TME085 - Compressible Flow

2019-08-21, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (1 p.)
What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T2. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T3. (2 p.)

(a) In shock tubes, unsteady contact discontinuities are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p, T, ρ, u, s is/are necessarily continuous across such a contact discontinuity?

(b) The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the "*weak*" type or the "*strong*" type. What is the main difference between these two shock types and which type is usually seen in reality?

T4. (2 p.)

(a) What simplifications are made when analyzing a convergent-divergent nozzle flow using a quasi-1D approach?

(b) What are the main limitations of such an analys?

(c) What is meant by an under-expanded or over-expanded nozzle flow?

T5. (2 p.)
Derive the special formula for the speed of sound a for a calorically perfect gas:

$$a = \sqrt{\gamma R T}$$

starting from the general formula:

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

T6. (1 p.)
Assume a steady-state 1D flow with a stationary normal shock. The fluid particles crossing the shock are subjected to

(a) a pressure drop

(b) a density increase

(c) an entropy increase

(d) a temperature drop

(e) a deceleration

Which statements are true and which are false?

T7. (1 p.)
In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

T8. (1 p.)
What is the difference between acoustic waves and other types of waves such as shock waves and expansion waves?

T9. (2 p.)

(a) When applying a time-marching flow solution scheme the so-called CFL number is an important parameter. Define the CFL number and describe its significance. What is a typical maximum CFL number for an explicit time stepping scheme.

(b) What is meant by the terms "*density-based*" and "*fully coupled*" when discussing CFD codes for compressible flow?

T10. (1 p.)
What is meant by choking the flow in a nozzle? Describe it.

T11. (2 p.)
Derive the continuity equation in non-conservation form from the corresponding relation on conservation form.

T12. (2 p.)
Prove, by using one of the non-conservation forms of the energy equation, that for steady-state, adiabatic flow with no body forces the total enthalpy is preserved along stream-lines.

T13. (1 p.)
How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

in total four problems worth 10 p. each

General instructions

Please note that some of the problems might be a bit time consuming to solve. Moreover, in general there is a significant risk that things go wrong along the way when solving problems iteratively using pen and paper, which might be needed. Therefore, please note that a correct description of the algorithms used and the physical principles involved goes a long way when it comes to the number of points rewarded for a specific problem so make sure to describe your work flow, simplifications and assumptions made, relevant physical principles, and relations used in detail especially if you are short of time.

Problem 1 - PIPE FLOW WITH FRICTION (10 p.)

Air is to be transported through a pipe a distance of 600.0 m. The average friction coefficient for this specific pipe flow is $\bar{f}$=0.004. At the entrance of the pipe the air temperature is 300 K and the pressure 350 kPa, respectively. Calculate the maximum pipe diameter given that the pipe needs to be able to handle a mass flow of $\dot{m}$=1.5 kg/s.

Problem 2 - HIGH-ENTHALPY MEASUREMENTS (10 p.)

A shock tube is used in a lab in order to establish high enthalpy conditions for a measurement. As you might remember from the course, such high-enthalpy conditions can be achieved by letting the incident shock, established when the diaphragm of the shock tube breaks, reflect at the right end of the tube. The reflection properties are defined by the need for the induced velocity behind the incident shock to be brought to rest as the flow cannot penetrate the end-wall of the tube. The time span available for measurements in this kind of configuration is very short. Let's assume that as the reflected shock meets the contact surface (the surface originally separating the two fluid states in the shock tube setup), the flow disturbances are too significant to be ignored and thus no measurements are done after that. Calculate the time span available for high-enthalpy measurements.

The shock tube is setup as follows:

- The gas in both the driver section and the driven section is air:
  γ=1.4, R=287.0 J/(kg K)
- Before the diaphragm is broken, the driver section temperature is T_4=27°C, the driven section temperature is T_1=27°C, and the driven section pressure is p_1=0.01 atm.
- The shock tube is designed to produce an incident shock with a Mach number of 5.0.
- The distance from the diaphragm location to the right end of the tube is 8.0 m.

Problem 3 - NOZLE FLOW (10 p.)

Air flows through a convergent divergent nozzle with an exit-to-throat area ratio of 1.6 and a nozzle exit diameter of D_e=0.2 m. The total temperature and pressure in the upstream reservoir are T_o=300.0 K and P_o=10.0 bar, respectively. Based on the given nozzle specifications answer the following questions.

(a) For which range of back pressures will the internal nozzle flow be non-isentropic? (4p.)

(b) For which range of back pressures will the nozzle flow be subsonic just downstream of the nozzle exit? (2p.)

(c) For which range of back pressures will the flow just downstream of the nozzle exit plane be supersonic? (2p.)

(d) What is the maximum achievable nozzle mass flow for the given conditions? (2p.)

Problem 4 - FLAT PLATE IN SUPERSONIC FLOW (10 p.)

Two researchers usually working with low-speed flows are testing a flat plate at an angle of attack (α=20°) in a high-speed freestream (M_∞=3.0) in a supersonic wind tunnel. The plate has a length in the flow direction L=0.4 m and is significantly longer in the spanwise direction thus the flow can be assumed to be fairly well represented using a 2D flow model. The researchers finds it rather strange that although the flow is attached to the surface of the flat plate, the force balance, the devise used for measuring forces on the object placed in the wind tunnel, indicates a large drag component in the total force exerted on the flat plate. Doing some quick hand calculations, they figure out that the force is way greater than forces associated with surface friction and thus they ask the operators of the wind tunnel to recalibrate the force balance. The operators simply laugh at them and asks them to go back and do their homework.

(a) What physical phenomena caused the drag component in the measured force? (1p.)

(b) Calculate the lift and drag components of the force exerted on the flat plate (3p.)

(c) Locally there will be a net turning of the flow by the presence of the flat plate. calculate the flow angle just downstream of the trailing edge of the plate. (6p.)

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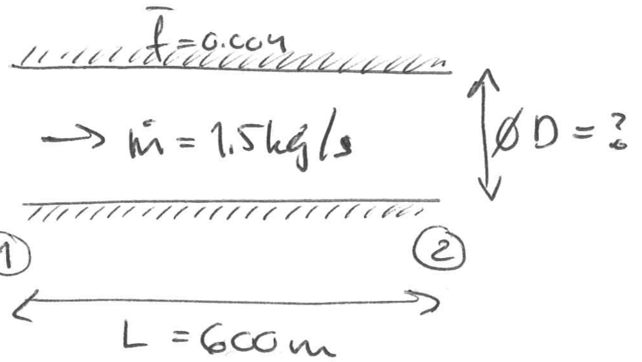
P1 (PIPE FLOW WITH FRICTION)

AIR IS TRANSPORTED A DISTANCE OF 600.0 m THROUGH A PIPE. THE AVERAGE FRICTION COEFFICIENT IS $\bar{f} = 0.004$. THE MASSFLOW IS $\dot{m} = 1.5$ kg/s

$T_1 = 300$ K
$P_1 = 350$ kPa



$$\rho_1 = \frac{P_1}{R T_1}$$

$$a_1 = \sqrt{\gamma R T_1}$$

CALCULATE THE MINIMUM DIAMETER THAT FULFILS THE SPECIFICATIONS GIVEN

GUESS $D = 0.1$

$$u_1 = \dot{m} / (\rho_1 \frac{\pi D^2}{4})$$

$$M_1 = u_1 / a_1$$

$$(3.107) \quad \frac{4 \bar{f} L^*}{D} = \frac{1 - M_1^2}{\gamma M_1^2} + \frac{\gamma + 1}{2\gamma} \ln \left(\frac{(\gamma + 1) M_1^2}{2 + (\gamma - 1) M_1^2} \right)$$

$$\Rightarrow L^* = 218 \text{ m}$$

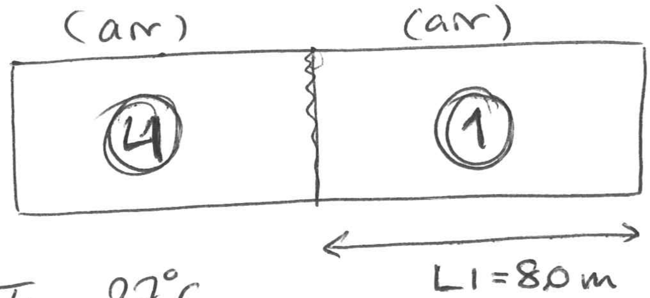
$L^* < L \Rightarrow D$ must be increased

UPDATE D AND ITERATE UNTIL $|L^* - L| < tol$

$$\Rightarrow D = 0.121 \text{ m}$$

P2 (HIGH-ENTHALPY MEASUREMENTS)

HIGH-TEMPERATURE MEASUREMENTS ARE DONE USING A SHOCK TUBE. THE POST-REFLECTION FLOW CONDITION IS USED FOR THE MEASUREMENTS. THE EXPERIMENT IS LIMITED IN TIME TO THE TIME FROM GENERATION OF THE REFLECTED SHOCK TO THE TIME WHEN THE REFLECTED SHOCK INTERACTS WITH THE CONTACT SURFACE



$T_1 = T_4 = 27°C$

$P_1 = 0.01$ atm

DESIGN CONDITIONS $\Rightarrow M_s = 5.0$!

(7.13) $$M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1}-1\right)+1} \Rightarrow \frac{P_2}{P_1} = 29.0$$

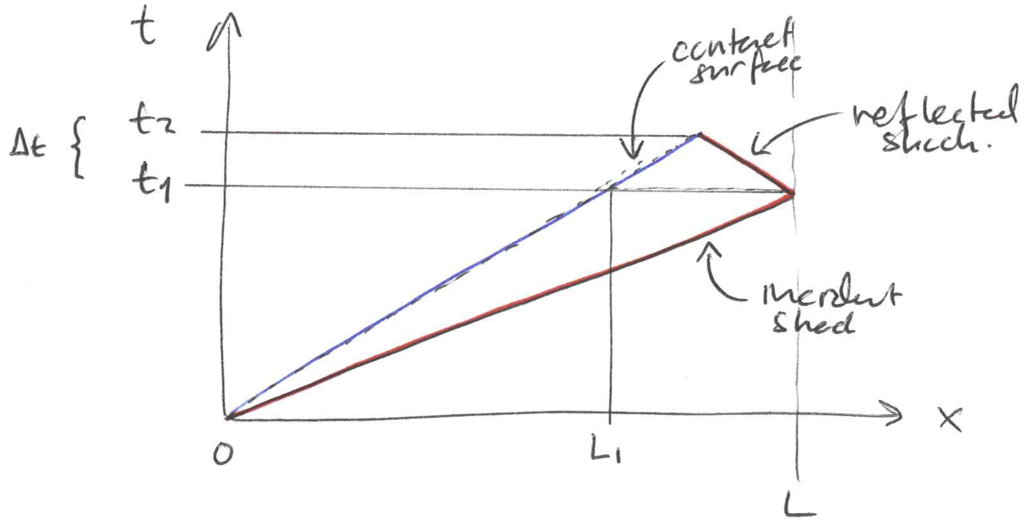
(7.16) $$u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1}-1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1}+\frac{\gamma-1}{\gamma+1}}\right)^{1/2} \Rightarrow u_p = 1388.8 \text{ m/s}$$

(7.23) $$\frac{M_R}{M_R^2-1} = \frac{M_s}{M_s^2-1}\sqrt{1+\frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2-1)\left(\gamma+\frac{1}{M_s^2}\right)}$$

$\Rightarrow M_R =$

$$\eta_R = \frac{W_R + u_p}{a_2} \Rightarrow W_R = 624{,}9 \text{ m/s}$$

$$W_s = \eta_s \cdot a_1 \Rightarrow W_s = 1735{,}9 \text{ m/s}$$



$$t_1 = L / W_s$$

$$L_1 = t_1 \cdot u_p$$

$$\Delta t = (t_2 - t_1) = (L - L_1) / (W_r + u_p)$$

$$\Rightarrow \underline{\Delta t = 0{,}79 \text{ ms}}$$

P3 (NOZZLE FLOW)

AIR FLOWS THROUGH A C-D-NOZZLE.

THE EXIT-TO-~~THROAT~~ AREA RATIO $A_e/A_t = 1.6$

NOZZLE EXIT DIAMETER $D_e = 0.2\,m$

UPSTREAM CONDITIONS: $T_0 = 300.0\,K$, $P_0 = 10.0\,bar = 1\,MPa$

a) FOR WHICH RANGE OF BACK PRESSURES WILL THE NOZZLE FLOW BE NON-ISENTROPIC?

NON-ISENTROPIC FLOW ⇒ SHOCK INSIDE THE NOZZLE

⇒ FROM CHOKED TO SHOCK-AT-EXIT

$$(5.20) \quad \left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUBSONIC SOLUTION: CHOKED FLOW (CRITICAL FLOW)

SUPERSONIC SOLUTION: SUPERCRITICAL FLOW

$M_{e_c} = 0.40$

$M_{esc} = 1.94$

$$(3.30) \quad \frac{P_0}{P_{e_c}} = \left(1+\frac{\gamma-1}{2}M_{e_c}^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_{e_c} = 897.1\,kPa$$

$$\frac{P_0}{P_{esc}} = \left(1+\frac{\gamma-1}{2}M_{esc}^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_{esc} = 141.3\,kPa$$

SHOCK AT EXIT:

$$\frac{P_{bnse}}{P_{esc}} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{esc}^2 - 1\right) \Rightarrow P_{bnse} = 593.9\,kPa$$

NON-ISENTROPIC NOZZLE FLOW:

$$593.9 < P_b < 897.1 \quad (kPa)$$

b) FOR WHICH RANGE OF BACK PRESSURES WILL THE FLOW BE SUBSONIC JUST DOWNSTREAM OF THE NOZZLE EXIT?

THE FLOW WILL BE SUPERSONIC AFTER SHOCK AT EXIT AND THUS.

SUBSONIC FLOW DOWNSTREAM OF NOZZLE:

$$P_0 > P_b \geq P_{bnse}$$

$$\underline{1.0\,MPa > P_b \geq 593.9\,kPa}$$

c) FOR WHICH RANGE OF BACK PRESSURES WILL THE FLOW BE SUPERSONIC JUST DOWNSTREAM OF THE NOZZLE EXIT

AFTER SHOCK AT EXIT:

$$P_{bnse} > P_b$$

$$\underline{593.9 > P_b} \quad (kPa)$$

d) WHAT IS THE MAXIMUM ACHIEVABLE MASSFLOW FOR THE GIVEN CONDITIONS?

(5.21)

$$\dot{m}_{choked} = \frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$\Rightarrow \underline{\dot{m}_{choked} = 45.82\ kg/s}$$

P4 (FLAT PLATE IN SUPERSONIC FLOW)

a) THE DRAG IS GENERATED BY THE SHOCKS THAT WILL FORMED AT THE ~~TRAI~~ LEADING EDGE OF THE PLATE AND THE PHENOMENON IS CALLED <u>WAVE DRAG</u>.

b)

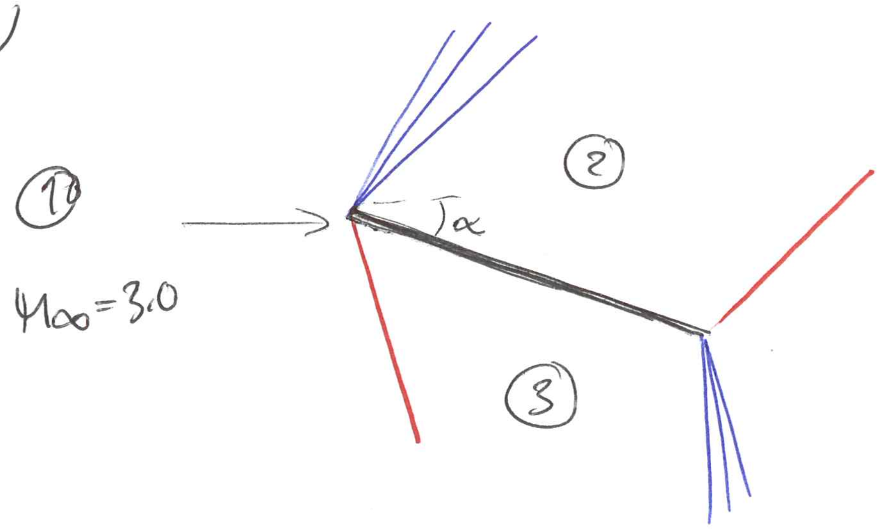


PLATE LENGTH = 0.9m

$\alpha = 20°$

UPPER SURFACE : EXPANSION FROM 1 → 2

PRANDTL–MEYER FUNCTION:

(4.44)
$$\nu_1 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_\infty^2-1)} - \tan^{-1}\sqrt{M_\infty^2-1}$$

$\nu_1 = 49.8°$

$\nu_2 = \nu_1 + \alpha = 69.8°$

$\nu_2 \Rightarrow M_2 = 4.32$ (USING 4.44)

$$\frac{P_2}{P_\infty} = \frac{P_2}{P_0}\frac{P_0}{P_\infty} = \left(\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\gamma/(\gamma-1)} = 0.16 \quad (3.30)$$

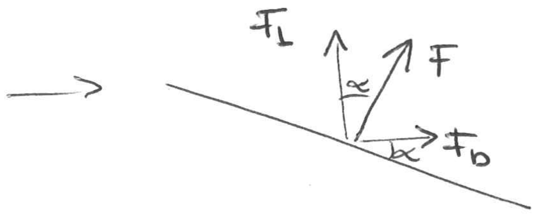
P_0 IS CONSTANT OVER THE EXPANSION (ISENTROPIC)

LOWER SURFACE: OBLIQUE SHOCK FROM $1 \to 3$

$\theta = \alpha$, $M_1 = M_\infty$: $\theta-\beta-M \Rightarrow \beta = 37.8°$

$$\left.\begin{array}{ll} (4.7) & M_{n_1} = M_\infty \sin\beta \\ (4.9) & \dfrac{p_3}{p_\infty} = 1 + \dfrac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \end{array}\right\} \Rightarrow \frac{p_3}{p_\infty} = 3.77$$

$$p_3 - p_2 = p_\infty\left(\frac{p_3}{p_\infty} - \frac{p_2}{p_\infty}\right) \simeq 3.6\, p_\infty$$

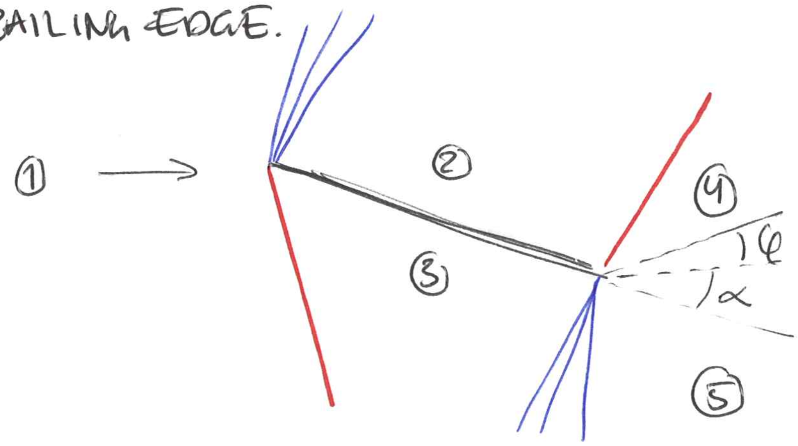


$F = (p_3 - p_2)L \simeq 3.6\, p_\infty L \simeq 1.47\, p_\infty$

$F_L \simeq 3.6\, p_\infty L \cos\alpha \simeq 1.36\, p_\infty$

$F_D \simeq 3.6\, p_\infty L \sin\alpha \simeq 0.49\, p_\infty$

c) CALCULATE THE FLOW ANGLE DOWNSTREAM OF THE TRAILING EDGE.



AFTER THE TRAILING EDGE THE FLOW ANGLE MUST BE THE SAME IN REGION 4 AND 5. ALSO, THE PRESSURE MUST BE THE SAME $p_4 = p_5$

SINCE THE FLOW FOLLOWING THE UPPER SURFACE HAS A DIFFERENT HISTORY THAN THE FLOW ALONG THE LOWER SURFACE, THERE WILL BE A NET TURNING OF THE

DOWNSTREAM OF THE TRAILING EDGE.

THE SHOCK ON THE LOWER SIDE WILL NOT HAVE THE SAME STRENGTH AS THE SHOCK ON THE UPPER SIDE OF THE PLATE SINCE THE UPSTREAM MACH NUMBERS ARE DIFFERENT.

ASSUME THAT THE FLOW WILL LOOK AS IN THE FIGURE ABOVE

OBLIQUE SHOCK $(2 \to 4)$:

$$\left.\begin{matrix} \epsilon = \alpha + \varphi \\ M_2 \end{matrix}\right\} \Rightarrow (\epsilon - \beta - M) \Rightarrow \beta$$

$$\left.\begin{matrix} M_{n_1} = M_2 \sin\beta \qquad (4.7) \\ \frac{p_4}{p_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \qquad (4.9) \end{matrix}\right\} \Rightarrow \frac{p_4}{p_2}$$

$$\frac{p_4}{p_\infty} = \frac{p_4}{p_2}\frac{p_2}{p_\infty}$$

EXPANSION $(3 \to 5)$

$$(4.44) \quad \nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}$$

$$\nu_1 = \nu(M_3)$$

$$\nu_2 = \nu_1 + \epsilon$$

$$\nu_2 = \nu(M_5) \Rightarrow M_5$$

$$\frac{p_5}{p_3} = \left(\frac{1 + \frac{\gamma-1}{2}M_3^2}{1 + \frac{\gamma-1}{2}M_5^2}\right)^{\gamma/(\gamma-1)} \qquad (3.30)$$

$$\frac{p_5}{p_\infty} = \frac{p_5}{p_3}\frac{p_3}{p_\infty}$$

GUESS A VALUE FOR φ

CALCULATE $\frac{P_4}{P_\infty}$ AND $\frac{P_5}{P_\infty}$ ACCORDING TO THE ALGORITHM ABOVE

UPDATE φ

ITERATE UNTIL $\left| \frac{P_4}{P_\infty} - \frac{P_5}{P_\infty} \right| < tol$

ITERATION GIVES $\underline{\varphi \simeq 0.89^\circ}$