TME085 - Compressible Flow

2019-06-10, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (2 p.)

(a) For a stationary normal shock, describe how the entropy, velocity, pressure and total temperature of a fluid particle is affected as it passes through the shock.

(b) What is the general definition (valid for any gas) of the "*total*" conditions p_0, T_0, ρ_0,... etc at some location in the flow?

T2. (1 p.)
An unsteady expansion wave is traveling inside a tube in which viscous effects are found to be negligible. Which of the following variables are constant throughout the expansion wave?

(a) pressure

(b) temperature

(c) entropy

(d) density

T3. (1 p.)
Derive the relation

$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

for calorically perfect gas from the energy equation form

$$h_0 = h + \frac{1}{2}V^2.$$

T4. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T5. (2 p.)

(a) In shock tubes, unsteady contact discontinuities are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p, T, ρ, u, s is/are necessarily continuous across such a contact discontinuity?

(b) The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the "*weak*" type or the "*strong*" type. What is the main difference between these two shock types and which type is usually seen in reality?

T6. (2 p.)
Derive the special formula for the speed of sound a for a calorically perfect gas:

$$a = \sqrt{\gamma RT}$$

starting from the general formula:

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

T7. (1 p.)
Assume a steady-state 1D flow with a stationary normal shock. The fluid particles crossing the shock are subjected to

(a) a pressure drop
(b) a density increase
(c) an entropy increase
(d) a temperature drop
(e) a deceleration

Which statements are true and which are false?

T8. (2 p.)
Derive the area-velocity relation in quasi-1D flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T9. (1p.)
When applying a CFD code for unsteady compressible flow, which of the following choices would you make: density-based or pressure-based, fully-coupled or segregated, conservative or non-conservative, explicit or implicit time stepping?

T10. (2 p.)

(a) In one-dimensional flow with heat addition, what is q^*?
(b) What happens in the flow when heat is added if the flow is initially supersonic and subsonic, respectively
(c) Describe how adding and/or removing heat from a one-dimensional flow in theory could be used to resemble the flow in a convergent-divergent nozzle

T11. (2 p.)

(a) Describe the choking of flow that occurs for pipe flow with friction. What happens if the real length L of a pipe is longer than L^* (for either subsonic flow or supersonic flow)?

(b) What is meant by choking the flow in a nozzle? Describe it.

T12. (1 p.)
Describe in words how a finite-volume spatial discretization can be achieved.

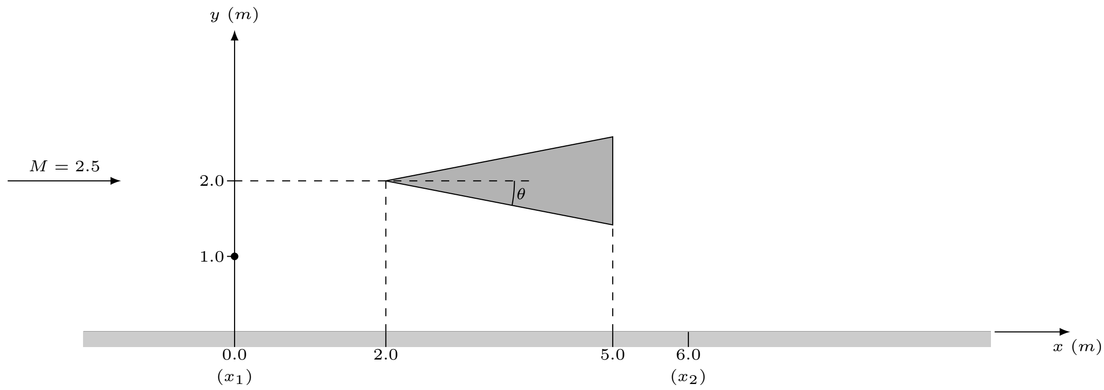
T13. (1 p.)
How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - WEDGE FLOW (10 p.)

A wedge-shaped object is placed over a flat surface according to the figure below. Air at 1.0 bar and 300.0 K is flowing passed the wedge. The freestream Mach number ahead of the object is 2.5. The wedge half angle (θ) is 11°.

Calculate and plot the pressure difference between locations along a streamline starting at (0.0,1.0), the black dot on the y-axis in the figure, and locations on the wall with the same axial coordinate, *i.e.* pressure difference as a function of axial coordinate, for $x_1 \leq x \leq x_2$. Locations of discrete changes in pressure difference should be justified by calculations.



Problem 2 - FLOW WITH FRICTION (10 p.)

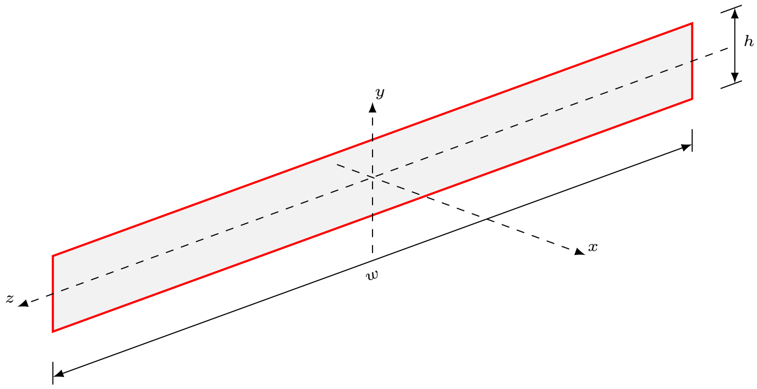
An experimental setup for estimation of the friction coefficient for supersonic air flow in a tube comprises a convergent divergent nozzle attached to a round tube. The nozzle inlet conditions are p_o=6.73 MPa and T_o=312 K. The nozzle throat diameter (d_{th}) and the nozzle exit diameter (D_e) are 0.0061 m and 0.0127 m, respectively. The tube attached to the nozzle has the same inner diameter as the nozzle at the nozzle exit plane, *i.e.* it is an axial extension of the nozzle. The nozzle flow is isentropic throughout and the flow in the convergent part of the nozzle is supersonic. The entire system can be assumed to be adiabatic.

Pressure is measured at two axial locations in the tube, see table below. Based on the given data, calculate the average friction coefficient for the specific tube and flow.

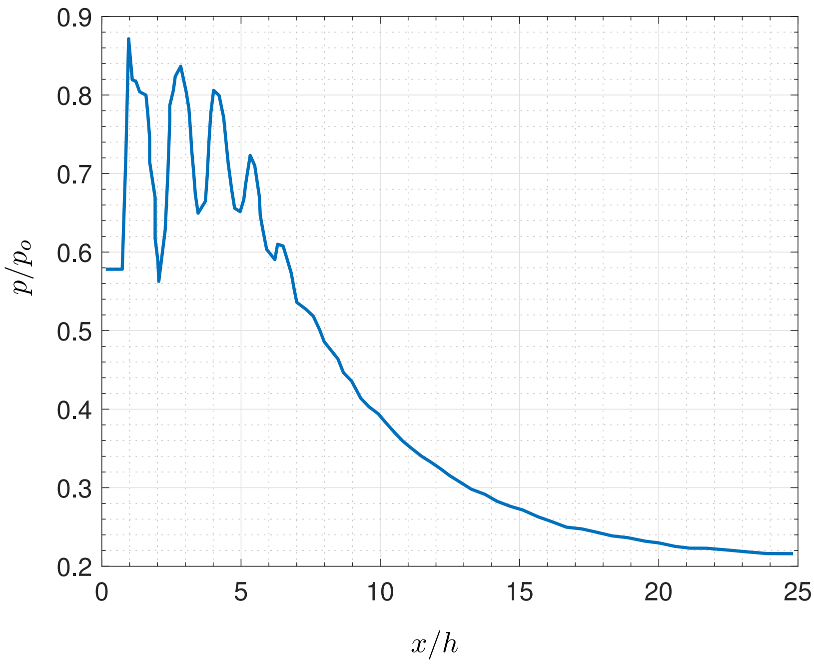
station	x/D_e	p [kPa]
1	1.75	238.0
2	29.6	485.0

Problem 3 - NOZZLE FLOW (10 p.)

The exit plane of a rectangular convergent divergent nozzle is depicted below. The width of the nozzle exit is significantly larger than the hight ($w \gg h$).



Just downstream of the nozzle exit plane the Mach number is 2.8. Pressure is measured along the x-axis resulting in the following profile.



(a) The measured pressure profile gives valuable information about the flow both upstream of the nozzle exit plane (inside the nozzle) and downstream. Based on the pressure profile, describe the nozzle expansion process in words (3p.)

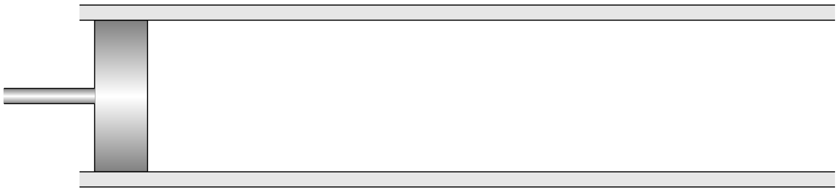
(b) The external co-flow (the surrounding flow outside of the nozzle and the downstream jet) is deflected as it is effected by the jet flow. Describe why the flow is deflected and calculate an estimate of the deflection angle (assumptions and simplifications needed should be justified) (7p.)

Problem 4 - TRAVELING WAVES (10 p.)

A horizontal tube contains stationary air at 1.0 atm and 300.0 K. The left end of the tube is closed by a movable piston, which at time t=0 is moved impulsively at a speed of 120.0 m/s (acceleration effects can be neglected). Calculate the pressure on the face of the piston for the two following scenarios:

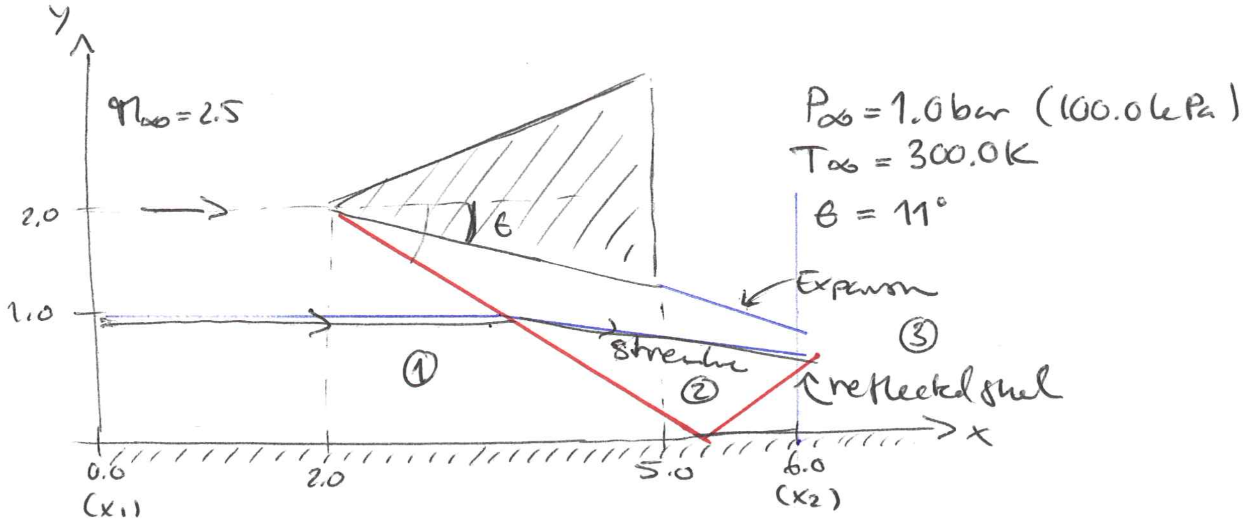
(a) the piston motion is to the left (5p.)

(b) the piston motion is to the right (5p.)



TME085 EXAM 19-06-10

P1 (WEDGE FLOW)



CALCULATE AND PLOT THE PRESSURE DIFFERENCE BETWEEN LOCATIONS ALONG A STREAMLINE STARTING AT (0.0; 1.0) AND LOCATIONS ON THE WALL (y=0) WITH THE SAME AXIAL COORDINATES.

$(\theta = 11°, M_\infty = 2.5)$ $(\theta-\beta-M) \Rightarrow \beta_1 = 32.8°$

(4.7) $M_{n_1} = M_1 \sin \beta_1$

(4.9) $\dfrac{p_2}{p_1} = 1 + \dfrac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$

(4.10) $M_{n_2}^2 = \dfrac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$

(4.12) $M_2 = M_{n_2} / \sin(\beta_1 - \theta)$

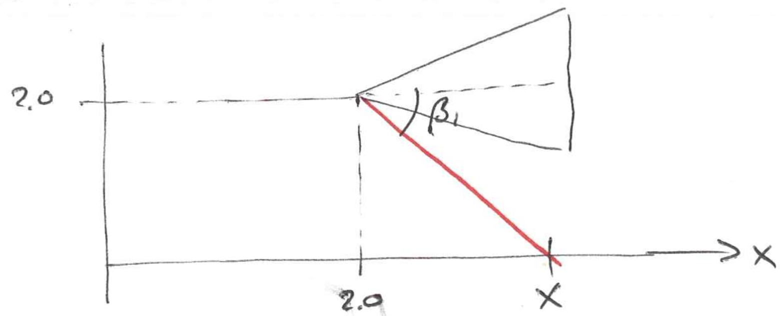
$\Rightarrow$ $M_2 = 2.04$, $\dfrac{p_2}{p_1} = 1.97$

SHOCK REFLECTION AT THE WALL:

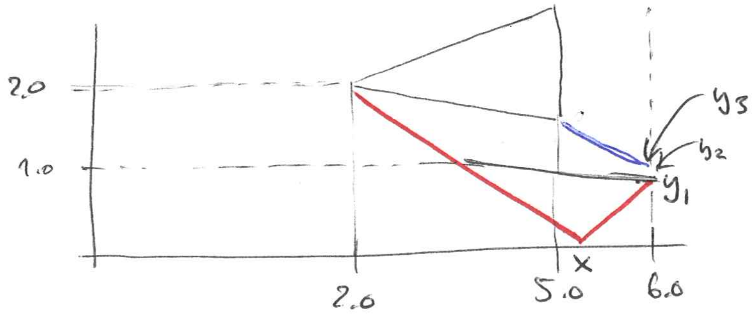
$(\theta = 11°, M_2 = 2.04 : \theta-\beta-M) \Rightarrow \beta_2 = 39.5°$

(4.7), (4.9), (4.10) & (4.12) $\Rightarrow$ $M_3 = 1.64$; $\dfrac{p_3}{p_2} = 1.81$

SHOCK REFLECTION LOCATION:



$$X = 2.0 / \tan(\beta_1) + 2.0 = 5.1 \text{ m}$$



REFLECTED SHOCK @ $x = 6.0$

$$y_1 = (6.0 - X) \cdot \tan(\beta_2 - \epsilon) = 0.488 \text{ m}$$

STREAMLINE @ $x = 6.0$

$$y_2 = 1.0 - (6.0 - ((X - 2.0)/2. + 2.0)) \tan(\epsilon) = 0.524 \text{ m}$$

EXPANSION @ $x = 6.0$

THE ANGLE OF THE EXPANSION FORMED AT THE REAR END OF THE WEDGE IS OBTAINED USING THE PRANDTL MEYER FUNCTION:

$$\nu(M_2) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2 - 1)} - \tan^{-1}\sqrt{M_2^2 - 1} \qquad (4.44)$$

$$\Rightarrow \nu(M_2) = 27.6^\circ$$

$$y_3 = 2.0 - 3.0 \tan(\epsilon) - (6.0 - 5.0) \tan(\nu(M_2)) = 0.894$$

$y_3 > y_2 > y_1 \Rightarrow$ NO INTERSECTION BEFORE $x = 6.0$

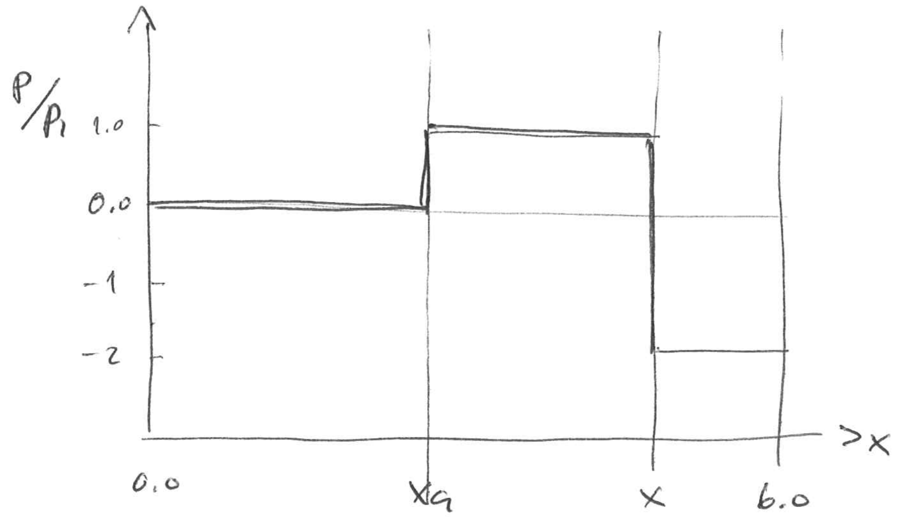
PRESSURE PLOT.

BEFORE THE STREAMLINE INTERSECTS THE FIRST SHOCK THE PRESSURE IS THE SAME ALONG THE STREMLINE AND ALONG THE WALL

THE STREAMLINE INTERSECTS THE SHOCK AT $X_a = (X - 2.0)/2 + 2.0$

BETWEEN X_a AND X, THE PRESSURE IS HIGHER ALONG THE STREAMLINE THAN ALONG THE WALL

FROM X TO 6.0, PRESSURE AT THE WALL IS P3 AND THE PRESSURE ALONG THE STREAMLINE IS P2



$0 \rightarrow X_a$:

$\Delta p = P_s - P_w = 0.$

$X_a \rightarrow X$:

$\Delta p = P_s - P_w = P_2 - P_1 \Rightarrow \frac{\Delta P_3}{P_1} = \frac{P_2}{P_1} - 1 = 0{,}97$

$X \rightarrow 6.0$:

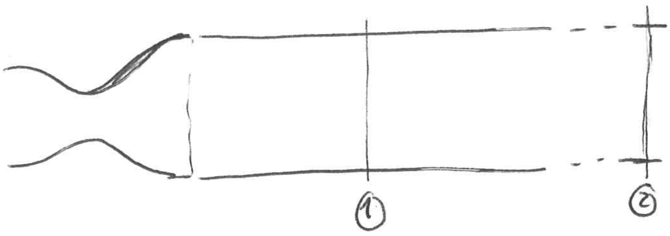
$\Delta P = P_s - P_w = P_2 - P_3 \Rightarrow \frac{\Delta P}{P_1} = \frac{P_2}{P_1} - \frac{P_3}{P_1} = \frac{P_2}{P_1} - \frac{P_2}{P_1}\frac{P_3}{P_2}$

$= \frac{P_2}{P_1}\left(1 - \frac{P_3}{P_2}\right) = -1{,}6$

P_2 (FLOW WITH FRICTION)

$P_0 = 6.73 \text{ MPa}$

$T_0 = 312 \text{ K}$



$D_{throat} = 0.0061 \text{ m}$

$D_{exit} = D_{tube} = 0.0127 \text{ m}$

STATION	1	2
x/D_e	1.75	29.6
P [kPa]	238.0	485.0

CALCULATE THE AVERAGE FRICTION COEFFICIENT $\bar{f}$

#1. CALCULATE NOZZLE-EXIT/PIPE-INLET CONDITION:

$$\frac{A_e}{A_t} = \left(\frac{D_e}{D_t}\right)^2 = 4.33$$

$$(5.20) \qquad \left(\frac{A}{A^*}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow M_e = 3.02$$

$$(3.30) \qquad \frac{P_0}{P_e} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_e = 176.6 \text{ kPa}$$

$$(3.104) \qquad \frac{P_e}{p^*} = \frac{1}{M_e}\left(\frac{\gamma+1}{2+(\gamma-1)M_e^2}\right)^{1/2} \Rightarrow p^* = 820.2 \text{ kPa}$$

$$\frac{P_1}{p^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2} \Rightarrow M_1 = 2.51$$

$$\frac{P_2}{p^*} = \frac{1}{M_2}\left(\frac{\gamma+1}{2+(\gamma-1)M_2^2}\right)^{1/2} \Rightarrow M_2 = 1.53$$

(3.107) $$\frac{4\bar{f}L^*}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma} \ln\left(\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right)$$

$\bar{f}L_e^* = 0.00167$

$\bar{f}L_1^* = 0.001379$

$\bar{f}L_2^* = 0.000466$

$$L_2^* = L_1^* - (x_2 - x_1) \Rightarrow \bar{f} = \frac{\bar{f}L_1^* - \bar{f}L_2^*}{x_2 - x_1} = 0.002581$$

$$\bar{f}L_e^* = 0.00167 \Rightarrow L_e^* = 50.9D_e > x_2$$

P3 (NOZZLE FLOW)

RECTANGULAR NOZZLE $W >> h$ => FLOW CAN BE ASSUMED TO BE TWO-DIMENSIONAL

JUST DOWNSTREAM OF THE NOZZLE EXIT, THE MACH NUMBER IS 2.8

a) DESCRIBE THE NOZZLE EXPANSION PROCESS BASED ON THE INFORMATION PROVIDED IN THE PRESSURE DATA

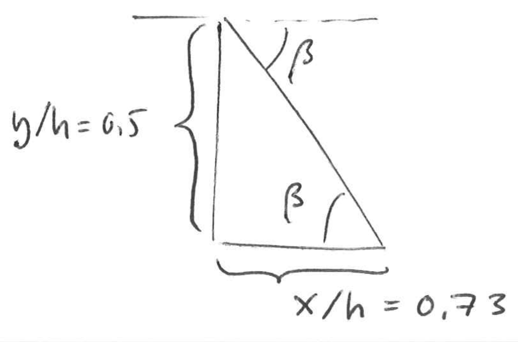
DOWNSTREAM OF THE NOZZLE ~~EX~~ EXIT, THE PRESSURE IS CONSTANT A SHORT DISTANCE THEN THE PRESSURE SUDDENLY INCREASES SIGNIFICANTLY..

THIS INDICATES THAT THE NOZZLE FLOW IS OVEREXPANDED => ISENTROPIC, SUPERSONIC FLOW IN THE DIVERGENT PART OF THE NOZZLE. THE FLOW REACHES DESIGN-MACH NUMBER AT THE NOZZLE EXIT

THE PRESSURE RATIO OVER THE OBLIQUE SHOCK DOWNSTREAM OF THE NOZZLE EXIT AND THE DISTANCE FROM THE NOZZLE EXIT TO THE PRESSURE INCREASE GIVES DATA THAT CAN BE USED TO ANALYZE THE OBLIQUE SHOCK.

THE MASSFLOW THROUGH THE NOZZLE IS THE CHOKED MASSFLOW. (NOT POSSIBLE TO CALCULATE - WOULD REQUIRE THE TOTAL TEMPERATURE AT THE NOZZLE INLET OR THE NOZZLE EXIT TEMPERATURE.)

b) THE OBLIQUE SHOCK FORMED DOWNSTREAM OF THE NOZZLE EXIT DEFLECTS THE FLOW TOWARDS THE CENTER OF THE NOZZLE AND THUS THE EXTERNAL FLOW WILL ALSO BE DEFLECTED TOWARDS THE NOZZLE CENTERLINE.

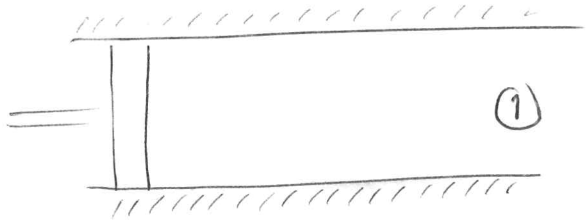


$\tan(\beta) = \frac{y/h}{x/h} \Rightarrow \beta = 34.3°$

NOZZLE EXIT MACH NUMBER: 2.8

$(\theta - \beta - M) \Rightarrow \theta = 15.4°$

P4 (TRAVELING WAVES)



BEFORE THE PISTON MOVES:

AIR @ $P = 101325$ Pa , $T = 300.0$ K

THE PISTON MOVES IMPULSIVELY AT A SPEED OF 120.0 m/s

(NEGLECT ACCELERATION EFFECTS)

CALCULATE THE PISTON PRESSURE.

a) PISTON MOVES TO THE LEFT

AN EXPANSION REGION WILL BE FORMED THAT MOVES TO THE RIGHT (INTO THE STAGNANT GAS) AND ADJUSTS THE VELOCITY AND PRESSURE GRADUALLY

(7.86)
$$\frac{P_{piston}}{P_1} = \left(1 - \frac{\gamma-1}{2}\left(\frac{u_{piston}}{a_1}\right)\right)^{2\gamma/(\gamma-1)}$$

WHERE $a_1 = \sqrt{\gamma R T_1}$

$\Rightarrow P_{piston} = 61.37$ kPa

b) PISTON MOVES TO THE RIGHT

A SHOCK WILL FORM AHEAD OF THE SHOCK

(7.16)
$$u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

ITERATION GIVES

($u_p = u\_piston$, $a_1 = \sqrt{\gamma R T_1}$) $\Rightarrow \frac{P_2}{P_1} = 1.59$

$\Rightarrow P_2 = P_{piston} = 161.57$ kPa