TME085 - Compressible Flow

2020-03-19, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Optional calculator/Valfri miniräknare
  (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Responsible teacher: Niklas Andersson tel.: 070 - 51 38 311

Good luck!

Instructions

General Info

Due to the extraordinary situation caused by the very high risk of the covid-19 infection spreading in Sweden, Chalmers' President has decided that all written exams for study period 3 will be carried out from home.

Exam Info

The exam consists of eight problems (each problem is a separate assignment in Canvas) Problems 1-2 can give a maximum of 6 points each and problems 3-8 can give a maximum of 8 points each. In total you can get 60 points on the exam. The points earned for the Compressible Flow Project is added to your exam result.

The total number of points on the exam (EP) and the bonus points earned from The Compressible Flow Project (BP) is translated into a course grade as follows:

- Fail: $EP < 24$ (i.e. the bonus points can not be used to pass the course)
- Grade 3: $24 \leq EP < (36 - BP)$
- Grade 4: $(36 - BP) \leq EP < (48 - BP)$
- Grade 5: $(48 - BP) \leq EP$

Niklas Andersson will be available for questions related to the exam from 8:30 until 13:30 the day of the exam (2020-03-19)

If you would like to get in contact with Niklas during the exam, you can send a Canvas message, call or send a mail

- mobile: 070-5138311
- mail: niklas.andersson@chalmers.se

Instructions

The written exam should be handed in through Canvas at the latest 10 minutes past the end of the exam time. If it is not possible to hand-in the exam through Canvas, it should be sent to the niklas.andersson@chalmers.se as soon as possible.

- The exam is divided into a number of separate assignments. You should submit a document with answers/solutions for each of these assignments. Do not wait until the last minute with the submission of files. It is better to submit files continuously as you solve the problems. You can always go back and update if you find mistakes later.
- Answers should be written in a text document (one document for each assignment). Calculations etc. may be written on paper and subsequently be photographed or scanned and included as images in the text document.
- If you use Matlab scripts, Python or any other programming languages to solve the problems you can paste your code snippets in the text document if you think that it will be helpful for the correction of the problems. Note! you will still have to explain what you have done in words, just code will not be sufficient.

- In case you have used some type of graphical representation of your solution (Matlab plots, matplotlib, gnu plot, ...), you could add these figures to your solution document if it adds value
- If you have used an iterative solution procedure using for example Matlab, you could add output from these iterations to your solution
- The exam is to be carried out individually, ie., collaboration is not allowed.
- Due to the current circumstances, all examination aids are allowed.
- Control for plagiarism will be carried out automatically for each of the problems.
- The exam cannot be written anonymously.

Note! By uploading your exam solutions you certify that you have solved the problems on your own without receiving any help from anyone else

General Exam Guide

- Always write down and justify your assumptions
- For some problems you may have to guess values on some properties that has not been given in the problem description
- Some problem descriptions may include data that you will not need for solving the problem
- It is not uncommon that an iterative solution process in needed to be able to solve a problem
- Even if it is difficult in some situations, always try to determine whether your results are realistic or not. An unrealistic solution is worth a bit more if you make a comment about the results and why you think that it is unrealistic.
- Always write down your planned solution process in words. If you do something wrong along the way or if you run out of time and leave the problem unfinished, a description of how to solve the problem goes a long way when it comes to the number of rewarded points (if it is correct of course)
- The header of each problem indicates the total number of points and the number of subtasks.

Problem 1 - NOZZLE FLOW (6 p., 3 subtasks)

As you know by now the convergent-divergent nozzle is a central element in the generation of supersonic gas flows. In the following example, pressurized air is expanded through a convergent-divergent nozzle. The length of the nozzle is 0.8 m and the throat is located 0.25 m from the nozzle inlet. When the nozzle pressure ratio (NPR) is 1.5, i.e. the inlet total pressure is 1.5 times the pressure downstream of the nozzle exit, there is a normal shock at the nozzle exit.

Note! you will most likely need to use iterative methods to solve the problems below

(a) (3p.) Calculate the nozzle area ratio (exit area over throat area)

(b) (2p.) Calculate the NPR (for the same nozzle geometry) for which choked conditions are reached but the flow through the entire nozzle is subsonic

(c) (1p.) Assume that we would have a NPR between the normal-shock-at-exit NPR (1.5) and the NPR defining lower limit of choked nozzle flow (the NPR obtained in the subtask above), would it be possible to use the area-mach-number relation throughout the nozzle? Justify and explain why or why not

Problem 2 - SYSTEM OF OBLIQUE SHOCKS (8 p., 2 subtasks)

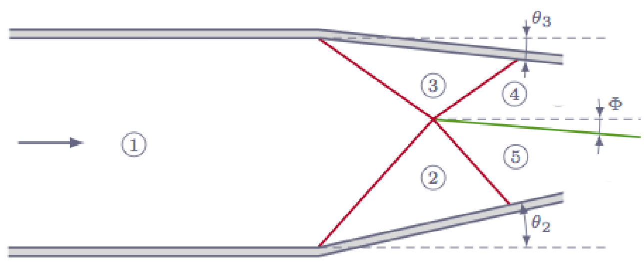
A uniform air flow enters a non-symmetric convergent inlet as depicted in the figure below ($\theta_2 \neq \theta_3$). The extent of the inlet channel in the direction normal to the plane shown in the figure is significantly longer than the channel height and thus it can be justified to assume the flow to be two dimensional.

Inlet Mach number $M_1 = 2.5$

deflection angles: $\theta_2 = 10°$, $\theta_3 = 15°$

(a) (2p.) What are the constrains that leads to the generation of the separating line between regions 4 and 5 (represented by a green line in the figure)? What is the reason for the need for this separating line.

(b) (6p.) Calculate the angle of the line separating regions 4 and 5 (weak shock solutions can be assumed everywhere)



Problem 3 - SHOCK-EXPANSION THEORY (8 p., 3 subtasks)

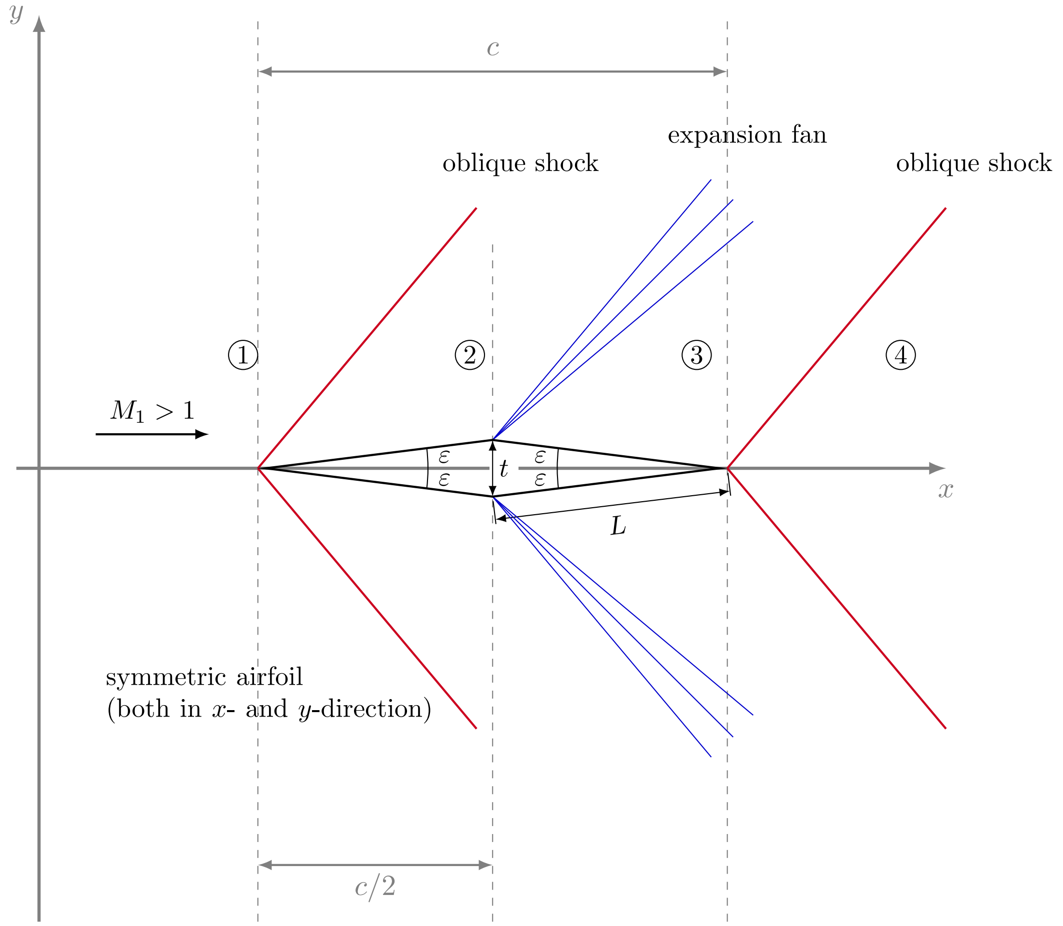
Air at supersonic speed ($M_1 = 2.5$) flows around a symmetric diamond-shaped wing. Assume two-dimensional inviscid flow.

With no angle of attack, $p_2 = 1.8 p_1$

(a) (3p.) Calculate the wave drag for the conditions given above

(b) (3p.) Calculate the wave drag and lift if the angle of attack is $\alpha = 5°$

(c) (2p.) At what angle of attack will the upper shock starting at the trailing edge be replaced by an expansion fan



Problem 4 - TUBE FLOW WITH FRICTION (8 p., 4 subtasks)

Air flows through a well-insulated tube with non-smooth walls. The diameter of the tube is $D = 0.2\ m$ and the length is $L = 2.0\ m$. At the tube exit, the temperature and pressure is $T_{exit} = 288.0\ K$ and $p_{exit} = 101.325\ kPa$, respectively. The air mass flow is through the tube is $\dot{m} = 15.7\ kg/s$.

(a) (2p.) Calculate the inlet conditions

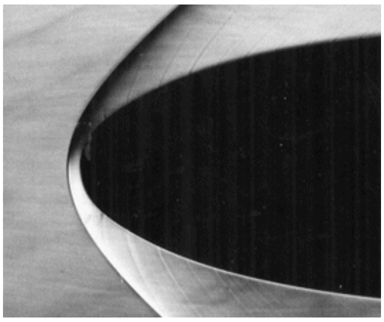
(b) (2p.) Calculate the maximum tube length to prevent choking

(c) (2p.) What would happen if the tube was extended longer than the length calculated above with everything else kept constant?

(d) (2p.) What could be done to the setup described above to allow for a somewhat longer tube (tube diameter and flow conditions kept constant). Explain and justify with calculations.

Problem 5 - BOW SHOCK (8 p., 4 subtasks)

The figure below is a Schlieren photograph obtained from a wind tunnel test where a blunt object is placed in a supersonic air stream. The supersonic flow is generated by letting the air in a large reservoir expand through a convergent-divergent nozzle. In the free stream the temperature is $T_\infty = 300.0\ K$, the pressure is $p_\infty = 101.325\ kPa$, and the upstream Mach number is $M_\infty = 11.0$



(a) (1p.) Why is the shock not attached to the leading edge of the object?

(b) (3p.) Calculate the flow conditions just ahead of the leading edge of the object (temperature, pressure, Mach number, behind the detached shock)

(c) (2p.) When comparing the calculated temperature with the measured, it turns out that the difference is quite significant (compare calculated density ratio over the shock with the density ratio in the graph below). What do you think is the root cause of this? (motivate your answer with a discussion that explains the physics behind this phenomenon)

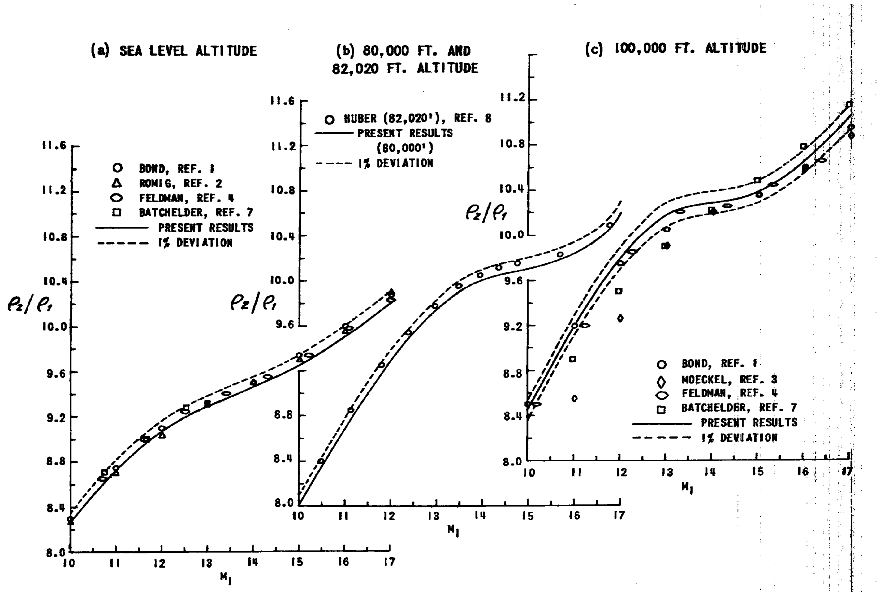


Figure 2 COMPARISON OF DENSITY RATIO

(d) (2p.) Will any parts of the flow field depicted in the figure be rotational? (explain why or why not using physical relations)

Problem 6 - COMBUSTION CHAMBER (6 p., 2 subtasks)

Air enters a combustion chamber at 80.0 m/s, 300.0 K and 76.0 kPa. The length of the combustion chamber is 0.5 m and the diameter is 0.15 m. Effects of friction can be neglected and the gas can be assumed to be calorically perfect.

(a) (3p.) Estimate the amount of heat added in the combustion chamber to achieve sonic conditions at the outlet (choked conditions)

(b) (3p.) If the combustion process adds 610.0 kJ/kg to the air flowing through the combustion chamber, calculate the exit conditions (fluid velocity, Mach number, temperature) and the drop in total pressure in the combustion chamber

Problem 7 - SHOCK TUBE (8 p., 5 subtasks)

The figure below shows the density field in a shock tube obtained from a quasi-1D simulation. The axial density distribution is shown for four time levels including the initial solution (before the shock tube is started) and three consecutive instances in time, each 250 solver iterations apart where the increment in time is 10^{-6} s per iteration. The length of the shock tube is one meter and the diaphragm is located at 0.5 m. The gas is air in both chambers at a temperature of 20.0 degrees Celsius. The pressure in the driven section before the burst of the separating diaphragm is 101.325 kPa.

From the shock tube specifications provided above and the figure, make an estimate of

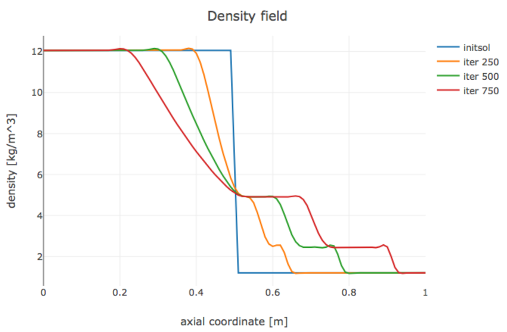
(a) (2p.) the induced flow velocity

(b) (2p.) the pressure ratio over the incident shock and the Mach number of the incident shock

(c) (2p.) calculate the propagation velocity of the head and tail of the expansion region

(d) (1p.) the Mach number of the reflected shock that will be generated when the incident shock reaches the right end wall

(e) (1p.) the driver section pressure before the shock tube is started



Problem 8 - CYLINDER WITH PISTON (8 p., 4 subtasks)

An insulated cylinder is filled with air at $T = 23.0°C$ and $p = 101.325\ kPa$. The right end of the tube is closed and in the left end there is a piston that is not moving initially. The piston is suddenly moved to the left with the fixed speed $u_{piston} = 275.0\ m/s$

For the calculations it can be assumed that acceleration effects and friction can be discarded.

(a) (3p.) The sudden movement of the piston will initiate an expansion region in the tube. At an instant in time after the movement of the piston has been initiated, calculate the pressure at the piston head and do a schematic graphical representation of the pressure along a line from the piston head to the right end wall.

(b) (1p.) Do a schematic graphical representation of the fluid velocity along the same line as above

(c) (1p.) Make $t - x$ diagram the shows the development of the expansion region and the movement of the piston in time and space.

(d) (3p.) Is it possible to make the tail of the expansion region stand still in the tube? How could that be done? (explain and make calculations)

TME085 EXAM 2020-03-19

P1 PRESSURIZED AIR IS EXPANDED THROUGH A CD-NOZZLE.
AT NPR = 1.5, THERE IS A NORMAL SHOCK AT THE NOZZLE EXIT PLANE.

a) CALCULATE THE NOZZLE AREA RATIO

$$NPR = \frac{p_{01}}{p_b} = 1.5$$

$$\frac{p_{01}}{p_{e\_sc}} = \left(1 + \frac{\gamma - 1}{2} M_{e_{sc}}^2\right)^{\gamma/(\gamma-1)} \quad (3.30)$$

SUPERCRITICAL NOZZLE EXIT CONDITION

$$\frac{p_b}{p_{e_{sc}}} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{e_{sc}}^2 - 1\right) \quad (3.57)$$

NORMAL SHOCK AT NOZZLE EXIT.

ITERATE TO FIND $M_{e_{sc}}$ $\Rightarrow$ $M_{e_{sc}} = 1.564$

$$(5.20) \quad \left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_{e_{sc}}^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_{e_{sc}}^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow \left(\frac{A_e}{A_t}\right) = 1.22$$

b) CALCULATE THE NPR FOR WHICH THE NOZZLE GETS CHOKED (SUBSONIC, CHOKED FLOW)

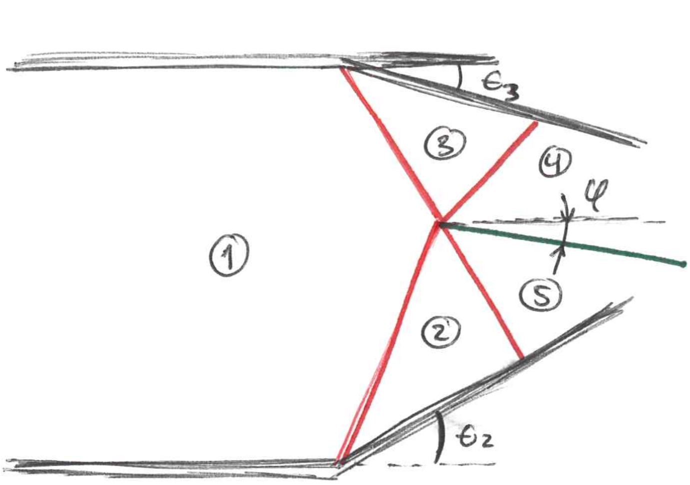
$$(5.20) \quad \left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_{e_c}^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_{e_c}^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUBSONIC SOLUTION: $M_{e_c} = 0.57$

$$(3.30) \quad \frac{p_0}{p_b} = \left(1 + \frac{\gamma-1}{2} M_{e_c}^2\right)^{\gamma/(\gamma-1)} \Rightarrow \frac{p_0}{p_b} = NPR_c = 1.25$$

c) FOR $NPR_c < NPR < NPR_{nse}$ THERE WILL BE A NORMAL SHOCK INSIDE OF THE NOZZLE. SINCE THE AREA-MACH-NUMBER RELATION (5.20) IS ONLY VALID FOR ISENTROPIC FLOW, IT CAN NOT BE USED OVER A NORMAL SHOCK. IT IS, HOWEVER, POSSIBLE TO USE THE RELATION BEFORE AND AFTER THE SHOCK. IT IS IMPORTANT TO NOTE THOUGH THAT A^* CHANGES OVER THE SHOCK.

P2 (SYSTEM OF OBLIQUE SHOCKS)



$M_1 = 2.5$

$\theta_2 = 10°$

$\theta_3 = 15°$

a) WHAT CONSTRAINS LEAD TO THE FORMATION OF THE SEPARATION LINE BETWEEN REGIONS 4 AND 5?

THE PRESSURE AND FLOW DIRECTION MUST BE THE SAME IN REGIONS 4 AND 5. DUE TO THE DIFFERENCE IN FLOW DEFLECTION FROM 1 TO 2 AND 1 TO 3, THE SHOCKS WILL BE OF DIFFERENT STRENGTH AND THUS THE ENTROPY (AND ALSO TEMPERATURE, DENSITY, ...) WILL DIFFER IN REGIONS 4 AND 5, WHICH IS WHY THE SEPARATION LINE (SLIP LINE) IS GENERATED.

b) CALCULATE THE ANGLE φ

THE FIRST STEP IS TO CALCULATE THE FLOW CONDITIONS IN REGIONS 2 AND 3 USING THE OBLIQUE SHOCK RELATIONS.

THE $(\theta-\beta-M)$-RELATION WITH $M_1 = 2.5$ AND $\theta = \theta_2$, $\theta = \theta_3$ =>

$\beta_2 = 31.85°$, $\beta_3 = 36.94°$

(4.7) $M_{n_1} = M_1 \sin\beta$

(4.9) $\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$

(4.10) $M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$

(4.12) $M_2 = \frac{M_{n2}}{\sin(\beta-\theta)}$

=>

$M_2 = 2.1$

$\frac{p_2}{p_1} = 1.86$

$M_3 = 1.87$

$\frac{p_3}{p_1} = 2.47$

IF φ IS DEFINED AS IN THE FIGURE WE GET

$\epsilon_4 = \theta_3 - \varphi$

$\epsilon_5 = \theta_2 + \varphi$

USING (4.7) AND (4.9) TOGETHER WITH THE $(\epsilon-\beta-M)$ RELATION

$$\left.\begin{array}{l}(\epsilon_4 \,\&\, M_3) \Rightarrow \beta_4 \\ (\epsilon_5 \,\&\, M_2) \Rightarrow \beta_5\end{array}\right\} (\epsilon-\beta-M)$$

$$\left.\begin{array}{l}(\beta_4 \,\&\, M_3) \Rightarrow p_4/p_3 \\ (\beta_5 \,\&\, M_2) \Rightarrow p_5/p_2\end{array}\right\} (4.7) \,\&\, (4.9)$$

$p_5/p_1 = (p_5/p_2)\cdot(p_2/p_1)$

$p_4/p_1 = (p_4/p_3)(p_3/p_1)$

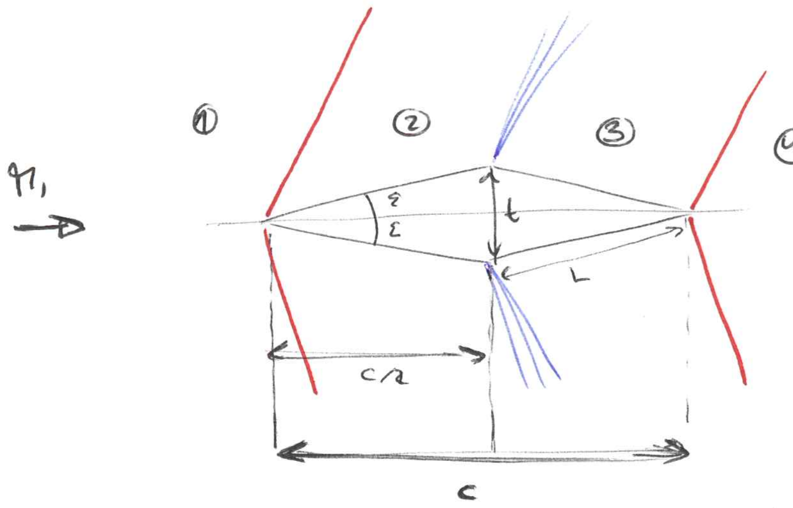
ITERATE UNTIL $p_5/p_1 \simeq p_4/p_1 \Rightarrow \varphi = 4.9°$

FLOW CONDITIONS IN REGIONS 4 AND 5

$\beta_4 = 42.2°$, $M_4 = 1.518$, $p_4/p_2 = 1.68$, $p_4/p_1 = 4.15$

$\beta_5 = 43.4°$, $M_5 = 1.527$, $p_5/p_3 = 2.22$, $p_5/p_1 = 4.15$

P3 (SHOCK-EXPANSION THEORY)



$M_1 = 2.5$

NO ANGLE OF ATTACK

$P_2 = 1.8 P_1$

a) CALCULATE THE WAVE DRAG.

1→2 (OBLIQUE SHOCK)

(4.9) $\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$ (4.7) $M_{n_1} = M_1 \sin(\beta)$ $\Big\} \Rightarrow \beta = 31.3°$

THE SHOCK WILL DEFLECT THE FLOW AN ANGLE $\theta = \varepsilon$

(θ-β-M) WITH $M = M_1 = 2.5$ AND $\beta = 31.3° \Rightarrow \theta = \varepsilon = 9.4°$

2→3 (EXPANSION)

(4.44) $$\nu_2 = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2-1)} - \tan^{-1}\sqrt{M_2^2-1}$$

(4.10) $$M_{n2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$$

(4.12) $$M_2 = M_{n2} / \sin(\beta - \theta) = 2.1$$

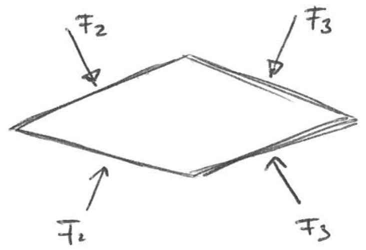
$$\nu_3 = \nu_2 + 2\varepsilon$$

(4.44) $$\nu_3 = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_3^2-1)} - \tan^{-1}\sqrt{M_3^2-1} \Rightarrow M_3 = 2.9$$

THE EXPANSION IS ISENTROPIC $\Rightarrow P_0$ IS CONSTANT.

$$\Rightarrow \frac{P_3}{P_2} = \left(\frac{1 + \frac{(\gamma-1)}{2} M_2^2}{1 + \frac{(\gamma-1)}{2} M_3^2} \right)^{\gamma/(\gamma-1)} = 0{,}29 \qquad (3.30)$$

THE DRAG FORCE (PER UNIT WIDTH) IS OBTAINED AS



$F_2 = P_2 L$
$F_3 = P_3 L$

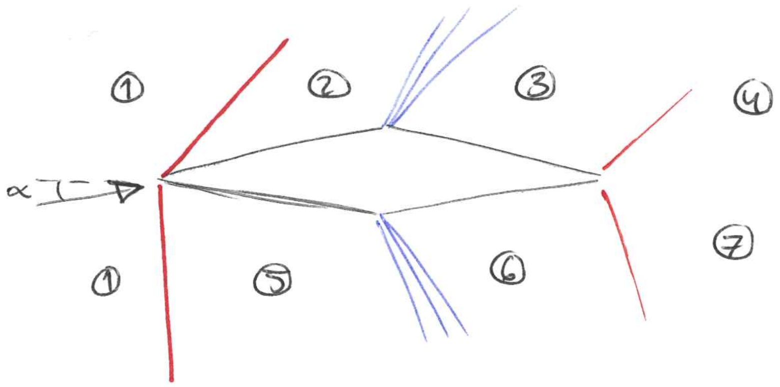
$$F_D = 2\,(F_2 \sin\varepsilon - F_3 \sin\varepsilon)$$

$$F_D = 2 \sin\varepsilon\, L \left(\frac{P_2}{P_1} P_1 - \frac{P_3}{P_1} P_1 \right) = 2 \sin\varepsilon\, L P_1 \left(\frac{P_2}{P_1} - \frac{P_3}{P_2} \frac{P_2}{P_1} \right)$$

$$= 2 \sin\varepsilon\, L P_1 \frac{P_2}{P_1} \left(1 - \frac{P_3}{P_2} \right) = 0{,}42\, P_1 L$$

b) CALCULATE LIFT AND DRAG FOR AN ANGLE OF ATTACK OF $\alpha = 5°$

$\alpha = 5° < \varepsilon \Rightarrow$ STILL A SHOCK ON THE UPPER SIDE OF THE WING AT THE ~~TRAILING~~ LEADING EDGE



THE PRESSURE IN REGIONS 2, 3, 5, AND 6 WILL BE CALCULATED USING (θ-β-M), (4.7) → (4.12), (4.44), (3.30)

$\theta_2 = \varepsilon - \alpha \Rightarrow (\theta\text{-}\beta\text{-}M) \Rightarrow \beta_2 = 26.9°$

$\theta_5 = \varepsilon + \alpha \Rightarrow (\theta\text{-}\beta\text{-}M) \Rightarrow \beta_5 = 36.3°$

(4.7) AND (4.9) $\Rightarrow \dfrac{P_2}{P_1} = 1.33\ ,\ \dfrac{P_5}{P_1} = 2.39$.

(4.10) AND (4.12) $\Rightarrow M_2 = 2.52 \;,\; M_5 = 1.9$

(4.44) $$\nu_2 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2-1)} - \tan^{-1}\sqrt{M_2^2-1}$$

$$\nu_3 = \nu_2 + 2\varepsilon$$

$$\nu_3 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_3^2-1)} - \tan^{-1}\sqrt{M_3^2-1} \Rightarrow M_3 = 3.2$$

$$\nu_5 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_5^2-1)} - \tan^{-1}\sqrt{M_5^2-1}$$

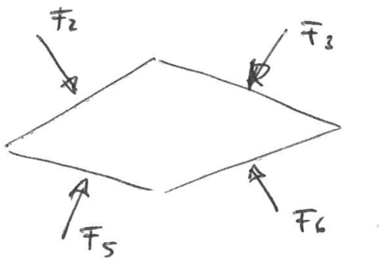
$$\nu_6 = \nu_5 + 2\varepsilon$$

$$\nu_6 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_6^2-1)} - \tan^{-1}\sqrt{M_6^2-1} \Rightarrow M_6 = 2.6$$

(3.30) $\Rightarrow$

$$\frac{p_3}{p_2} = \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_3^2}\right)^{\gamma/(\gamma-1)} = 0.26$$

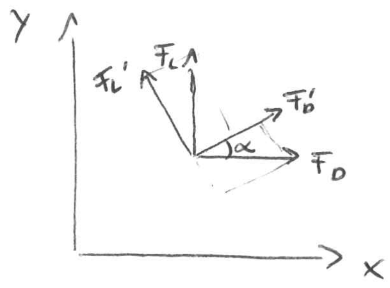
$$\frac{p_6}{p_5} = \left(\frac{1+\frac{\gamma-1}{2}M_5^2}{1+\frac{\gamma-1}{2}M_6^2}\right)^{\gamma/(\gamma-1)} = 0.31$$



$$F_D = F_2\sin\varepsilon - F_3\sin\varepsilon + F_5\sin\varepsilon - F_6\sin\varepsilon =$$
$$= p_1 L\sin\varepsilon\left(\frac{p_2}{p_1}\left(1-\frac{p_3}{p_2}\right) + \frac{p_5}{p_1}\left(1-\frac{p_6}{p_5}\right)\right) = 0.42\, p_1 L$$

$$F_L = -F_2\cos\varepsilon - F_3\cos\varepsilon + F_5\cos\varepsilon + F_6\cos\varepsilon =$$
$$= p_1 L\cos\varepsilon\left(-\frac{p_2}{p_1}\left(1+\frac{p_3}{p_2}\right) + \frac{p_5}{p_1}\left(1+\frac{p_6}{p_5}\right)\right) = 1.44\, p_1 L$$

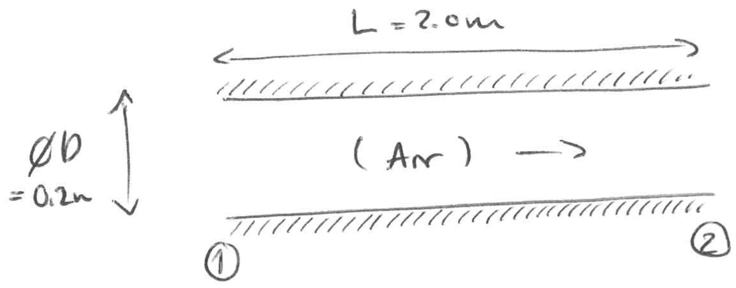
IT MIGHT BE MORE RELEVANT TO CALCULATE DRAG AND LIFT IN THE FLOW DIRECTION.



$$F_D' = F_D \cos\alpha + F_L \sin\alpha$$
$$F_L' = -F_D \sin\alpha + F_L \cos\alpha$$

$$\Rightarrow F_D' = 0.55 \, P_i L$$
$$F_L' = 1.40 \, P_i L$$

P4 (TUBE FLOW WITH FRICTION)



$T_2 = 288\,K$
$P_2 = 101.325\,kPa$

$\dot{m} = 15.7\,kg/s$

Assume: CALORICALLY PERFECT, $\bar{f} = 0.005$

a) CALCULATE INLET CONDITIONS:

FIRST WE NEED TO CALCULATE THE MACH NUMBER AT THE EXIT M_2

$$\dot{m} = \rho_2 u_2 \frac{\pi D^2}{4} = \frac{P_2}{RT_2} u_2 \frac{\pi D^2}{4} \Rightarrow u_2 = 407.7\ m/s$$

$$\left.\begin{array}{l} a_2 = \sqrt{\gamma R T_2} \\ M_2 = u_2 / a_2 \end{array}\right\} \Rightarrow M_2 = 1.2 \Rightarrow \text{SUPERSONIC AT INLET } (M_1 > 1)$$

(3.107)

$$\frac{4\bar{f}L_2^*}{D} = \frac{1-M_2^2}{\gamma M_2^2} + \frac{\gamma+1}{2\gamma} \ln\left(\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2}\right) \Rightarrow L_2^* = 0.33\ m$$

$$L_1^* = L_2^* + L = 2.33\ m$$

(3.107)

$$\frac{4\bar{f}L_1^*}{D} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma} \ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

SUPERSONIC SOLUTION $\Rightarrow M_1 = 1.77$

(3.103)

$$\frac{T_2}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_2^2} \qquad \frac{T_1}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_1^2}$$

$$\frac{P_2}{P^*} = \frac{1}{M_2}\left(\frac{\gamma+1}{2+(\gamma-1)M_2^2}\right)^{1/2} \qquad \frac{P_1}{P^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2}$$

p^* AND T^* ARE CONSTANT $\Rightarrow$

$$P_1 = 60.8\ kPa$$
$$T_1 = 227.5\ K$$

$$\dot{m} = \rho_1 u_1 \frac{\pi D^2}{4} = \frac{p_1}{RT_1} u_1 \frac{\pi D^2}{4} \Rightarrow u_1 = 536.4\ m/s$$

b) CALCULATE THE MAXIMUM TUBE LENGTH IF CHOKING SHOULD BE PREVENTED.

$$L_{max} = L_1^* = 2.33\ m$$

c) WHAT WOULD HAPPEN IF THE TUBE WAS EXTENDED MORE THAN L_{max}?

A SHOCK WOULD BE GENERATED IN THE TUBE SUCH THAT THE TUBELENGTH WOULD BE THE CHOKED FLOW LENGTH FOR THE SUBSONIC CONDITION AFTER THE SHOCK AND THE SHOCK POSITION. THE LONGER THE TUBE, THE FURTHER UPSTREAM THE SHOCK WOULD BE GENERATED. EVENTUALLY THE SHOCK WOULD MOVE OUT FROM THE TUBE AND MAKE THE UPSTREAM FLOW SUBSONIC. STATIC FLOW PROPERTIES WOULD THAN HAVE TO BE CHANGED AT THE INLET SUCH THAT THE LENGTH OF THE TUBE EQUALS THE CHOKE LENGTH (IE. THE MASS FLOW WOULD CHANGE)

d) WHAT COULD BE DONE TO THE SETUP TO ALLOW FOR SOMEWHAT LONGER TUBE (WITHOUT CHANGING ANY FLOW PROPERTIES)

IF THE TUBE IS POLISHED AND THUS $\bar{f}$ IS LOWERED THE CHOKING LENGTH IS LONGER.

$$L_{max}(\bar{f} = 0.005) = 2.33\ m$$
$$L_{max}(\bar{f} = 0.002) = 5.83\ m$$

CALCULATED USING (3.107) WITH $M = M_1 = 1.77$

P5 (BOW SHOCK)

$P_\infty = 101.325\,kPa$
$T_\infty = 300.0\,K$
$M_\infty = 11.0$

a) WHY IS THE SHOCK NOT ATTACHED TO THE LEADING EDGE OF THE OBJECT

IT IS NOT POSSIBLE TO DEFLECT THE FLOW WITH ANY OBLIQUE SHOCK SUCH THAT THE FLOW IS BENT AROUND THE OBJECT

A FLOW DEFLECTION OF 90° FALLS OUTSIDE OF THE MAXIMUM FLOW DEFLECTION ANGLE FOR ANY MACH NUMBER..

INSTEAD A DETACHED SHOCK WILL BE FORMED UPSTREAM OF THE OBJECT, GENERATING A ZONE OF SUBSONIC FLOW AHEAD OF THE LEADING EDGE ALLOWING FOR THE FLOW TO BE REDIRECTED TO FOLLOW THE SURFACE OF THE OBJECT.

b) CALCULATE FLOW PROPERTIES JUST UPSTREAM OF THE OBJECT.

AT THE LEADING EDGE, THE SHOCK WILL BE A NORMAL SHOCK => WE CAN USE THE NORMAL SHOCK RELATIONS.

(3.51) $$M_2^2 = \frac{1 + ((\gamma-1)/2)M_1^2}{\gamma M_1^2 - (\gamma-1)/2}$$

(3.57) $$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

(3.59) $$\frac{T_2}{T_1} = \frac{P_2}{P_1}\left(\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right)$$

$M_2 = 0,39$
$P_2 = 14,28 \, MPa$
$T_2 = 7341,3 \, K$
$\rho_2 = 6,78 \, kg/m^3$

c) COMPARISONS WITH EXPERIMENTAL DATA SHOWS QUITE SIGNIFICANT DEVIATIONS. WHAT IS THE ROOT CAUSE OF THIS..

USING THE FORMULAS ~~[illegible]~~ FOR NORMAL SHOCKS (3.51)-(3.59) WE HAVE MADE THE ASSUMPTION THAT THE GAS IS CALORICALLY PERFECT. AT THE TEMPERATURES ASSOCIATED WITH SHOCKS AT THE MACH NUMBER IN THIS PROBLEM, THE GAS IS NO LONGER CALORICALLY PERFECT. EQUILIBRIUM GAS WOULD GIVE A BETTER RESULT AS REACTIONS AND IONIZATION ARE ACCOUNTED FOR.

d) WILL ANY PARTS OF THE FLOW BE IRROTATIONAL?

CROCCO'S THEOREM:

(6.59) $$T \nabla s = \nabla h_0 - \mathbb{V} \times (\nabla \times \mathbb{V}) + \frac{\partial \mathbb{V}}{\partial t}$$

h_0 CONSTANT OVER SHOCK

$\frac{\partial \mathbb{V}}{\partial t} = 0$ (STEADY STATE)

$$\Rightarrow T \nabla s = -\mathbb{V} \times (\nabla \times \mathbb{V})$$

THE SHOCK STRENGTH WILL VARY FROM THE NORMAL SHOCK AT THE CENTER TO WEAKER OBLIQUE SHOCKS FOR OFF-CENTER LOCATIONS $\Rightarrow$ BEHIND THE SHOCK $\nabla s \neq 0$
$\Rightarrow \mathbb{V} \times (\nabla \times \mathbb{V}) \neq 0 \Rightarrow$ FLOW BEHIND SHOCK WILL BE ROTATIONAL

P6 (COMBUSTION CHAMBER)

AIR ENTERS A COMBUSTION CHAMBER AT 80 m/s, 300.0 K AND 76.0 kPa.

a) ESTIMATE THE AMOUNT OF HEAT ADDED TO CHOKE THE FLOW ($M = 1$ AT EXIT)

GET INLET MACH NUMBER

$$M_1 = u_1 / a_1 = u_1 / \sqrt{\gamma R T_1} = 0.23$$

$$(3.28) \quad \frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \Rightarrow T_{01} = 303.2 \text{ K}$$

$$(3.89) \quad \frac{T_{01}}{T_0^*} = \frac{(\gamma+1) M_1^2}{(1+\gamma M_1^2)^2} \left(2 + (\gamma - 1) M_1^2\right)$$

$$q_1^* = C_p (T_0^* - T_{01}) = 1060.2 \text{ kJ/kg}$$

$\Rightarrow$ THE AMOUNT OF ADDED HEAT TO GET THERMAL CHOKING IS 1060.2 kJ/kg

b) ADDED HEAT $q_{12} = 610.0$ kJ/kg
CALCULATE EXIT CONDITIONS

$$q_{12} = C_p (T_{02} - T_{01}) \Rightarrow T_{02} = 910.5 \text{ K}$$

$$(3.89) \quad \frac{T_{02}}{T_0^*} = \frac{(\gamma+1) M_2^2}{(1+\gamma M_2^2)^2} \left(2 + (\gamma - 1) M_2^2\right) \Rightarrow M_2 = 0.49$$

$$(3.86) \quad \left.\begin{aligned} \frac{T_1}{T^*} &= M_1^2 \left(\frac{1+\gamma}{1+\gamma M_1^2}\right)^2 \\ \frac{T_2}{T^*} &= M_2^2 \left(\frac{1+\gamma}{1+\gamma M_2^2}\right)^2 \end{aligned}\right\} \Rightarrow \underline{T_2 = 869.4 \text{ K}}$$

(3.85)

$$\left.\begin{array}{l} \dfrac{p_1}{p^*} = \dfrac{1+\gamma}{1+\gamma M_1^2} \\[2ex] \dfrac{p_2}{p^*} = \dfrac{1+\gamma}{1+\gamma M_2^2} \end{array}\right\} \Rightarrow \underline{p_2 = 61.4\ \text{kPa}}$$

$$u_2 = M_2 a_2 = M_2\sqrt{\gamma R T_2} = \underline{287.1\ \text{m/s}}$$

(3.30)

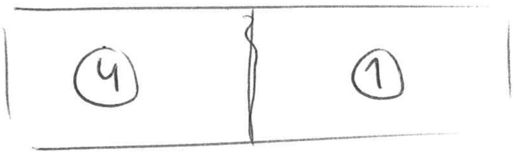
$$\frac{p_{01}}{p_1} = \left(1 + \frac{(\gamma-1)}{2} M_1^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_{02}}{p_2} = \left(1 + \frac{(\gamma-1)}{2} M_2^2\right)^{\gamma/(\gamma-1)}$$

$$\underline{p_{01} - p_{02} = 6.74\ \text{kPa}}$$

P7 (SHOCK TUBE)

PROVIDED FIGURE SHOWS DENSITY FIELDS FROM A Q1D-SIMULATION OF A SHOCK TUBE.



$T_1 = T_4 = 293\,K$

$p_1 = 101.325\,kPa$

$NSTEP = 250$

$\Delta t = 10^{-6}\,s$

a) ESTIMATE THE INDUCED FLOW VELOCITY:

FROM THE FIGURE, THE DISTANCE THAT THE CONTACT SURFACE TRAVELS IS ESTIMATED TO $\Delta x = 0.07$ (BETWEEN TWO SOLUTIONS)

$\Rightarrow u_p = \Delta x / (NSTEP \cdot \Delta t) = \underline{280.0\,m/s}$

b) OBTAIN THE SHOCK PRESSURE RATIO AND THE SHOCK MACH NUMBER.

$$(7.16) \quad u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

WHERE $a_1 = \sqrt{\gamma R T_1}$

$\Rightarrow p_2/p_1 = \underline{2.83}$

$$(7.13) \quad M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \quad \Rightarrow \underline{M_s = 1.6}$$

c) CALCULATE THE PROPAGATION SPEED OF THE HEAD AND TAIL OF THE EXPANSION

$$(7.85) \quad \frac{T_3}{T_4} = \left(1 - \frac{\gamma-1}{2}\left(\frac{u_3}{a_4}\right)\right)^2$$

WHERE $u_3 = u_2 = u_p$ AND $a_4 = \sqrt{\gamma R T_4}$

$\Rightarrow T_3 = 205.2\,K$

$$u_{HEAD} = -a_4 = -\sqrt{\gamma R T_4} = -343.1 \text{ m/s}$$

$$u_{TAIL} = u_3 - a_3 = u_3 - \sqrt{\gamma R T_3} = u_p - \sqrt{\gamma R T_3} = -7.1 \text{ m/s}$$

d) CALCULATE THE MACH NUMBER OF THE REFLECTED SHOCK GENERATED WHEN THE SHOCK REACHES THE RIGHT WALL OF THE SHOCK TUBE.

$$(7.23) \quad \frac{M_R}{M_R^2 - 1} = \frac{M_S}{M_S^2 - 1}\sqrt{1 + \frac{2(\gamma-1)}{(\gamma+1)^2}(M_S^2 - 1)\left(\gamma + \frac{1}{M_S^2}\right)}$$

$$\Rightarrow \underline{M_R = 1.5}$$

e) CALCULATE THE DRIVER SECTION PRESSURE (P_4)

$$(7.94) \quad \frac{P_4}{P_1} = \frac{P_2}{P_1}\left(1 - \frac{(\gamma-1)(a_1/a_4)(P_2/P_1 - 1)}{\sqrt{2\gamma(2\gamma + (\gamma+1)(P_2/P_1 - 1))}}\right)^{-2\gamma/(\gamma-1)}$$

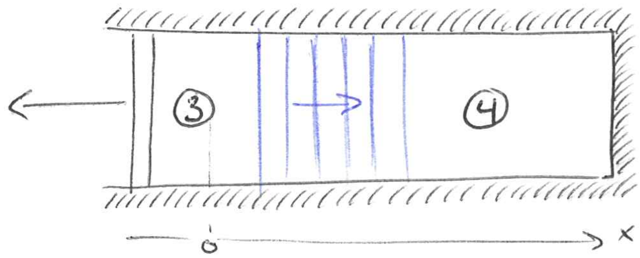
$$\Rightarrow \underline{P_4 = 998.7 \text{ kPa}}$$

P8 (CYLINDER WITH PISTON)

AN INSULATED CYLINDER IS FILLED WITH AIR AT T = 23°C (300 K) AND P = 101,325 kPa. A PISTON AT THE LEFT END OF THE TUBE IS SUDDENLY MOVED AT A VELOCITY OF 275.0 m/s TO THE LEFT.

UNSTEADY EFFECTS (START UP TRANSIENTS) ARE NEGLECTED. FRICTION CAN BE DISCARDED.

a) AN EXPANSION TRAVELING TO THE RIGHT IS GENERATED AS THE PISTON STARTS TO MOVE. CALCULATE THE PRESSURE AT THE PISTON HEAD AND SHOW HOW PRESSURE VARIES THROUGH THE EXPANSION.



$T_4 = 300\,K$

$P_4 = 101{,}325\,kPa$

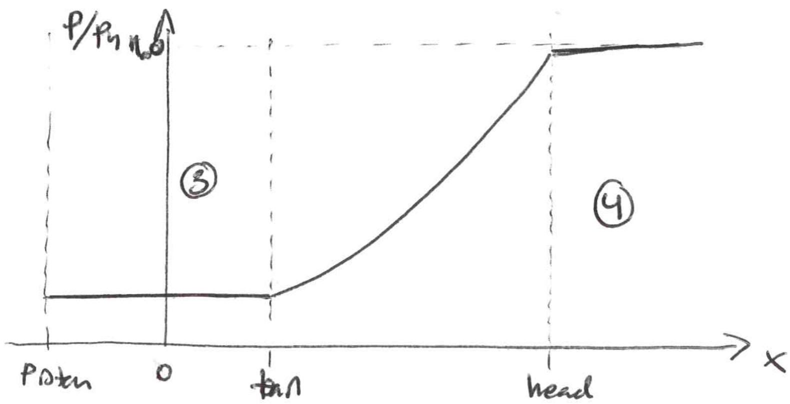
$u_3 = u_{piston}$ (THE VELOCITY IN REGION 3 MUST MOVE TO THE LEFT WITH THE SAME SPEED AS THE PISTON)

(7.86) $$\frac{p_3}{p_4} = \left(1 - \frac{\gamma - 1}{2}\left(\frac{u_3}{a_4}\right)\right)^{2\gamma/(\gamma-1)}$$

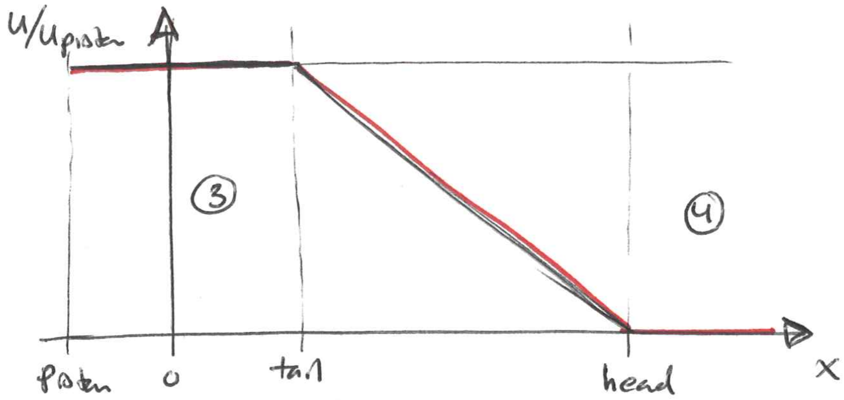
WHERE $a_4 = \sqrt{\gamma R T_4}$

$\Rightarrow \; p_3 = 30.0\ kPa$

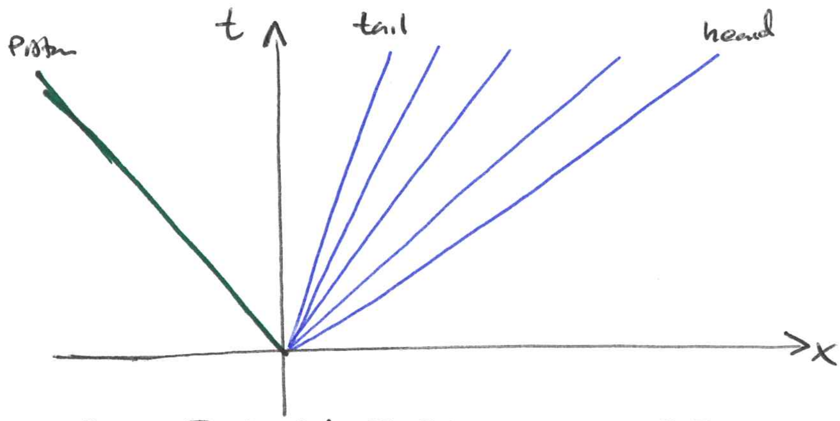
PRESSURE DISTRIBUTION IN EXPANSION REGION (SCHEMATIC)



b) SHOW HOW VELOCITY VARIES THROUGH THE EXPANSION (SCHEMATICALLY)



c) SHOW THE MOVEMENT OF THE EXPANSION ~~tail~~ AND THE PISTON IN A xt - DIAGRAM (SCHEMATICALLY)



d) IS IT POSSIBLE TO MAKE THE TAIL OF THE EXPANSION REGION STAND STILL IN THE TUBE. EXPLAIN AND MAKE CALCULATIONS.

$$u_{tail} = a_3 - u_3$$

$$u_{tail} = 0 \Rightarrow u_3 = a_3$$

$u_3 = u_{piston}$ => THE PISTON SHOULD MOVE TO THE LEFT AT A VELOCITY EQUAL TO THE SPEED OF SOUND IN REGION 3

(7.85) $$\frac{T_3}{T_4} = \left(1 - \frac{\gamma - 1}{2}\left(\frac{u_3}{a_4}\right)\right)^2$$

$$u_3 = a_3 \quad , \quad a_3 = \sqrt{\gamma R T_3} \quad , \quad a_4 = \sqrt{\gamma R T_4} \quad \Rightarrow$$

$$\frac{T_3}{T_4} = \left(1 - \frac{\gamma - 1}{2}\sqrt{\frac{T_3}{T_4}}\right)^2$$

$$\sqrt{\frac{T_3}{T_4}} = 1 - \frac{\gamma - 1}{2}\sqrt{\frac{T_3}{T_4}} \Rightarrow \left(1 + \frac{\gamma-1}{2}\right)\sqrt{\frac{T_3}{T_4}} = 1$$

$$\left(\frac{\gamma+1}{2}\right)\sqrt{\frac{T_3}{T_4}} = 1$$

$$\frac{T_3}{T_4} = \left(\frac{2}{\gamma+1}\right)^2 \Rightarrow T_3 = 205.6\,K$$

$$a_3 = \sqrt{\gamma R T_3} = 287.4 \text{ m/s}$$

$u_{piston} = 287.4$ m/s TO THE LEFT.