TME085 - Compressible Flow

2017-06-07, 08.30-13.30, M-building

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (1 p.)
What conditions must be satisfied for a steady-state compressible flow to be isentropic?

T2. (1 p.)
For a stationary normal shock, describe how the entropy, velocity, pressure and total temperature of a fluid particle is affected as it passes through the shock.

T3. (1 p.)
For a steady-state adiabatic compressible flow of calorically perfect gas, which of the variables p_0 (total pressure) and T_0 (total temperature) is/are constant along streamlines? Why?

T4. (1 p.)
An unsteady expansion wave is traveling inside a tube in which viscous effects are found to be negligible. Which of the following variables are constant throughout the expansion wave?

(a) pressure
(b) temperature
(c) entropy
(d) density

T5. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?
(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T6. (2 p.)

(a) In shock tubes, unsteady contact discontinuities are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p, T, ρ, u, s is/are necessarily continuous across such a contact discontinuity?
(b) The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the "*weak*" type or the "*strong*" type. What is the main difference between these two shock types and which type is usually seen in reality?

T7. (2 p.)
Assume a steady-state flow in a convergent-divergent nozzle. Describe what characterizes the following operating conditions:

(a) Sub-critical nozzle flow
(b) Over-expanded nozzle flow
(c) Under-expanded nozzle flow

T8. (2 p.)
Derive the area-velocity relation in quasi-1D flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T9. (1 p.)
When applying a CFD code for unsteady compressible flow, which of the following choices would you make: density-based or pressure-based, fully-coupled or segregated, conservative or non-conservative, explicit or implicit time stepping?

T10. (1 p.)
Describe the choking of flow that occurs for pipe flow with friction. What happens if the real length L of a pipe is longer than L^* (for either subsonic flow or supersonic flow)?

T11. (2 p.)
Derive the continuity equation in non-conservation form from the corresponding conservation form.

T12. (2 p.)
Prove, by using one of the non-conservation forms of the energy equation, that for steady-state, adiabatic flow with no body forces the total enthalpy is preserved along stream-lines.

T13. (2 p.)

(a) Describe in words the significance of Crocco's equation

(b) Prove that a steady-state irrotational flow with constant total enthalpy must also be isentropic

(c) What does Crocco's relation say about the flow behind a curved shock

Part II - Problems (40 p.)

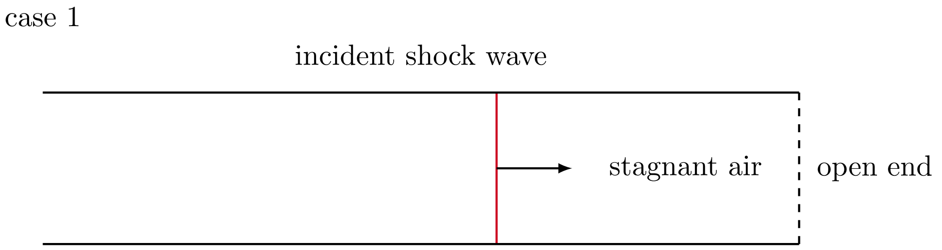
Problem 1 - Moving Shock (10 p.)

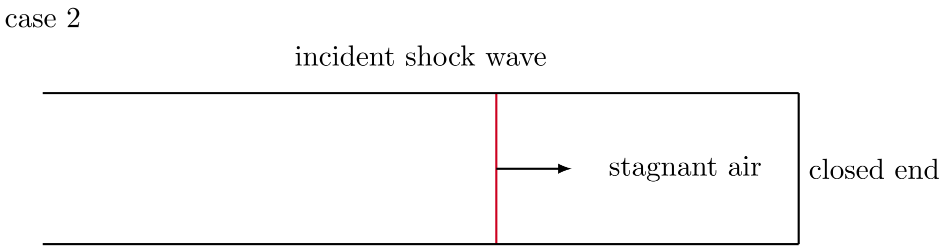
The figure below shows two different shock tube configurations. In the upper figure, the right end of the tube is open and in lower figure, the right end is closed. In both cases a shock moves downstream in the tube with the velocity (relative to the stagnant air into which it propagates) of 415 m/s. The pressure and temperature in the air in front of the shock is 101 kPa and 288 K, respectively. When the incident shock reaches the right end, we will in case 1 get an expansion wave moving to the left and in case 2 we will get a reflected shock wave.

Calculate:

(a) The temperature and pressure behind the reflected wave in case 1

(b) The temperature and pressure behind the reflected wave in case 2





Problem 2 - Combustion Chamber (10 p.)

A fuel-air mixture (approximated as air with γ=1.4) enters a duct combustion chamber at u_1 = 75 [m/s], p_1 = 150 [kPa], and T_1 = 300 [K]. The heat addition from the combustion is 900 kJ per kg of mixture.

Compute:

(a) the exit properties u_2, p_2, and T_2

(b) the total heat addition which would have caused a sonic exit flow

Solution:

First we calculate the inlet conditions

$$\frac{T_{o1}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$M_1 = \frac{u_1}{\sqrt{\gamma R T_1}}$$

$$T_{o1} = T_1 + \frac{\gamma - 1}{2\gamma R} u_1^2 = T_1 + \frac{u_1^2}{2C_p}$$

The calculated inlet total temperature and the speciifed added heat gives us the outlet total temperature according to

$$T_{o2} = T_{o1} + q/C_p$$

Now, calculate T_o^* using eqn 3.89

$$\frac{T_o}{T_o^*} = \frac{(\gamma + 1)M_1^2}{(1 + \gamma M_1^2)^2} \left(2 + (\gamma - 1)M_1^2\right)$$

At the outlet we get T_{o2}/T_o^*=0.7886

Table A.3
T_o/T_o^*=7.96478e^{-01} $\Rightarrow$ M=0.58
T_o/T_o^*=8.18923e^{-01} $\Rightarrow$ M=0.6

Interpolation gives M_2=0.573

$$\frac{T_{o2}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2$$

gives T_2=1124.9 K

$$a_2 = \sqrt{\gamma R T_2}$$

$$u_2 = M_2 a_2$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

gives: u_2=385.2 m/s, p_2=109.5 kPa

The amount needed in order to choke the flow is calculated using

$$q^* = C_p(T_o^* - T_o) = C_p T_o \left(\frac{T_o^*}{T_o} - 1\right)$$

which gives q^*=1.2 [MJ/kg]

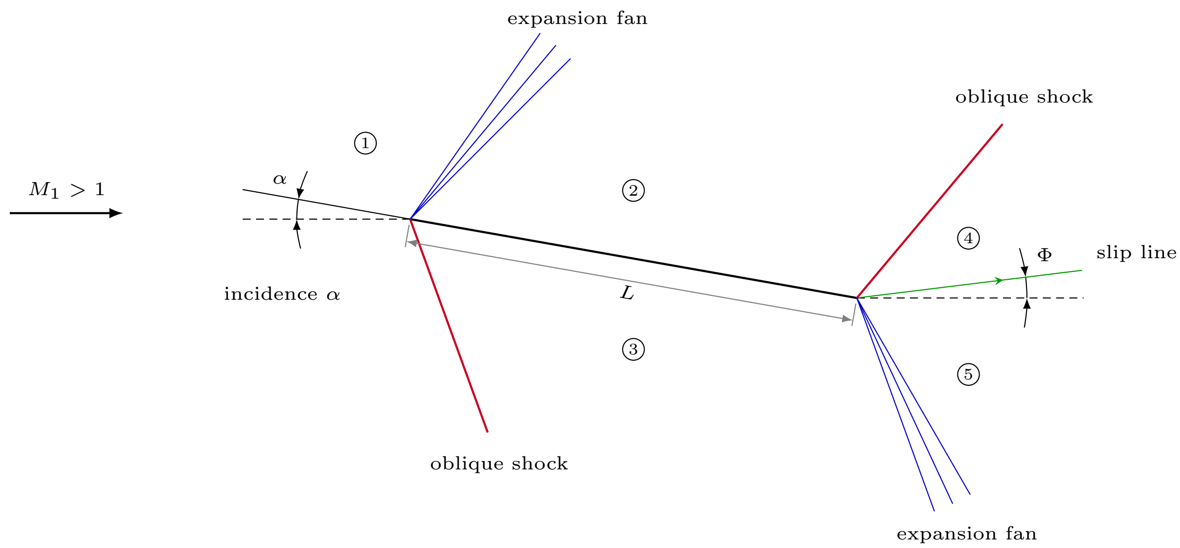
Problem 3 - Shock-Expansion Theory (10 p.)

A thin plate is placed in a supersonic flow of air. The plate has an angle relative to the free stream of 22° (angle of attack) and the freestream Mach number is 2.8

Calculate:

(a) the lift and drag per unit length of the plate if the width (L) is 0.5 [m]

(b) the direction of the flow leaving the trailing edge of the plate (Φ)
Hint: in the figure below, the angle Φ is largely exaggerated. In reality it is very small. Calculating the angle will require an iterative solution process. In order to not spend to much time on that, start by assuming that Φ is zero (or at least close to zero).



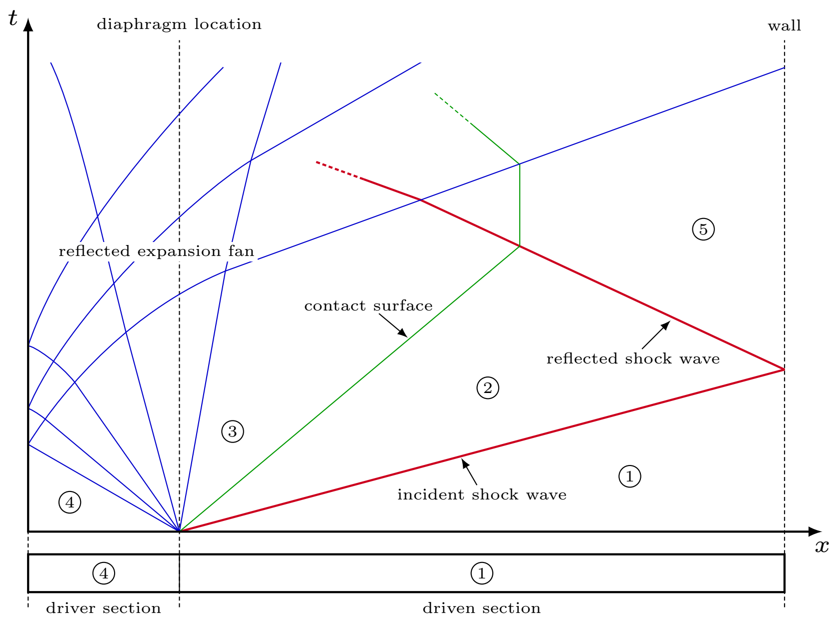
Solution: Follow example 4.16 in course book (pp. 178-180)

Problem 4 - Shock-Tube (10 p.)

A shock tube is to be constructed for experimental purposes. The desired conditions at the right end of the tube, after the incident shock wave has reflected at the right end wall, are T_5=1500 [K] and p_5=56 [bar]. In order to get this result, the pressure ratio over the incident shock wave must be at least p_2/p_1=11. Before the diaphragm (the membrane separating conditions 1 and 4) breaks the gas in the driver section (region 4) is heated to T_4=680 [K] and the pressure and temperature in the driven section (region 1) are p_1=1.0 [bar] and T_1=300 [K], respectively. The maximum allowed pressure in the driver section is 40 [bar] but to be on the safe side it it decided to not let the pressure be higher than 32.0 [bar]. The driven gas is air assumed to be calorically perfect.

(a) It is decided to use helium as the driver gas. Will it be possible to achieve the desired test conditions using helium? *justify your answer with calculations*

(b) In order to save money, the design team would like to replace the pre-heater in region 4 to one with lower power. What is the lowest temperature allowed in the driver section (region 4).



Gas/Vapor	Ratio of specific heats (γ)	Gas constant R
Acetylene	1.23	319
Air (standard)	1.40	287
Ammonia	1.31	530
Argon	1.67	208
Benzene	1.12	100
Butane	1.09	143
Carbon Dioxide	1.29	189
Carbon Disulphide	1.21	120
Carbon Monoxide	1.40	297
Chlorine	1.34	120
Ethane	1.19	276
Ethylene	1.24	296
Helium	1.67	2080
Hydrogen	1.41	4120
Hydrogen chloride	1.41	230
Methane	1.30	518
Natural Gas (Methane)	1.27	500
Nitric oxide	1.39	277
Nitrogen	1.40	297
Nitrous oxide	1.27	180
Oxygen	1.40	260
Propane	1.13	189
Steam (water)	1.32	462
Sulphur dioxide	1.29	130

http://www.engineeringtoolbox.com

Solution:

This problem is solved using one of the shock-tube relations (eqn 7.94)

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left\{1 - \frac{(\gamma_4 - 1)(a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma_1\left[2\gamma_1 + (\gamma_1 + 1)(p_2/p_1 - 1)\right]}}\right\}^{\frac{-2\gamma_4}{\gamma_4 - 1}}$$

given:

- $p2/p1$=11
- p_1=1 bar
- gas in driven section: air $\Rightarrow$ γ_1=1.4, R_1=287
- T_1=300 K $\Rightarrow a_1 = \sqrt{\gamma_1 R_1 T_1}$=347 m/s
- T_4=600 K
- p_{4max}= **32** bar

solution:

- chose gas for driver section $\Rightarrow \gamma_4$ and $R_4 \Rightarrow a_4$
- use equation 7.94 to calculate p_4
- verify that $p_4/p_1 <$ **32**

example:

- driver section gas: Helium
- γ_4=1.67, R_4=2080
- p_4=30.7 bar

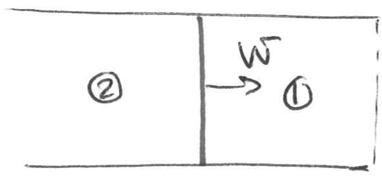
P3 (MOVING SHOCK)

SHOCK WAVE PROPAGATING AT THE VELOCITY $W = 415$ m/s TO THE RIGHT IN A TUBE

FLOW CONDITION AHEAD OF THE MOVING SHOCK

$u_1 = 0, \quad P_1 = 101 \text{ kPa}, \quad T_1 = 288 \text{ K}$

a) OPEN END:



WHEN THE SHOCK REACHES THE OPEN END, AN EXPANSION TRAVELING TO THE LEFT WILL BE FORMED SUCH THAT THE PRESSURE MATCHES THE AMBIENT PRESSURE (IF THE INDUCED FLOW VELOCITY BEHIND THE SHOCK IS SUBSONIC)

SHOCK MACH NUMBER

$$M_s = W / \sqrt{\gamma R T_1} = 1.22$$

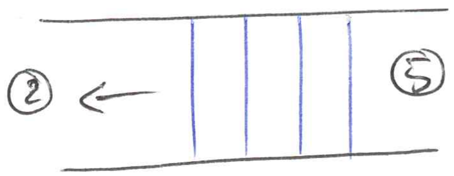
$$(7.13) \quad M_s = \sqrt{\frac{\gamma+1}{2}\left(\frac{P_2}{P_1} - 1\right) + 1} \Rightarrow \frac{P_2}{P_1} = 1{,}57$$

$$(7.16) \quad u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2} = 113.5 \text{ m/s}$$

$$(7.10) \quad \frac{T_2}{T_1} = \frac{P_2}{P_1}\left(\frac{\frac{\gamma+1}{\gamma-1} + \frac{P_2}{P_1}}{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{P_2}{P_1}\right)}\right) \Rightarrow T_2 = 328{,}5 \text{ K}$$

INDUCED FLOW MACH NUMBER:

$$M_2 = u_p / \sqrt{\gamma R T_2} = 0.31 \quad \text{(SUBSONIC)}$$



$P_5 = P_1$

THE EXPANSION IS ISENTROPIC:

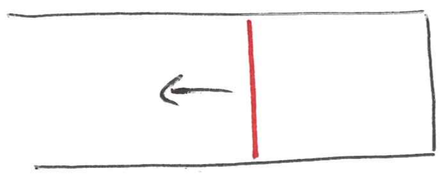
$$\left(\frac{P_5}{P_2}\right) = \left(\frac{T_5}{T_2}\right)^{\gamma/(\gamma-1)}$$

$$P_5 = P_1$$

$$\frac{T_5}{T_2} = \left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma}$$

$\Rightarrow$ $P_5 = 100$ kPa, $T_5 = 288.8$ K

b)



REFLECTED SHOCK

$$(7.23) \quad \frac{M_R}{M_R^2 - 1} = \frac{M_s}{M_s^2 - 1}\sqrt{1 + \frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2 - 1)\left(\gamma + \frac{1}{M_s^2}\right)}$$

$$(3.57) \quad \frac{P_5}{P_2} = 1 + \frac{2\gamma}{\gamma+1}(M_R^2 - 1)$$

$$\frac{P_5}{P_1} = \frac{P_5}{P_2}\frac{P_2}{P_1} \Rightarrow P_5 = 242.1 \text{ kPa}$$

$$(3.59) \quad \frac{T_5}{T_2} = \frac{P_5}{P_2}\left[\frac{2 + (\gamma-1)M_R^2}{(\gamma+1)M_R^2}\right], \quad \frac{T_5}{T_1} = \frac{T_5}{T_2}\frac{T_2}{T_1} \Rightarrow T_5 = 371.5 \text{ K}$$