TME085 - Compressible Flow

2017-03-16, 08.30-13.30, M-building

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (2 p.)

(a) For a stationary normal shock, describe how the entropy, velocity, pressure and total temperature of a fluid particle is affected as it passes through the shock.

(b) What is the general definition (valid for any gas) of the "*total*" conditions p_0, T_0, ρ_0,... etc at some location in the flow?

T2. (1 p.)
Derive the relation

$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

for calorically perfect gas from the energy equation form

$$h_0 = h + \frac{1}{2}V^2.$$

T3. (1 p.)
What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T4. (2 p.)

(a) In shock tubes, unsteady contact discontinuities are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p, T, ρ, u, s is/are necessarily continuous across such a contact discontinuity?

(b) The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the "*weak*" type or the "*strong*" type. What is the main difference between these two shock types and which type is usually seen in reality?

T5. (1 p.)
What is the difference between a calorically perfect gas and a thermally perfect gas?

T6. (2 p.)
Derive the special formula for the speed of sound a for a calorically perfect gas:

$$a = \sqrt{\gamma R T}$$

starting from the general formula:

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

T7. (1 p.)
Assume a steady-state 1D flow with a stationary normal shock. The fluid particles crossing the shock are subjected to

(a) a pressure drop
(b) a density increase
(c) an entropy increase
(d) a temperature drop
(e) a deceleration

Which statements are true and which are false?

T8. (1 p.)
In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

T9. (2 p.)
Derive the area-velocity relation in quasi-1D flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T10. (2 p.)

(a) When applying a time-marching flow solution scheme the so-called CFL number is an important parameter. Define the CFL number and describe its significance. What is a typical maximum CFL number for an explicit time stepping scheme.
(b) What is meant by the terms "*density-based*" and "*fully coupled*" when discussing CFD codes for compressible flow?

T11. (1 p.)
What is it meant by choking the flow in a nozzle? Describe it.

T12. (1 p.)
How does the absolute Mach number change after a weak/strong stationary oblique shock?

T13. (2 p.)

(a) Describe in words the significance of Crocco's equation

(b) Prove that a steady-state irrotational flow with constant total enthalpy must also be isentropic

(c) What does Crocco's relation say about the flow behind a curved shock

T14. (1 p.)

How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - Shock tube (10 p.)

Figure 1 below shows the distribution of density in a shock tube at a time $t = 0.0025$ seconds after the diaphragm was broken in a shock tube test. Unfortunately the original test data was lost and the density values are not to scale due to a bug in the data processing program. In order to be able to rerun the test, the temperature in the driven section is needed, *i.e.* the temperature to the right of the diaphragm before it was broken.

Help the lab engineers to calculate the temperature in the driven section using the information available in Figure 1 and your profound knowledge on shock tube physics.

The gas in the shock tube is air and we can assume calorically perfect gas.

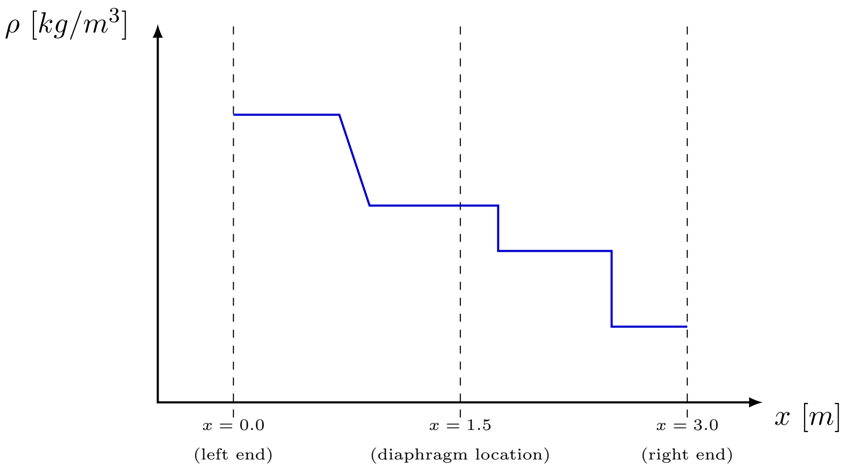

Figure 1: density as a function of axial coordinate at time $t = 0.0025s$

Problem 2 - Nozzle flow (10 p.)

A convergent divergent nozzle with the exit-to-throat area ratio of 4.0 ($A_t = 0.5\ m^2$, $A_e = 2.0\ m^2$) is operated such that a normal shock is standing at the nozzle exit plane (Figure 2). At this operating condition the pressure and temperature just downstream of the nozzle exit are $p_b = 1.0$ bar and $T_b = 300.0$ K, respectively. The gas that flows through the nozzle is air that can be assumed to be calorically perfect.

(a) calculate flow Mach number and static pressure just upstream of the nozzle exit plane

(b) calculate total pressure p_o and total temperature T_o upstream of the nozzle (plenum chamber)

(c) what is the maximum mass flow through the nozzle at the given conditions

(d) assume that a pipe with a friction factor of $f = 0.005$ is attached to the nozzle as shown in Figure 3. What is the maximum possible length of this extension pipe without altering the nozzle flow conditions?

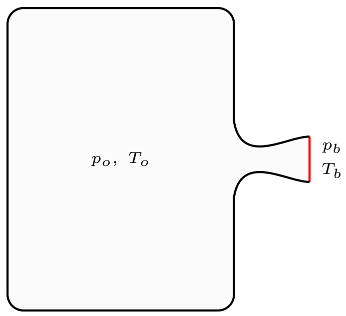

Figure 2: convergent-divergent nozzle with a shock at the nozzle exit plane

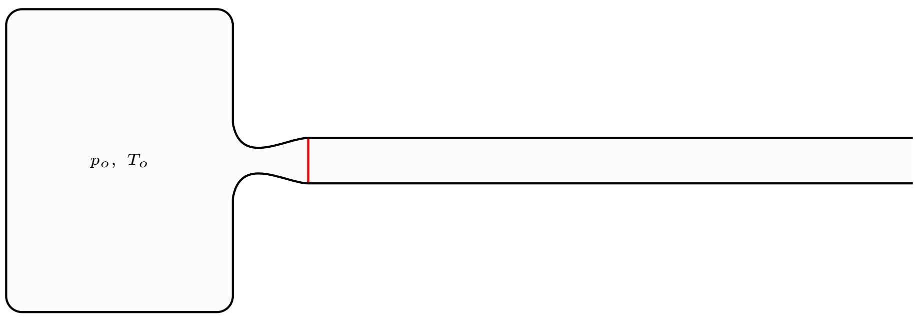

Figure 3: convergent-divergent nozzle with an attached pipe

Problem 3 - Wedge flow (10 p.)

A 20° wedge with a 10° shoulder (depicted in Figure 4 below) is situated in a flow with a free stream Mach number of $M = 2.0$.

(a) draw a schematic sketch of the important flow features in the flow over the wedge

(b) calculate the Mach numbers in regions 2 and 3

(c) assume that the flow would pass a simple 10° wedge (without the shoulder), the resulting flow direction would be the same. Would the total pressure in the directed flow be greater or lower than in the case with the shoulder? What is the reason for this?

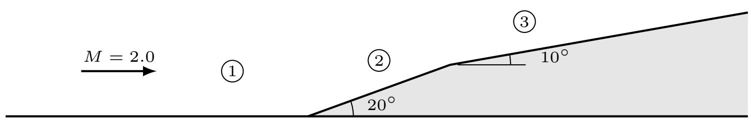

Figure 4: supersonic flow over wedge

Problem 4 - Nozzle expansion (10 p.)

Figure 5 shows a Schlieren photograph of an overexpanded jet. The Schlieren image shows a snap shot the instantaneous flow field. In order to be able to analyze the flow, a schematic representation of the shock system has been added to the figure. The angles of the oblique shocks are $\alpha_1 = 40°$ and $\alpha_2 = 45°$, respectively. Between region 1 and 4 there is a normal shock.

Calculate the exit Mach number, *i.e.* the Mach number in region 1.

The gas is air and we can assume calorically perfect gas.

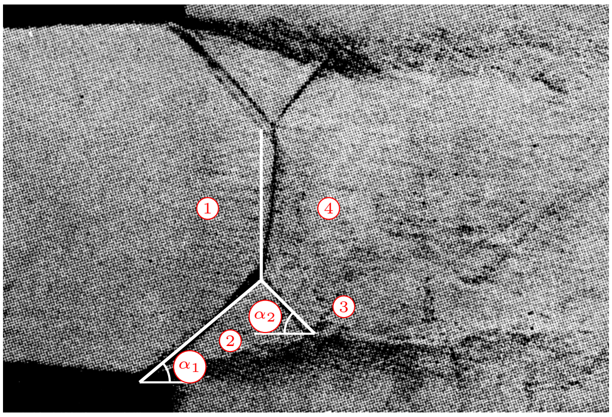

Figure 5: overexpanded nozzle flow

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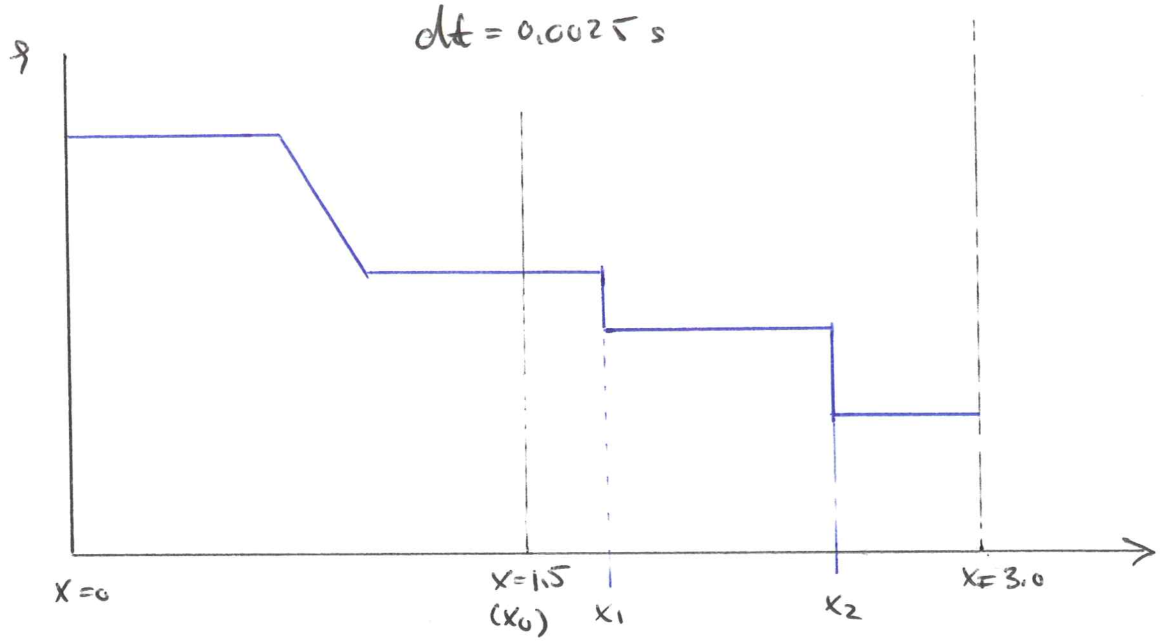
P1 (SHOCK TUBE)



THE DISCONTINUITY ~~AT~~ $x = x_2$ REPRESENTS THE INCIDENT SHOCK AND THE ONE AT $x = x_1$, THE CONTACT SURFACE.

$$\Rightarrow \quad W = \frac{x_2 - x_0}{dt}, \quad u_p = \frac{x_1 - x_0}{dt}$$

$$(7{,}13) \quad M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1}, \quad M_s = \frac{W}{a_1} \qquad (1)$$

WHERE $a_1 = \sqrt{\gamma R T_1}$

$$(7{,}16) \quad u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2} \qquad (2)$$

$$(1): \quad (M_s^2 - 1)\left(\frac{2\gamma}{\gamma+1}\right) + 1 = \frac{p_2}{p_1} \qquad (3)$$

(3) IN (2) $\Rightarrow$

$$u_p = \frac{a_1}{\gamma}\left((M_s^2 - 1)\frac{2\gamma}{\gamma+1}\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{(M_s^2-1)\left(\frac{2\gamma}{\gamma+1}\right) + 1 + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

$$u_p = \frac{a_1}{\gamma}\left((M_s^2-1)\frac{2\gamma}{\gamma+1}\right)\left(\frac{2\gamma}{(M_s^2-1)2\gamma+\gamma+1+\gamma-1}\right)^{1/2}$$

$$u_p = \frac{a_1}{\gamma}\left((M_s^2-1)\frac{2\gamma}{\gamma+1}\right)\left(\frac{2\gamma}{(M_s^2-1)2\gamma\ +2\gamma}\right)^{1/2}$$

$$u_p = \frac{a_1}{\gamma}\left((M_s^2-1)\frac{2\gamma}{\gamma+1}\right)\frac{1}{M_s}$$

$$u_p = a_1\,\frac{2(M_s^2-1)}{M_s(\gamma+1)} \qquad (4)$$

$$M_s = \frac{W}{a_1} = \frac{W}{\sqrt{\gamma R T_1}}$$

$$u_p = \sqrt{\gamma R T_1}\ \frac{2\left(W^2/(\gamma R T_1)-1\right)}{W/\sqrt{\gamma R T_1}\ (\gamma+1)}$$

$$u_p = (\gamma R T_1)\ \frac{2\left(W^2/(\gamma R T_1)-1\right)}{W(\gamma+1)}$$

$$u_p = \left(\frac{2}{\gamma+1}\right)\left(W - \frac{\gamma R T_1}{W}\right) \qquad (5)$$

$$u_p\left(\frac{\gamma+1}{2}\right) = W - \frac{\gamma R T_1}{W}$$

$$\frac{\gamma R T_1}{W} = W - u_p\left(\frac{\gamma+1}{2}\right)$$

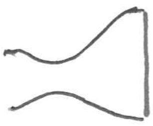
$$T_1 = \frac{W}{\gamma R}\left(W - u_p\left(\frac{\gamma+1}{2}\right)\right)$$

ESTIMATED FROM THE PROVIDED FIGURE GIVES

$$\underline{W} = \frac{1.0}{dt} \quad , \quad u_p = \frac{0.25}{dt}$$

$$T_1 = \frac{W}{\gamma R}\left(W - u_p\left(\frac{\gamma+1}{2}\right)\right) = \underline{278.75\ K}$$

P_2 (NOZZLE FLOW)



SHOCK AT EXIT
DOWNSTREAM PRESSURE : 1 bar (= 100.0 kPa)
DOWNSTREAM TEMPERATURE : 300.0 K

$$\frac{A_e}{A_t} = 4.0 \quad (A_e = 2.0\,,\ A_t = 0.5)\ (m^2)$$

a) CALCULATE MACH NUMBER AND PRESSURE JUST UPSTREAM OF THE SHOCK

$$(5.20)\quad \left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_{e_{sc}}^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_{e_{sc}}^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUPERSONIC SOLUTION..

$$M_{e_{sc}} = 2.9$$

NORMAL SHOCK:

$$(3.51)\quad M_2^2 = \frac{1+((\gamma-1)/2)M_{e_{sc}}^2}{\gamma M_{e_{sc}}^2-(\gamma-1)/2} = 0.48 \text{ (WILL BE NEEDED LATER)}$$

$$(3.57)\quad \frac{P_b}{P_e} = 1+\frac{2\gamma}{\gamma+1}\left(M_{e_{sc}}^2-1\right)$$

$$P_b = 100.0\,kPa \Rightarrow P_e = 10.1\,kPa$$

b) CALCULATE P_0 AND T_0 UPSTREAM OF THE NOZZLE

$$(3.30)\quad \frac{P_0}{P_e} = \left(1+\frac{\gamma-1}{2}M_{e_{sc}}^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_0 = 338.5\,kPa$$

$$(3.59)\quad \frac{T_b}{T_e} = \frac{P_b}{P_e}\left(\frac{2+(\gamma-1)M_{e_{sc}}^2}{(\gamma+1)M_{e_{sc}}^2}\right) \Rightarrow T_e = 115\,K$$

$$(3.28)\quad \frac{T_0}{T_e} = 1+\frac{\gamma-1}{2}M_{e_{sc}}^2 \Rightarrow T_0 = 313.8\,K$$

c) CALCULATE THE MAXIMUM MASSFLOW THROUGH THE NOZZLE FOR THE GIVEN CONDITIONS.

THE MAXIMUM MASSFLOW IS THE CHOKED MASSFLOW:

$$(5.21) \qquad \dot{m} = \frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/(\gamma-1)}} = 386 \text{ kg/s}$$

d) A PIPE WITH THE AVERAGE FRICTION FACTOR $\bar{f} = 0.005$ (FOR THE FLOW PROVIDED BY THE NOZZLE..) IS ATTACHED TO THE NOZZLE. WHAT IS THE MAXIMUM LENGTH OF THE PIPE POSSIBLE WITHOUT ALTERING THE NOZZLE FLOW..

SINCE WE ARE ASKED TO CALCULATE THE MAXIMUM LENGTH WE WILL ASSUME THAT THE PRESSURE AT THE PIPE EXIT WILL MATCH THE PIPE-EXIT-PRESSURE..

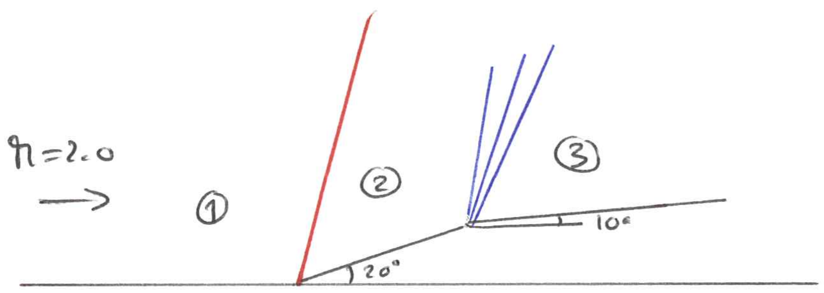
WITH THAT ASSUMPTION, THE MAXIMUM LENGTH IS THE LENGTH THAT WILL CHOKE THE FLOW (L^*)

$$(3.107) \qquad \frac{4\bar{f}L^*}{D_e} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma} \ln \left(\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2} \right)$$

FROM BEFORE (a) WE HAVE THAT $M = 0.48$

$\Rightarrow L^* = 100{,}26 \text{ m}$

P3 (WEDGE FLOW)



b) CALCULATE THE MACH NUMBER IN REGIONS 2 AND 3

1→2 (OBLIQUE SHOCK)

($\theta-\beta-M$, WITH $M=2.0$ AND $\theta=20°$) ⇒ $\beta=53.4°$

(4.7) $M_{n_1}=M_1 \sin\beta$

(4.10) $M_{n_2}^2=\dfrac{M_{n_1}^2+(2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2-1}$ ⇒ $M_2=1.2$

(4.12) $M_2=\dfrac{M_{n_2}}{\sin(\beta-\theta_1)}$

2→3 (EXPANSION)

(4.44) $\nu(M_2)=\sqrt{\dfrac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\dfrac{\gamma-1}{\gamma+1}(M_2^2-1)}-\tan^{-1}\sqrt{M_2^2-1}$

$=3.8°$

$\nu(M_3)=\nu(M_2)+\theta_2=13.8°$

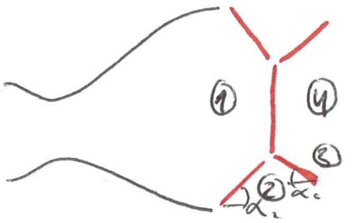
(4.44) ⇒ $M_3=1.56$

c)

THE REDUCTION OF TOTAL PRESSURE IS HIGHER IN THE SHOULDER CASE THAN IN THE SIMPLE WEDGE CASE THE REASON IS THAT A 20° DEFLECTION RESULTS IN A STRONGER SHOCK THAN A 10° DEFLECTION WITH GREATER REDUCTION OF TOTAL PRESSURE AS A CONSEQUENCE THE EXPANSION IS ISENTROPIC AND THUS THE SHOULDER EXPANSION PROCESS DOES NOT AFFECT THE TOTAL PRESSURE.

P4 (NOZZLE EXPANSION)

OVEREXPANDED JET



$\alpha_1 = 40°$

$\alpha_2 = 45°$

THE FIRST OBLIQUE SHOCK WILL TURN THE FLOW INWARDS AN ANGLE θ AND THE REFLECTED SHOCK WILL TURN THE FLOW BACK AGAIN (NOT NECESSARILY THE SAME ANGLE)

THE PRESSURE AFTER THE SECOND OBLIQUE SHOCK SHOULD MATCH THE PRESSURE IN REGION 4 (AFTER THE NORMAL SHOCK)

1 → 4 NORMAL SHOCK

$$(3.57) \quad \frac{p_4}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)$$

1 → 2 OBLIQUE SHOCK $(\theta\text{-}\beta\text{-}M,\ \beta_1 = \alpha_1,\ M = M_1) \Rightarrow \theta_1 = ?$

$$(4.7) \quad M_{n_1} = M_1 \sin(\beta_1)$$

$$(4.9) \quad \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)$$

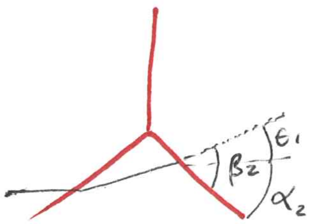
$$(4.10) \quad M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$$

$$(4.12) \quad M_2 = \frac{M_{n2}}{\sin(\beta_1 - \theta_1)}$$

2 → 3 OBLIQUE SHOCK $(\theta\text{-}\beta\text{-}M,\ \beta = \alpha_2 + \theta_1,\ M = M_2) \Rightarrow \theta_2 = ?$

$$(4.7) \quad M_{n_1} = M_2 \sin(\beta_2)$$

$$(4.9) \quad \frac{p_3}{p_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_2}^2 - 1\right)$$



$\beta_2 = \alpha_2 + \theta_1$

(4.10) $M_{n2}^2 = \dfrac{M_{n1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n1}^2 - 1}$

(4.12) $M_3 = \dfrac{M_{n2}}{\sin(\beta_2 - \theta_2)}$

1 → 3 (TWO OBLIQUE SHOCKS)

$$\frac{P_3}{P_1} = \frac{P_3}{P_2}\frac{P_2}{P_1}$$

ITERATE UNTIL $P_3 \simeq P_4$ ⇒

$M_1 = 2.4$

$\theta_1 = 16.5$

$\theta_2 = 17.2$

$M_2 = 1.7$

$M_3 \simeq 1.0$