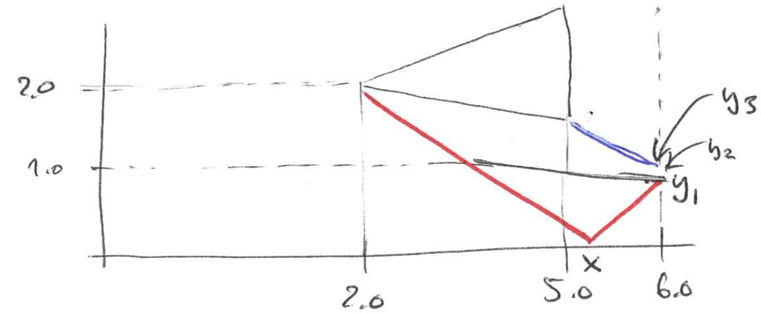
$(\theta = 11°, M_2 = 2.04 : \theta-\beta-M) \Rightarrow \beta_2 = 39.5°$

(4.7), (4.9), (4.10) & (4.12) $\Rightarrow$ $M_3 = 1.64$; $\dfrac{p_3}{p_2} = 1.81$

SHOCK REFLECTION LOCATION:



$$X = 2.0/\tan(\beta_1) + 2.0 = 5.1\ m$$



REFLECTED SHOCK @ $x = 6.0$

$$y_1 = (6.0 - X) \cdot \tan(\beta_2 - \epsilon) = 0.488\ m$$

STREAMLINE @ $x = 6.0$

$$y_2 = 1.0 - (6.0 - ((X - 2.0)/2. + 2.0))\tan(\epsilon) = 0.524\ m$$

EXPANSION @ $x = 6.0$

THE ANGLE OF THE EXPANSION FORMED AT THE REAR END OF THE WEDGE IS OBTAINED USING THE PRANDTL MEYER FUNCTION:

$$\nu(M_2) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2 - 1)} - \tan^{-1}\sqrt{M_2^2 - 1} \qquad (4.44)$$

$$\Rightarrow \nu(M_2) = 27.6^\circ$$

$$y_3 = 2.0 - 3.0\tan(\epsilon) - (6.0 - 5.0)\tan(\nu(M_2)) = 0.894$$

$y_3 > y_2 > y_1 \Rightarrow$ NO INTERSECTION BEFORE $x = 6.0$

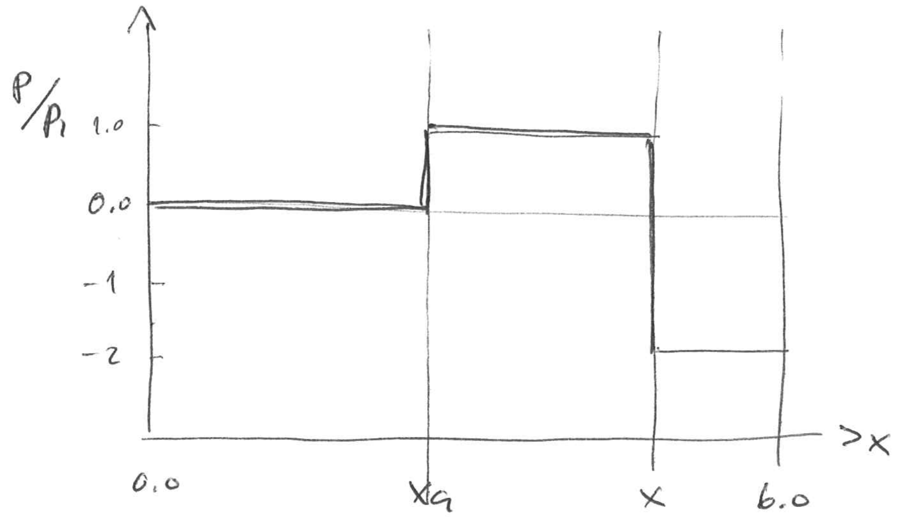
PRESSURE PLOT.

BEFORE THE STREAMLINE INTERSECTS THE FIRST SHOCK THE PRESSURE IS THE SAME ALONG THE STREAMLINE AND ALONG THE WALL

THE STREAMLINE INTERSECTS THE SHOCK AT $X_a = (X - 2.0)/2 + 2.0$

BETWEEN X_a AND X, THE PRESSURE IS HIGHER ALONG THE STREAMLINE THAN ALONG THE WALL

FROM X TO 6.0, PRESSURE AT THE WALL IS P3 AND THE PRESSURE ALONG THE STREAMLINE IS P2



$0 \rightarrow X_a$:

$\Delta p = P_s - P_W = 0.$

$X_a \rightarrow X$:

$\Delta p = P_s - P_W = P_2 - P_1 \Rightarrow \frac{\Delta P_s}{P_1} = \frac{P_2}{P_1} - 1 = 0{,}97$

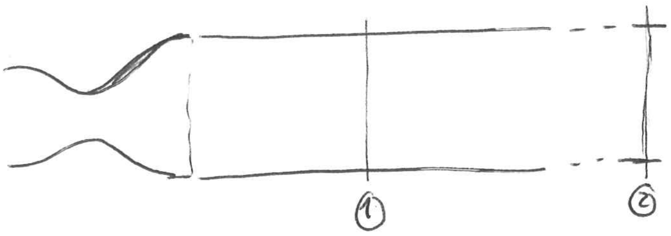
$X \rightarrow 6.0$:

$\Delta P = P_s - P_W = P_2 - P_3 \Rightarrow \frac{\Delta P}{P_1} = \frac{P_2}{P_1} - \frac{P_3}{P_1} = \frac{P_2}{P_1} - \frac{P_2}{P_1}\frac{P_3}{P_2}$

$= \frac{P_2}{P_1}\left(1 - \frac{P_3}{P_2}\right) = -1{,}6$

P_2 (FLOW WITH FRICTION)

$P_0 = 6.73\,MPa$
$T_0 = 312\,K$



$D_{throat} = 0.0061\,m$

$D_{exit} = D_{tube} = 0.0127\,m$

STATION	1	2
x/De	1.75	29.C
p [kPa]	238.0	485.0

CALCULATE THE AVERAGE FRICTION COEFFICIENT $\bar{f}$

#1. CALCULATE NOZZLE-EXIT/PIPE-INLET CONDITION:

$$\frac{A_e}{A_t} = \left(\frac{D_e}{D_t}\right)^2 = 4.33$$

(5.20) $$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow M_e = 3.02$$

(3.30) $$\frac{P_0}{P_e} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_e = 176.6\,kPa$$

(3.104) $$\frac{P_e}{p^*} = \frac{1}{M_e}\left(\frac{\gamma+1}{2+(\gamma-1)M_e^2}\right)^{1/2} \Rightarrow p^* = 820.2\,kPa$$

$$\frac{P_1}{p^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2} \Rightarrow M_1 = 2.51$$

$$\frac{P_2}{p^*} = \frac{1}{M_2}\left(\frac{\gamma+1}{2+(\gamma-1)M_2^2}\right)^{1/2} \Rightarrow M_2 = 1.53$$

(3.107) $$\frac{4\bar{f}L^*}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right)$$

$$\bar{f}L_e^* = 0.00167$$
$$\bar{f}L_1^* = 0.001379$$
$$\bar{f}L_2^* = 0.000466$$

$$L_2^* = L_1^* - (x_2 - x_1) \Rightarrow \bar{f} = \frac{\bar{f}L_1^* - \bar{f}L_2^*}{x_2 - x_1} = 0.002581$$

$$\bar{f}L_e^* = 0.00167 \Rightarrow L_e^* = 50.9D_e > x_2$$

P4 (FLAT PLATE IN SUPERSONIC FLOW)

a) THE DRAG IS GENERATED BY THE SHOCKS THAT WILL FORMED AT THE ~~TRAI~~ LEADING EDGE OF THE PLATE AND THE PHENOMENON IS CALLED WAVE DRAG.

b)

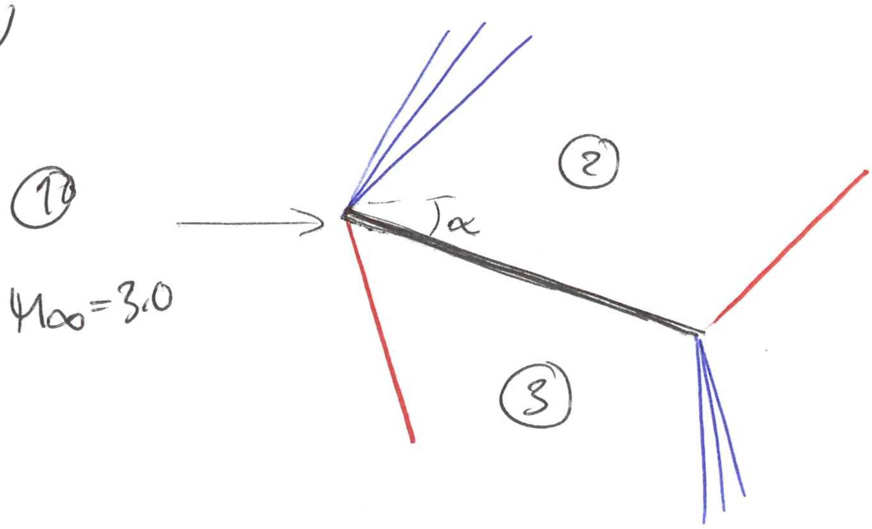


PLATE LENGTH = 0.9m

$\alpha = 20°$

UPPER SURFACE : EXPANSION FROM 1 → 2

PRANDTL-MEYER FUNCTION:

(4.44)
$$\nu_1 = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_\infty^2-1)} - \tan^{-1}\sqrt{M_\infty^2-1}$$

$\nu_1 = 49.8°$

$\nu_2 = \nu_1 + \alpha = 69.8°$

$\nu_2 \Rightarrow M_2 = 4.32$ (using 4.44)

$$\frac{P_2}{P_\infty} = \frac{P_2}{P_0}\frac{P_0}{P_\infty} = \left(\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\gamma/(\gamma-1)} = 0.16 \quad (3.30)$$

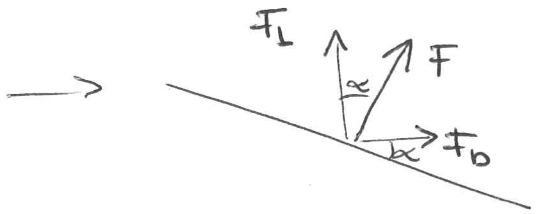
P_0 IS CONSTANT OVER THE EXPANSION (ISENTROPIC)

LOWER SURFACE: OBLIQUE SHOCK FROM $1 \rightarrow 3$

$\theta = \alpha$, $M_1 = M_\infty$: $\theta - \beta - M \Rightarrow \beta = 37.8^\circ$

$$\left.\begin{array}{ll} (4.7) & M_{n_1} = M_\infty \sin\beta \\ (4.9) & \dfrac{p_3}{p_\infty} = 1 + \dfrac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \end{array}\right\} \Rightarrow \frac{p_3}{p_\infty} = 3.77$$

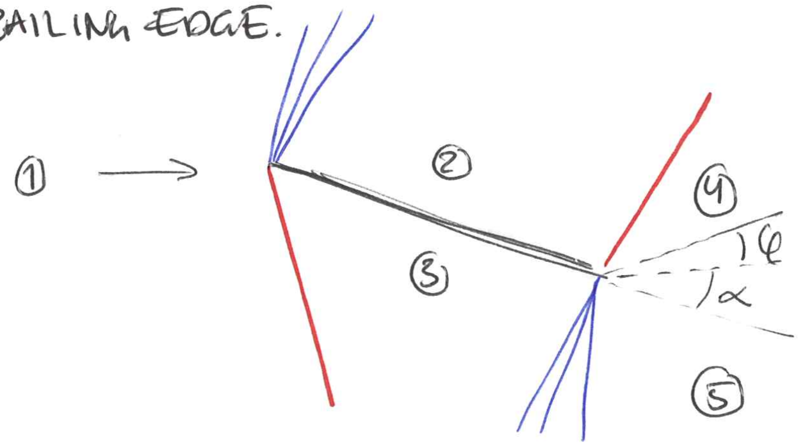
$$p_3 - p_2 = p_\infty \left(\frac{p_3}{p_\infty} - \frac{p_2}{p_\infty} \right) \simeq 3.6\, p_\infty$$



$$F = (p_3 - p_2) L \simeq 3.6\, p_\infty L \simeq 1.47\, p_\infty$$

$$F_L \simeq 3.6\, p_\infty L \cos\alpha \simeq 1.36\, p_\infty$$
$$F_D \simeq 3.6\, p_\infty L \sin\alpha \simeq 0.49\, p_\infty$$

c) CALCULATE THE FLOW ANGLE DOWNSTREAM OF THE TRAILING EDGE.



AFTER THE TRAILING EDGE THE FLOW ANGLE MUST BE THE SAME IN REGION 4 AND 5. ALSO, THE PRESSURE MUST BE THE SAME $p_4 = p_5$

SINCE THE FLOW FOLLOWING THE UPPER SURFACE HAS A DIFFERENT HISTORY THAN THE FLOW ALONG THE LOWER SURFACE, THERE WILL BE A NET TURNING OF THE

DOWNSTREAM OF THE TRAILING EDGE.

THE SHOCK ON THE LOWER SIDE WILL NOT HAVE THE SAME STRENGTH AS THE SHOCK ON THE UPPER SIDE OF THE PLATE SINCE THE UPSTREAM MACH NUMBERS ARE DIFFERENT.

ASSUME THAT THE FLOW WILL LOOK AS IN THE FIGURE ABOVE

OBLIQUE SHOCK $(2 \to 4)$:

$$\left.\begin{array}{l} \epsilon = \alpha + \varphi \\ M_2 \end{array}\right\} \Rightarrow (\epsilon - \beta - M) \Rightarrow \beta$$

$$\left.\begin{array}{ll} M_{n_1} = M_2 \sin\beta & (4.7) \\ \frac{p_4}{p_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) & (4.9) \end{array}\right\} \Rightarrow \frac{p_4}{p_2}$$

$$\frac{p_4}{p_\infty} = \frac{p_4}{p_2}\frac{p_2}{p_\infty}$$

EXPANSION $(3 \to 5)$

$$(4.44) \quad \nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}$$

$$\nu_1 = \nu(M_3)$$

$$\nu_2 = \nu_1 + \epsilon$$

$$\nu_2 = \nu(M_5) \Rightarrow M_5$$

$$\frac{p_5}{p_3} = \left(\frac{1 + \frac{\gamma-1}{2}M_3^2}{1 + \frac{\gamma-1}{2}M_5^2}\right)^{\gamma/(\gamma-1)} \quad (3.30)$$

$$\frac{p_5}{p_\infty} = \frac{p_5}{p_3}\frac{p_3}{p_\infty}$$

GUESS A VALUE FOR φ

CALCULATE $\frac{P_4}{P_\infty}$ AND $\frac{P_5}{P_\infty}$ ACCORDING TO THE ALGORITHM ABOVE

UPDATE φ

ITERATE UNTIL $\left| \frac{P_4}{P_\infty} - \frac{P_5}{P_\infty} \right| < tol$

ITERATION GIVES $\underline{\varphi \simeq 0.89^\circ}$