TME085 - Compressible Flow

2019-03-21, 08.30-13.30

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T2. (2 p.)

(a) What simplifications are made when analyzing a convergent-divergent nozzle flow using a quasi-1D approach?

(b) What are the main limitations of such an analys?

(c) What is meant by an under-expanded or over-expanded nozzle flow?

T3. (2 p.)
Derive the area-velocity relation in quasi-1D flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T4. (2 p.)

(a) When applying a time-marching flow solution scheme the so-called CFL number is an important parameter. Define the CFL number and describe its significance. What is a typical maximum CFL number for an explicit time stepping scheme.

(b) What is meant by the terms "*density-based*" and "*fully coupled*" when discussing CFD codes for compressible flow?

T5. (2 p.)

(a) What is it meant by choking the flow in a nozzle? Describe it.

(b) How does the absolute Mach number change after a *weak* and a *strong* stationary oblique shock, respectively?

T6. (2 p.)

(a) In one-dimensional flow with heat addition, what is q^*?

(b) What happens in the flow when heat is added if the flow is initially supersonic and subsonic, respectively

(c) Describe how adding and/or removing heat from a one-dimensional flow in theory could be used to resemble the flow in a convergent-divergent nozzle

T7. (2 p.)
Prove, by using one of the non-conservation forms of the energy equation, that for steady-state, adiabatic flow with no body forces the total enthalpy is preserved along stream-lines.

T8. (4 p.)

(a) Derive Crocco's relation starting from the momentum equation and the energy equation (the first and second law of thermodynamics)

(b) Describe in words the significance of Crocco's equation

(c) Prove that a steady-state irrotational flow with constant total enthalpy must also be isentropic

(d) What does Crocco's relation say about the flow behind a curved shock

T9. (1 p.)
Describe in words how a finite-volume spatial discretization can be achieved.

T10. (1 p.)
How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - NOZZLE FLOW (10 p.)

In a rocket propulsion system, a convergent divergent nozzle with area ratio 3.0 is used for the expansion of a compound gas from a combustion chamber. In the plenum chamber upstream of the nozzle, the gas mixture has a temperature of 2500.0 K and the pressure is 20.0 bar. The gas mixture can be approximated as a single species gas with a molar mass of 33.5 kg/kmol and ratio of specific heats $\gamma = 1.2$. Although not appropriate, assume that the gas can be considered to be calorically perfect.

- Why is the calorically perfect gas assumption not appropriate? What would be a better alternative? (1p)
- Assuming that the expansion process is isentropic all the way through the divergent part of the nozzle, estimate the momentum thrust $\dot{m}V_e$,where V_e is the exhaust velocity, *i.e.* the velocity at the nozzle exit plane. The nozzle throat diameter is 40.0 cm. (9p)

Solution:

Given:

Area ratio = 3.0

Nozzle throat diameter D_{th}=0.4 m

Gas mixture properties:
molar mass 33.5 kg/kmol $\Rightarrow R = R_{univ}/33.5 = 248J/(kgK)$, γ=1.2

Plenum conditions: Temperature 2500.0 K, Pressure 20.0 bar

P1a.

The given temperature (2500.0 K) is way over the range where calorically perfect gas is applicable. The temperature is even over the range where thermally perfect gas can be used. Using calorically perfect gas means that neither vibrational energy modes nor chemical reactions will be taken into account and that is probably important for an accurate prediction of temperature which means that equilibrium gas would be a better model (depending on the rate of chemical reactions in relations to flow time scales other models could be even better)

P1b.

Assume choked nozzle flow, *i.e.* supersonic flow in the divergent part, since it is stated that the flow is isentropic all the way through the divergent section. To be able to calculate the momentum thrust, we need to calculate the mass flow and the exit velocity.

$$\dot{m} = \frac{P_o A^*}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} \tag{5.21}$$

which gives $\dot{m} = 206.9$ kg/s.

Now, lets try to calculate the exit velocity. Since the flow is isentropic, we can use the Area-Mach-number relation to get the exit Mach number. The exit Mach number together with the total temperature and gives us the temperature and thus the speed of sound and with that it should be possible to get the flow velocity at the exit.

$$AR(M)^2 = \left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right)^{(\gamma+1)/(\gamma-1)} \tag{5.20}$$

Since the gas is not air we can not use the tabulated values and thus we need to do some number crunching. We now what the area ratio is at the exit and thus we can solve the equation by trail end error. I will use Newton-Raphson to solve the problem in order to get a more accurate result but that is not needed.

$$f(M^n) = 3.0 - AR(M^n)$$

$$f'(M^n) = \frac{f(M^n + \Delta M) - f(M^n - \Delta M)}{2\Delta M}$$

where ΔM is a small number $\Delta M \ll M$

$$M^{n+1} = M^n - \frac{f(M^n)}{f'(M^n)}$$

iterate until $f(M)$ converges to zero

iteration	M_e	A/A^*	$f(M_e)$	$f'(M_e)$
1	2.0	1.8837	1.1163	-2.0183
2	2.5531	3.6651	-0.66506	-4.7957
3	2.4144	3.0661	-0.066078	-3.8744
4	2.3974	3.0009	-0.00086289	-3.7736
5	2.3971	3	-1.525e-07	-3.7723

The exit temperature can now be calculated using

$$\frac{T_o}{T} = \left(1 + \frac{\gamma-1}{2}M^2\right) \tag{3.28}$$

which gives T_e=1587.7 K

$$a_e = \sqrt{\gamma R T_e}$$

$$V_e = a_e M_e = 1648.3 \ m/s$$

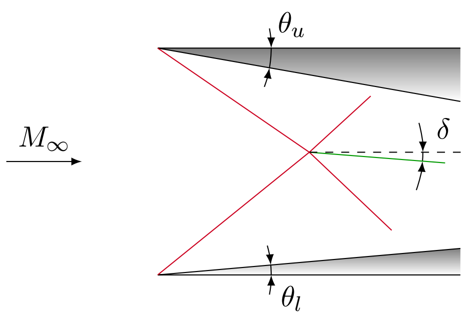
And finally, the momentum thrust $\dot{m}V_e$=0.34 MN

Problem 2 - ENGINE INTAKE (10 p.)

A supersonic flow enters an engine intake schematically represented in the figure below. Since the deflection angle of the upper wall is different from that of the lower wall, a slip line will form at the intersection of the oblique shocks. Calculate the slip line angle δ. The free stream Mach number, M_∞, is 2.0, the deflection angle of the upper wall is $\theta_u = 10°$ and the lower wall deflection angle is $\theta_l = 5°$. The flow can be assumed to be two dimensional.

a Why is there a need for the formation of a slip line at the shock intersection point (1p)

b Calculate the slip line angle δ (9p)
Note: you would most likely need to solve this problem iteratively



Solution:

Given:

Free stream Mach number M_∞=2.0

Lower wall deflection angle θ_u=5°

Upper wall deflection angle θ_u=10°

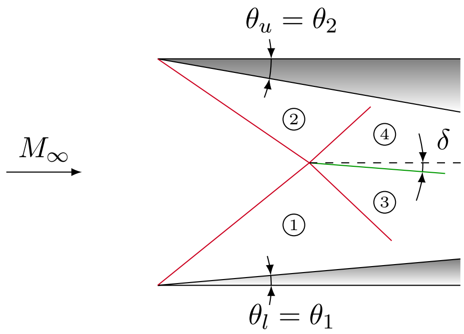
P2a.

The fact that the deflection angles of the upper and lower wall are different means that the angles of the oblique shocks will differ according to the θ-β-Mach relation. This further implies that the strength of the shocks will differ and thus the entropy generation. Furthermore, the Mach number behind the shock will be different and thus the strength and angle of the oblique shock that follows (the shock that turns the flow back again). The deflection angles defining the second upper and lower shocks will make the flow direction and pressure equal in the upper and lower flow path and a slip line will form between the two regions. Over a slip line the pressure and flow direction must be the same all other flow quantities may change discontinuously. To summarize, the reason for the formation of the slip line is the fact that fluid particles moving through different shock systems will have different history and it is usually not possible to find two different shock systems that will generate the exact same downstream flow state. The slip line reduces the requirement on the the downstream flow significantly and is nature's elegant way of solving this problem.

P2b.

Calculate the slip line angle δ

Defining flow regions 1-4 according to the figure below



Start by calculating the conditions downstream of the first two shocks using the θ-β-Mach relation and the normal shock relations

$$\tan\theta = 2\cot\beta\left(\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right) \tag{4.17}$$

or in this case

$$\tan\theta_1 = 2\cot\beta_1\left(\frac{M_\infty^2\sin^2\beta_1 - 1}{M_\infty^2(\gamma + \cos 2\beta_1) + 2}\right)$$

and

$$\tan\theta_2 = 2\cot\beta_2\left(\frac{M_\infty^2\sin^2\beta_2 - 1}{M_\infty^2(\gamma + \cos 2\beta_2) + 2}\right)$$

These equations are rather difficult to solve so the preferred alternative is to use the graphical representation provided with the exam, which gives $\beta_1 \simeq 34.3°$ and $\beta_2 \simeq 39.4°$

Now, we will use the angles β_1 and β_2 and the oblique shock relations to get the flow properties in regions 1 and 2, respectively.

$$M_{n_1} = M_1\sin\beta \tag{4.7}$$

$$M_{n_2}^2 = \frac{M_{n_1}^2 + [2/(\gamma - 1)]}{[2\gamma/(\gamma - 1)]M_{n_1}^2 - 1} \tag{4.10}$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} \tag{4.12}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{n_1}^2 - 1\right) \tag{4.9}$$

Region 1

$$M_{n_{11}} = M_\infty \sin\beta_1$$

$$M_{n_{12}}^2 = \frac{M_{n_{11}}^2 + [2/(\gamma - 1)]}{[2\gamma/(\gamma - 1)]M_{n_{11}}^2 - 1}$$

$$M_1 = \frac{M_{n_{12}}}{\sin(\beta_1 - \theta_1)} = 1.8233$$

$$\frac{p_1}{p_\infty} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{n_{11}}^2 - 1\right) = 1.3139$$

Region 2

$$M_{n_{21}} = M_\infty \sin\beta_2$$

$$M_{n_{22}}^2 = \frac{M_{n_{21}}^2 + [2/(\gamma - 1)]}{[2\gamma/(\gamma - 1)]M_{n_{21}}^2 - 1}$$

$$M_2 = \frac{M_{n_{22}}}{\sin(\beta_2 - \theta_2)} = 1.6374$$

$$\frac{p_2}{p_\infty} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{n_{21}}^2 - 1\right) = 1.7097$$

Iteration algorithm

With regions 1 and 2 defined, we will now go finding the flow state in regions 3 and 4. This part of the solution needs to be done iteratively since there is no straight forward method for solving this problem. The requirement is that after the second set of oblique shocks the flow direction and pressure must be equal in regions 3 and 4.

The algorithm will be as follows:

guess a value of θ_3, the flow deflection of the second oblique shock in the lower flow path. The angle of the slip line (and the direction of the flow) is then given by

$$\delta = \theta_3 - \theta_1$$

The upper flow path must follow the same direction, which implies

$$\theta_4 = \theta_2 - \delta$$

The θ-β-Mach together with the flow deflection angles θ_3 and θ_4 gives us the oblique shock angles β_3 and β_4

$$\tan\theta_3 = 2\cot\beta_3\left(\frac{M_1^2\sin^2\beta_3 - 1}{M_1^2(\gamma + \cos 2\beta_3) + 2}\right)$$

and

$$\tan\theta_4 = 2\cot\beta_4\left(\frac{M_2^2\sin^2\beta_4 - 1}{M_2^2(\gamma + \cos 2\beta_4) + 2}\right)$$

with values for the shock wave angles β_3 and β_4, we can use the oblique shock relations to the the pressure downstream of the two shock systems as

$$M_{n_{31}} = M_1 \sin\beta_3$$

$$\frac{p_3}{p_\infty} = \frac{p_3}{p_1}\frac{p_1}{p_\infty} = \frac{p_1}{p_\infty}\left[1 + \frac{2\gamma}{\gamma+1}\left(M_{n_{31}}^2 - 1\right)\right]$$

$$M_{n_{41}} = M_2 \sin\beta_4$$

$$\frac{p_4}{p_\infty} = \frac{p_4}{p_2}\frac{p_2}{p_\infty} = \frac{p_4}{p_\infty}\left[1 + \frac{2\gamma}{\gamma+1}\left(M_{n_{41}}^2 - 1\right)\right]$$

when

$$\frac{p_3}{p_\infty} = \frac{p_4}{p_\infty}$$

we have found the solution to the problem!

Iteration 1: as a start guess I use δ=0

$$\theta_3 = 5.0°$$

$$\delta = \theta_3 - \theta_1 = 0.0$$

$$\theta_4 = \theta_2 - \delta = 10.0°$$

The oblique shock relations gives:

$$\frac{p_3}{p_\infty} = 1.7041$$

$$\frac{p_4}{p_\infty} = 2.8119$$

The pressure is higher in the upper flow path, which indicates that the flow should be deflected downwards

Iteration 2: deflect the flow downwards

$$\theta_3 = 8.0^\circ$$

$$\delta = \theta_3 - \theta_1 = 3.0$$

$$\theta_4 = \theta_2 - \delta = 7.0^\circ$$

The oblique shock relations gives:

$$\frac{p_3}{p_\infty} = 1.9765$$

$$\frac{p_4}{p_\infty} = 2.4138$$

This is an improvement but the pressure is still higher in the upper flow path, which indicates that the flow should be deflected downwards even more

Iteration 3: deflect the flow downwards even more

$$\theta_3 = 10.0^\circ$$

$$\delta = \theta_3 - \theta_1 = 5.0$$

$$\theta_4 = \theta_2 - \delta = 5.0^\circ$$

The oblique shock relations gives:

$$\frac{p_3}{p_\infty} = 2.1853$$

$$\frac{p_4}{p_\infty} = 2.1914$$

Now we are getting close. The pressure is still slightly higher in the upper flow path, which indicates that the flow should be deflected downwards a little bit more. Let's do some fine tuning ...

Iteration 4: deflect the flow downwards a little bit more

$$\theta_3 = 10.0098^\circ$$

$$\delta = \theta_3 - \theta_1 = 5.0098$$

$$\theta_4 = \theta_2 - \delta = 4.9902^\circ$$

The oblique shock relations gives:

$$\frac{p_3}{p_\infty} = 2.1944$$

$$\frac{p_4}{p_\infty} = 2.1924$$

This is getting ridiculous! I will stop here. The angle of the slip line is slightly more than 5.0°

Problem 3 - WALL FRICTION (10 p.)

A tube is connected to reservoir containing air at 101.35 kPa and 300.0 K via a frictionless bell-mouth entrance. The tube is 3.0 m long and has a diameter of 5.0 cm. The average friction coefficient, $\bar{f}$, is 0.005. The tube is perfectly insulated.

(a) Calculate the maximum mass flow through the pipe (3p)

(b) What back pressure will result in a mass flow that is 90% of the maximum mass flow (5p)

(c) What is the exit velocity when the mass flow is 90% of the maximum mass flow (2p)

Solution:

Given:

Reservoir conditions: p_r=101.35 kPa, T_r 300.0 K

Tube dimensions: L=3.0 m, D= 5.0e^{-2} m

Tube friction coefficient: $\bar{f}$=0.005

The connection between the tube and the reservoir is friction free bell mouth $\Rightarrow T_o = T_r$ and $p_o = P_r$

P3a.

We will get the maximum mass flow when the flow is choked, which implies $L^* = L$.

$$\dot{m}_{max} = \rho^* u^* A = \rho^* a^* \frac{\pi D^2}{4}$$

We need to get values for ρ^* and a^*

$$\frac{4\bar{f}L^*}{D} = 1.2$$

interpolating in Table A.4 between $4\bar{f}L^*/D$=1.06906 and $4\bar{f}L^*/D$=1.24534 gives

M_1	0.48514
T_1/T^*	1.1459
p_1/p^*	2.2074

With the Mach number at the inlet of the tube, we can get the inlet temperature and pressure

$$\frac{T_o}{T_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right) \tag{3.28}$$

$$\frac{p_o}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)} \tag{3.30}$$

We can now calculate p^* and T^* as

$$T^* = \frac{T^*}{T_1} T_1$$

$$p^* = \frac{p^*}{p_1} p_1$$

T_1	286.513	K
p_1	86.2789	kPa
T^*	250.0255	K
p^*	39.0868	kPa

Now, using the ideal gas law and the relation between temperature and speed of sound for calorically perfect gases gives

$$\rho^* = \frac{p^*}{RT^*}$$

$$a^* = \sqrt{\gamma RT^*}$$

The maximum mass flow is now obtained as

$$\dot{m}_{max} = \rho^* a^* \frac{\pi D^2}{4} = 0.34\ kg/s$$

P3b.

In order calculate the exit pressure that results in a mass flow that is 90% of, the previously calculated, $\dot{m}_{max}$, we first need to obtain the corresponding inlet conditions
using the ideal gas law and the relation between temperature and speed of sound for calorically perfect gases we get

$$\dot{m} = \rho_1 u_1 A$$

$$\dot{m} = \frac{p_1}{RT_1} M_1 a_1 A$$

$$\dot{m} = \frac{p_1}{RT_1} M_1 \sqrt{\gamma R T_1} A$$

$$\dot{m} = \frac{p_1}{\sqrt{T_1}} M_1 \sqrt{\frac{\gamma}{R}} A$$

$$\dot{m} = \frac{p_1}{p_o} \sqrt{\frac{T_o}{T_1}} M_1 P_o \sqrt{\frac{\gamma}{RT_o}} A$$

Both p_o/p_1 and T_o/T_1 are functions of M_1 and this expression above can be solved using a trial and error approach or, as here, a Newton-Raphson solver

$$f(M^n) = 0.9 * \dot{m}_{max} - \dot{m}(M^n)$$

$$f'(M^n) = \frac{f(M^n + \Delta M) - f(M^n - \Delta M)}{2\Delta M}$$

where ΔM is a small number $\Delta M \ll M$

$$M^{n+1} = M^n - \frac{f(M^n)}{f'(M^n)}$$

iterate until $f(M)$ converges to zero

iteration	M_1	$\dot{m}$	$f(M_1)$	$f'(M_1)$
1	0.48514	0.33912	-0.034028	-0.51046
2	0.41848	0.30286	0.002235	-0.57676
3	0.42236	0.30509	7.2066e-06	-0.57304
4	0.42237	0.30509	7.3629e-11	-0.57303

Interpolate in Table A.4 between M1=0.42 and M1=0.44 and using the total condition relations

$$\frac{T_o}{T_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right) \tag{3.28}$$

$$\frac{p_o}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)} \tag{3.30}$$

and the ideal gas and relation between temperature and speed of sound for calorically perfect gases gives

L^*	4.8521	m
T_1/T^*	1.1587	
p_1/p^*	2.5491	
T_1	289.6649	K
p_1	89.6468	kPa
T^*	250.0011	K
p^*	35.1682	kPa
ρ^*	0.49015	kg/m^3
a^*	316.9391	m/s

At the end of the tube (L=3.0 m), $L_2^* = L^* - L$=1.8521 m, which gives

$$\frac{4\bar{f}L_2^*}{D} = 0.7408$$

interpolating in Table A.4 between $4\bar{f}L^*/D$=0.673571 and $4\bar{f}L^*/D$=0.786625 gives

M_2	0.5481
T_2/T^*	1.132
p_2/p^*	2.5368

The starred quantities are constants which means that the exit conditions can now be obtained

T_2	282.9938	K
p_2	89.2153	kPa

P3c.

The exit velocity is simply calculated as $u_2 = M_2 a_2$

$$a_2 = \sqrt{\gamma R T_2} = 337.2046\ m/s$$

and thus u_2=184.8 m/s

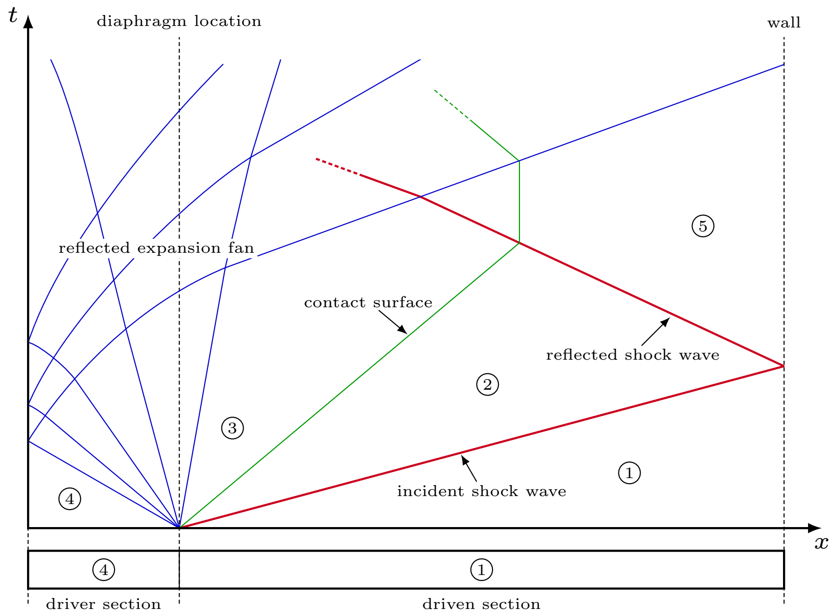
Problem 4 - SHOCK TUBE (10 p.)

A shock tube is classic application of compressible flow where two gases are initially separated by a diaphragm (see figure below). The gas in the chamber to the left of the diaphragm (the driver section) is higher than the pressure in the chamber to the right (the driven section). The gases can also differ in terms of molecular weight and temperature. As the diaphragm is removed a series of events is initiated. A right-running shock wave and a left-running expansion region is generated. A contact surface separates the part of the fluid in the driven section that has been passed by the incident shock wave (region 2) and the fluid in the driver section that has been passed by the expansion wave (region 3).

In the following, let's assume that both the driver gas and the driven gas is air (*i.e.* $\gamma_4 = \gamma_1 = \gamma = 1.4$). Initially, before the diaphragm breaks, the pressure and temperature in the driven section, p_1 and T_1, are 1.0 bar and 293.0 K, respectively, and the pressure in the driver section, p_4, is 10.0 bar.

(a) The tail of the left-running expansion region (the boundary to region 3) can, although the expansion region is a left-running wave, travel to the right (as indicated in the figure below) - explain why (1p)

(b) Under specific circumstances the tail of the expansion wave stands still in the shock tube. For that specific condition, calculate the Mach number of the incident shock wave, M_s, and the induced velocity behind it, u_p (9p)



Solution:

Given:

Initial conditions

driver section: p_4=10.0 bar

driven section: p_1=1.0 bar, T_1=293.0 K

Gas properties: both chambers are filled with air γ_1=γ_4=γ=1.4

P4a.

The expansion region is a left-running wave and the tail of the region (the "border" to region 3) propagates to the left with the speed $a_3 - u3$. The flow velocity in region 3 is the same as the flow velocity in region 2 since there can not be a discontinuity in flow velocity over the contact discontinuity surface, which means that the flow velocity in region 3 equals the induced velocity behind the incident shock u_p. This means that if the induced velocity u_p exceeds speed of sound in region 3, the tail of the expansion region will propagate to the right although it is a left-running wave.

P4b.

If the tail of the expansion region is standing still that implies that the induced velocity $u_p = u_2 = u_3$ equals the speed of sound in region 3 (a_3).
Start by examine one of the relations for the expansion region

$$\frac{T}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}\right]^2 \tag{7.85}$$

Evaluated at the tail of the expansion region 7.85 becomes

$$\frac{T_3}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u_3}{a_4}\right]^2$$

but $u_3 = a_3$ which gives

$$\frac{T_3}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{a_3}{a_4}\right]^2$$

with $a_3 = \sqrt{\gamma R T_3}$ and $a_4 = \sqrt{\gamma R T_4}$, we get

$$\frac{T_3}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\sqrt{\frac{T_3}{T_4}}\right]^2$$

$$\sqrt{\frac{T_3}{T_4}} = 1 - \frac{1}{2}(\gamma - 1)\sqrt{\frac{T_3}{T_4}}$$

$$(1 + \frac{1}{2}(\gamma - 1))\sqrt{\frac{T_3}{T_4}} = 1$$

$$\sqrt{\frac{T_3}{T_4}} = (1 + \frac{1}{2}(\gamma - 1))^{-1}$$

$$\frac{T_3}{T_4} = (1 + \frac{1}{2}(\gamma - 1))^{-2}$$

The expansion process is isentropic which means that we can use the isentropic relations and thus

$$\frac{p_3}{p_4} = (1 + \frac{1}{2}(\gamma - 1))^{-2\gamma/(\gamma-1)}$$

There is no change in pressure over the contact discontinuity which means that $p_3 = p_2$ and thus

$$\frac{p_2}{p_4} = (1 + \frac{1}{2}(\gamma - 1))^{-2\gamma/(\gamma-1)}$$

$$p_2 = p_4(1 + \frac{1}{2}(\gamma - 1))^{-2\gamma/(\gamma-1)}$$

In order to be able to calculate the induced Mach number we need the pressure ration over the moving shock p_2/p_1. p_1 is known and thus we can divide the the expression above to get the pressure ratio

$$\frac{p_2}{p_1} = \frac{p_4}{p_1}(1 + \frac{1}{2}(\gamma - 1))^{-2\gamma/(\gamma-1)}$$

With the pressure ratio p_2/p_1 obtained we can now finally calculate the Mach number of the incident shock wave using the following relation:

$$M_s = \sqrt{\frac{\gamma + 1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \tag{7.13}$$

which gives $M_s = 1.6$

The induced velocity is calculated as:

$$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right]^{1/2} \tag{7.16}$$

with $a_1 = \sqrt{\gamma R T_1}$ it is possible to calculate the induced velocity.

The Mach number of the induced flow $M_2 = 0.68$

The induced velocity $u_p = 275.7$ m/s