TME085 - Compressible Flow

2018-06-05, 08.30-13.30, M-building

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (2 p.)

(a) What is the general definition (valid for any gas) of the "*total*" conditions p_0, T_0, ρ_0,... etc at some location in the flow?

(b) For a steady-state adiabatic compressible flow of calorically perfect gas, which of the variables p_0 (total pressure) and T_0 (total temperature) is/are constant along streamlines? Why?

T2. (1 p.)
Derive the relation

$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

for calorically perfect gas from the energy equation form

$$h_0 = h + \frac{1}{2}V^2.$$

T3. (1 p.)
In shock tubes, unsteady contact discontinuities are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p, T, ρ, u, s is/are necessarily continuous across such a contact discontinuity?

T4. (1 p.)
A stationary normal shock with upstream Mach number M_1 ($M_1 > 1$) is compared to a moving normal shock, traveling with Mach number M_S into quiescent (non-moving) air. If $M_1 = M_S$, is there any physical difference between the two shock waves apart from the fact that they have different speeds relative to the observer?

T5. (2 p.)
What is the difference between a calorically perfect gas and a thermally perfect gas? For what conditions are the two gas models applicable?

T6. (1 p.)
Assume a steady-state 1D flow with a stationary normal shock. The fluid particles crossing the shock are subjected to

(a) a pressure drop

(b) a density increase

(c) an entropy increase

(d) a temperature drop

(e) a deceleration

Which statements are true and which are false?

T7. (2 p.)
Assume a steady-state flow in a convergent-divergent nozzle. Describe what characterizes the following operating conditions:

(a) Sub-critical nozzle flow

(b) Over-expanded nozzle flow

(c) Under-expanded nozzle flow

T8. (1 p.)
In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

T9. (2 p.)
Derive the area-velocity relation in quasi-1D flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T10. (2 p.)

(a) When applying a time-marching flow solution scheme the so-called CFL number is an important parameter. Define the CFL number and describe its significance. What is a typical maximum CFL number for an explicit time stepping scheme.

(b) What is meant by the terms "*density-based*" and "*fully coupled*" when discussing CFD codes for compressible flow?

T11. (2 p.)

(a) What is it meant by choking the flow in a nozzle? Describe it.

(b) How does the absolute Mach number change after a *weak* and a *strong* stationary oblique shock, repectively?

T12. (2 p.)
Prove, by using one of the non-conservation forms of the energy equation, that for steady-state, adiabatic flow with no body forces the total enthalpy is preserved along stream-lines.

T13. (1 p.)
How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - Ramjet engine (10 p.)

An engineer wants to build and test a very simple ramjet engine for supersonic propulsion. The engine consists of a cylindrical inlet section; followed by a cylindrical heater section, and finally an axisymmetric diverging nozzle (see Figure below). The design speed is Mach 2.5 at sea level. The inlet is designed for 'straight inflow', i.e. the flow should go straight into the inlet without any change of direction or velocity. Inside the inlet section a stationary normal shock will be positioned, in order to decelerate the flow to subsonic conditions at station 2. In the heater section heat is added (by combustion) in order to achieve sonic conditions at station 3. In the expansion section the flow is further accelerated (due to the increasing duct area) to supersonic speeds. The expansion is designed to achieve pressure matching at the nozzle exit (station 4), i.e. that the exit pressure equals the ambient pressure. The ambient pressure and temperature are assumed to be 1.0 bar and 288 K respectively. The tube diameter in the inlet and heater sections is 10 cm.

(a) In order to be able to solve the problem, some simplifications needs to be made. List the simplifications that you make and justify them.

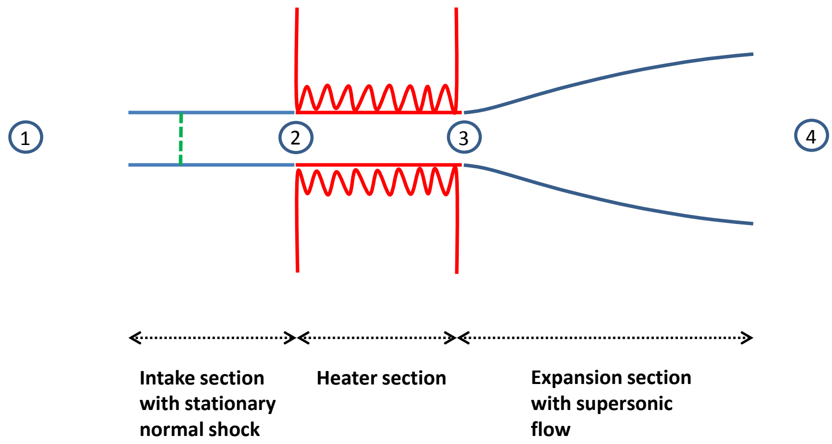
Calculate:

(b) the inflow velocity u_1 and the mass flow through the engine

(c) the exit velocity u_4

(d) the heating power needed for the engine operation

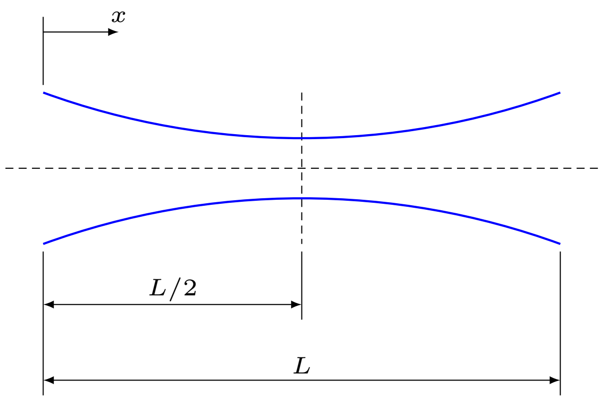
(e) the net thrust of the engine - draw some conclusions about the practical application of the engine



Problem 2 - Convergent-Divergent Nozzle (10 p.)

A converging-diverging nozzle with an exit to throat area ratio, A_e/A_t, of 1.633, is designed to operate with atmospheric pressure at the exit plane, $p_e = p_{atm}$. The converging-diverging nozzle area, A, varies with position, x, as:

$$\frac{A(x)}{A_t} = \left(\frac{A_e}{A_t} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1$$



(a) Determine the range(s) of pressure ratios (p_o/p_{atm}) for which the nozzle will be free from normal shocks

(b) Will there be a normal shock in the nozzle if nozzle pressure ratio is p_o/p_{atm}=1.5? If so, at what position (x/L) will the normal shock occur?
Hint: calculating the exit Mach number in case of existence of an internal normal shock is a good starting point

Problem 3 - Pipe flow with friction (10 p.)

A pipe with circular cross-section has a constant inner diameter $D = 1$ cm and a length L=9.45 m. The pipe is insulated to prevent any heat transfer to the air flowing inside. The inflow end of the pipe is connected to a reservoir of air with total pressure p_o and total temperature T_o. The outflow end of the pipe is open to the ambient air, with pressure $p_{amb} = 1.0$ bar. The air massflow in the pipe is 9.5^{-3} kg/s and the flow is subsonic all the way to the exit where the static flow temperature is 300.0 K.

Compute:

(a) the Mach number at the inflow end of the pipe

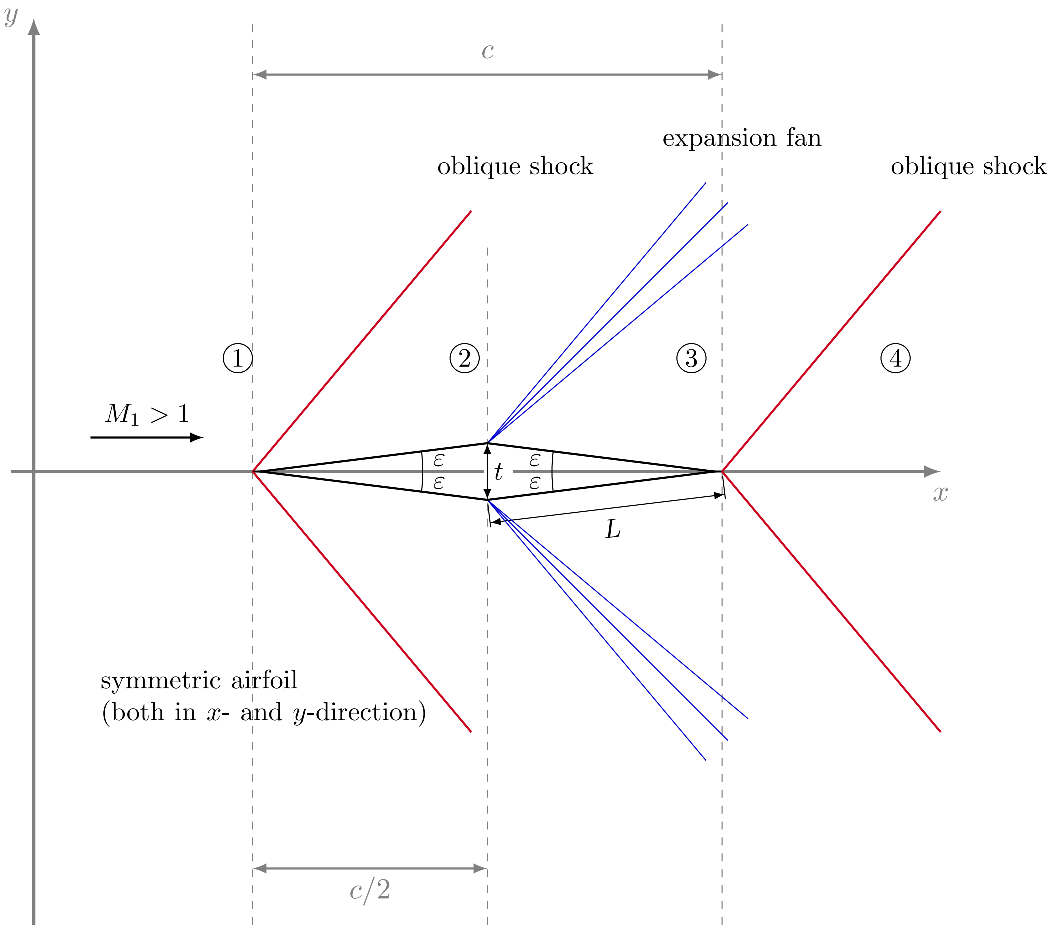
(b) the reservoir pressure p_o

The air may be treated as a calorically perfect gas with γ=1.4 and R=287 J/kg/K. The friction coefficient of the pipe is found to be f = 0.005.

Problem 4 - Shock-Expansion Theory (10 p.)

A symmetric diamond-shaped airfoil is placed in a supersonic flow of air. The airfoil is oriented with zero angle of attack relative to the freestream flow. The extent of the airfoil on the z-direction is long in comparison to the extent in the x- and y-directions, hence an assumption of two-dimensional flow can be justified. Calculate the drag (*wave drag*) per unit length if the freestream Mach number is M_1=2.0.

airfoil parameters			
half angle	ε	5	[degrees]
coord length	c	1.0	[m]



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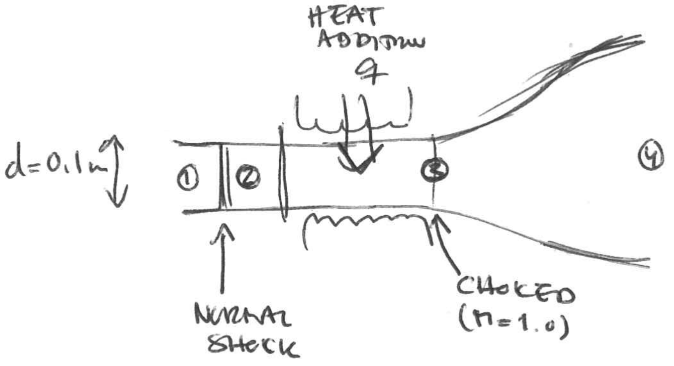
P1 (RAMJET ENGINE)



$M_1 = 3.0$
$p_1 = 100.0\,kPa$
$T_1 = 300.0\,K$
$p_4 = 100.0\,kPa$
$M_3 = 1.0$

$u_1 = M_1 a_1 = M_1\sqrt{\gamma R T_1} = 1041.6\,m/s$
$\dot{m} = u_1 \rho_1 A_1 = u_1 \frac{p_1}{RT_1}\frac{\pi d^2}{4} = 9.5\,kg/s$

1 → 2 NORMAL SHOCK

(3.51) $$M_2^2 = \frac{1+((\gamma-1)/2)M_1^2}{\gamma M_1^2-(\gamma-1)/2} \Rightarrow M_2 = 0.48$$

(3.57) $$\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma+1}(M_1^2-1) \Rightarrow p_2 = 1033.33\,kPa$$

(3.59) $$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left(\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right) \Rightarrow T_2 = 803.7\,K$$

(3.28) $$\frac{T_0}{T_2} = 1+\frac{\gamma-1}{2}M_2^2 \Rightarrow T_{02} = 840.0\,K$$

(3.30) $$\frac{p_0}{p_2} = \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)} \Rightarrow p_{02} = 1206.1\,kPa$$

2 → 3 HEAT ADDITION

$M_3 = 1.0$ (choked) $\Rightarrow T_3 = T^*$, $p_3 = p^*$

(3.85) $$\frac{p_2}{p^*} = \frac{1+\gamma}{1+\gamma M_2^2} \Rightarrow p^* = p_3 = 566.7\,kPa$$

(3.86) $$\frac{T_2}{T^*} = M_2^2\left(\frac{1+\gamma}{1+\gamma M_2^2}\right)^2 \Rightarrow T^* = T_3 = 1070.4\,K$$

(3.28) $\quad \frac{T_{03}}{T_3} = 1 + \frac{\gamma - 1}{2} M_3^2 = \frac{\gamma + 1}{2} \quad \Rightarrow T_{03} = 1284.9 \text{ K}$

(3.30) $\quad \frac{P_{03}}{P_3} = \left(1 + \frac{\gamma - 1}{2} M_3^2\right)^{\gamma/(\gamma-1)} = \left(\frac{\gamma + 1}{2}\right)^{\gamma/(\gamma-1)} \Rightarrow P_{03} = 1072.7 \text{ kPa}$

ADDED HEAT: $\quad q_{23} = q_2^* = C_p \left(T_{03} - T_{02}\right) = 446.4 \text{ kJ/kg}$

HEAT POWER: $\quad P_{23} = q_{23} \cdot \dot{m}_F = 4.29 \text{ MW}$

3 -> 4 SUPERSONIC EXPANSION

$P_4 = P_1$, $P_{04} = P_{03}$ (ISENTROPIC EXPANSION), $T_{04} = T_{03}$

(3.30) $\quad \frac{P_{04}}{P_4} = \left(1 + \frac{\gamma - 1}{2} M_4^2\right)^{\gamma/(\gamma-1)} \Rightarrow M_4 = 2.2$

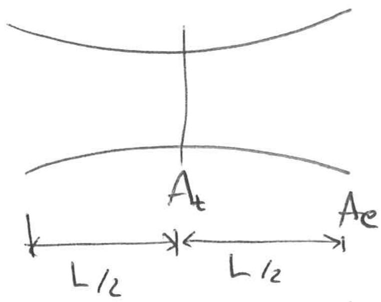
(3.28) $\quad \frac{T_{04}}{T_4} = \left(1 + \frac{\gamma - 1}{2} M_4^2\right)^{\gamma/(\gamma-1)} \Rightarrow T_4 = 652.1 \text{ K}$

$u_4 = M_4 a_4 = M_4 \sqrt{\gamma R T_4} = 1127.1 \text{ m/s}$

$F_{THRUST} = \dot{m} \left(u_4 - u_1\right) = 813.0 \text{ N}$

P2 (CONVERGENT - DIVERGENT NOZZLE)

$A_e / A_t = 1.633$



$$\frac{A(x)}{A_t} = \left(\frac{A_e}{A_t} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1$$

a) DETERMINE THE RANGES OF NOZZLE PRESSURE RATIOS FOR WHICH THE FLOW IS FREE FROM NORMAL SHOCKS.

FREE FROM NORMAL SHOCKS!

- SUBSONIC FLOW => UP TO CHOKED FLOW
- SUPERSONIC FLOW (AFTER SHOCK AT EXIT)

$$(5.20) \quad \left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUBSONIC SOLUTION: $M_{e_c} = 0{,}39$

SUPERSONIC SOLUTION: $M_{esc} = 1.96$

(3.30)

$$NPR_c = \frac{P_{oc}}{P_b} = \left(1 + \frac{\gamma-1}{2}M_{e_c}^2\right)^{\gamma/(\gamma-1)} = 1.11$$

$$NPR_{sc} = \frac{P_{osc}}{P_b} = \left(1 + \frac{\gamma-1}{2}M_{esc}^2\right)^{\gamma/(\gamma-1)} = 7.36$$

SHOCK AT EXIT :

$$\frac{P_b}{P_e} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{esc}^2 - 1\right)$$

$$NPR_{nse} = \frac{P_{onse}}{P_{b_{nse}}} = \frac{P_{osc}}{P_e}\frac{P_e}{P_b} = 1{,}70$$

SHOCK-FREE FLOW:

$1.0 < NPR < 1.11$

$NPR > 1.70$

b) NPR = 1.5 1.11 < NPR < 1.70 => NORMAL SHOCK IN NOZZLE

$$(5.28) \quad M_e^2 = \frac{-1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}\left(\frac{P_{01}A_t}{P_e A_e}\right)^2}$$

$$\Rightarrow M_e = 0.52$$

CHOKED FLOW:

(5.21)

$$\dot{m} = \frac{P_{01}A_1^*}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$\dot{m} = \frac{P_{02}A_2^*}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$\dot{m} = \text{const} \Rightarrow P_{01}A_1^* = P_{02}A_2^*$$

$$\frac{P_{02}}{P_{01}} = \frac{A_1^*}{A_2^*}$$

$$\left(\frac{A_e}{A_2^*}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

WITHOUT SHOCK: $\frac{A_e}{A_1^*} = \frac{A_e}{A_t} = 1.633$

FIND P_{02}/P_{01} IN THE NOZZLE:

FOR A LOCATION x:

$$(3.57) \quad \frac{P_y}{P_x} = 1 + \frac{2\gamma}{\gamma+1}(M_x^2-1)$$

$$(3.51) \quad M_y^2 = \frac{1+((\gamma-1)/2)M_x^2}{\gamma M_x^2 - (\gamma-1)/2}$$

$$(3.30) \quad \frac{P_{01}}{P_x} = \left(1+\frac{\gamma-1}{2}M_x^2\right)^{\gamma/(\gamma-1)}$$

$$(3.30) \quad \frac{P_{02}}{P_y} = \left(1+\frac{\gamma-1}{2}M_y^2\right)^{\gamma/(\gamma-1)}$$

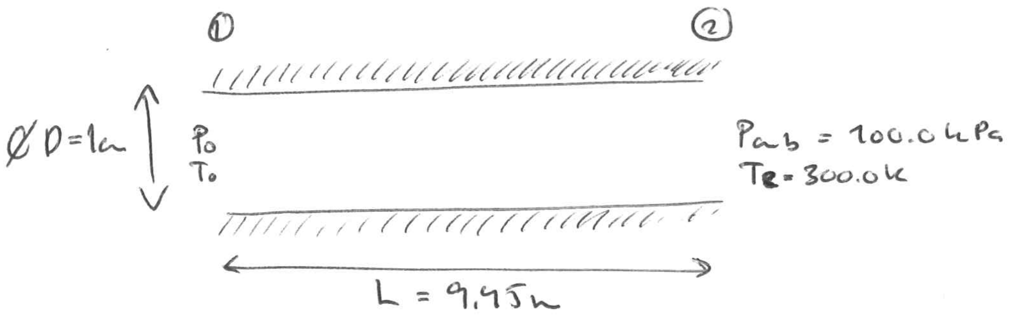
ITERATE TO FIND $M_x = 1.88$ (MACHNUMBER UPSTREAM OF SHOCK)

$$\left(\frac{A_x}{A_1^*}\right)^2 = \frac{1}{M_x^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_x^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow A_x / A^*_1 = 1{,}47$$

$$\& \ \frac{A(x)}{A_t} = \left(\frac{A_c}{A_t} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1 \Rightarrow x/L = 0{,}93$$

P3 (PIPE FLOW WITH FRICTION)



$$\dot{m} = 9.5 \cdot 10^{-3} \ kg/s$$
$$\bar{f} = 0.005$$

$$\dot{m} = 9.5 \cdot 10^{-3} = \frac{P_e}{RT_e} u_e \frac{\pi D^2}{4} = u_e = 104.1 \ m/s$$

$$M_2 = \frac{u_2}{a_2} = \frac{u_2}{\sqrt{\gamma R T_2}} = 0.3$$

(3.107) $$\frac{4\bar{f}L_2^*}{D} = \frac{1-M_2^2}{\gamma M_2^2} + \frac{\gamma+1}{2\gamma} \ln\left(\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2}\right)$$

$$\Rightarrow L_2^* = 2.65 \ m$$

$$L_1^* = L_2^* + L = 12.1 \ m$$

(3.107) $$\frac{4\bar{f}L_1^*}{D} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma} \ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$$

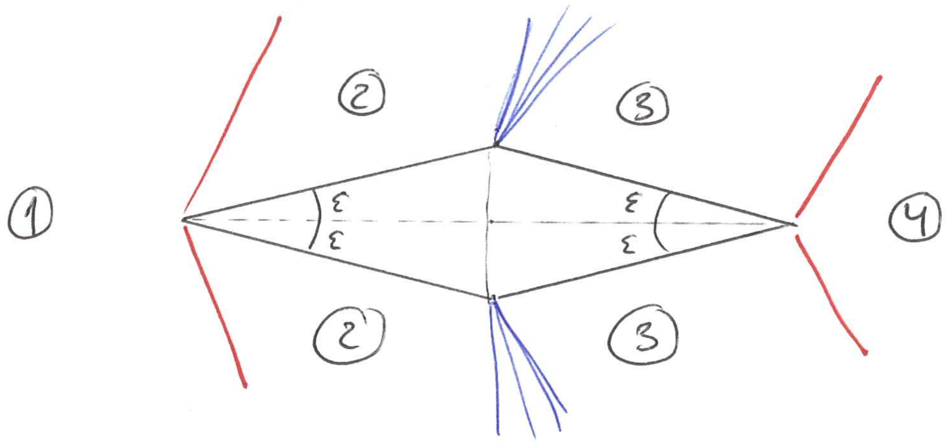
$$\Rightarrow M_1 = 0.16$$

(3.104) $$\frac{P_2}{P^*} = \frac{1}{M_2}\left(\frac{\gamma+1}{2+(\gamma-1)M_2^2}\right)^{1/2} \Rightarrow P^* = 27.6 \ kPa$$

$$\frac{P_1}{P^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2} \Rightarrow P_1 = 188.7 \ kPa$$

(3.30) $$\frac{P_{01}}{P_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_0 = 192.1 \ kPa$$

P3 (SHOCK-EXPANSION THEORY)



$\varepsilon = 5°$

Symmetric wing chord $c = 1.0\,m$

$M_1 = 2.0$

$P_1 = 101385\,Pa$

OBLIQUE SHOCK 1 → 2

$$\left.\begin{aligned}\theta &= \varepsilon \\ M &= M_1\end{aligned}\right\} (\theta-\beta-M) \Rightarrow \beta = 34.3°$$

$$\left.\begin{aligned}
&(4.7) \quad M_{n_1} = M_1 \sin\beta \\
&(4.9) \quad \frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \\
&(4.10) \quad M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1} \\
&(4.12) \quad M_2 = \frac{M_{n_2}}{\sin(\beta-\theta)}
\end{aligned}\right\}
\quad
\begin{aligned}
&M_2 = 1.82 \\
&\frac{P_2}{P_1} = 1.32 \Rightarrow P_2 = 133.3\,kPa
\end{aligned}$$

EXPANSION $2 \to 3$

(4.44) $\nu_2 = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2-1)} - \tan^{-1}\sqrt{M_2^2-1}$

FROM $2 \to 3$, THE FLOW MUST BE TURNED BACK ε DEGREES AND THEN ANOTHER ε DEGREES TO FOLLOW THE REAR PART OF THE WING, THUS $\Delta\theta = 2\varepsilon$

$\Rightarrow \nu_3 = \nu_2 + \Delta\theta = 31.34°$

(4.44) $\nu_3 = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_3^2-1)} - \tan^{-1}\sqrt{M_3^2-1}$

$\Rightarrow M_3 = 2.18$

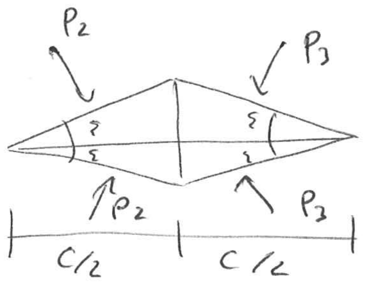
(3.30) $$\frac{p_{02}}{p_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_{03}}{p_3} = \left(1 + \frac{\gamma-1}{2}M_3^2\right)^{\gamma/(\gamma-1)}$$

$p_{02} = p_{03}$ (EXPANSION IS ISENTROPIC)

$$\Rightarrow \frac{p_3}{p_2} = \left(\frac{1+\frac{(\gamma-1)}{2}M_2^2}{1+\frac{(\gamma-1)}{2}M_3^2}\right)^{\gamma/(\gamma-1)} = 0.57$$

$$\frac{p_3}{p_1} = \frac{p_3}{p_2}\frac{p_2}{p_1} = 0.75 \Rightarrow p_3 = 75.77 \text{ kPa}$$



$$F_D = 2\left((p_2 - p_3)\frac{c}{2}\tan(\varepsilon)\right)$$

$$= (p_2 - p_3)\, c \tan(\varepsilon)$$

$$= 5.0 \text{ kN}$$