TME085 - Compressible Flow

2016-04-04, 08.30-13.30, M-building

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (2 p.)

(a) What is the general definition (valid for any gas) of the "*total*" conditions p_0, T_0, ρ_0,... etc at some location in the flow?

(b) For a steady-state adiabatic compressible flow of calorically perfect gas, which of the variables p_0 (total pressure) and T_0 (total temperature) is/are constant along streamlines? Why?

T2. (1 p.)
Derive the relation

$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

for calorically perfect gas from the energy equation form

$$h_0 = h + \frac{1}{2}V^2.$$

T3. (1 p.)
What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T4. (1 p.)
An engineer wants to apply a numerical solution scheme for compressible flow. The flow he is interested in contains shocks. He has to choose between two different solution methods – one which is based on the conservation form of the governing equations and one which is based on the non-conservation form of the governing equations. Which method should he choose and why?

T5. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T6. (1 p.)
In shock tubes, unsteady contact discontinuities are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p, T, ρ, u, s is/are necessarily continuous across such a contact discontinuity?

T7. (2 p.)

(a) What is meant by an under-expanded or over-expanded nozzle flow?

(b) What is it meant by choking the flow in a nozzle? Describe it.

T8. (2 p.)
Derive the special formula for the speed of sound a for a calorically perfect gas:

$$a = \sqrt{\gamma R T}$$

starting from the general formula:

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

T9. (1 p.)
Assume a steady-state 1D flow with a stationary normal shock. The fluid particles crossing the shock are subjected to

(a) a pressure drop
(b) a density increase
(c) an entropy increase
(d) a temperature drop
(e) a deceleration

Which statements are true and which are false?

T10. (2 p.)

(a) When applying a time-marching flow solution scheme the so-called CFL number is an important parameter. Define the CFL number and describe its significance. What is a typical maximum CFL number for an explicit time stepping scheme.
(b) What is meant by the terms "*density-based*" and "*fully coupled*" when discussing CFD codes for compressible flow?

T11. (1 p.)
Describe the choking of flow that occurs for pipe flow with friction. What happens if the real length L of a pipe is longer than L^* (for either subsonic flow or supersonic flow)?

T12. (1 p.)
How does the absolute Mach number change after a weak/strong stationary oblique shock?

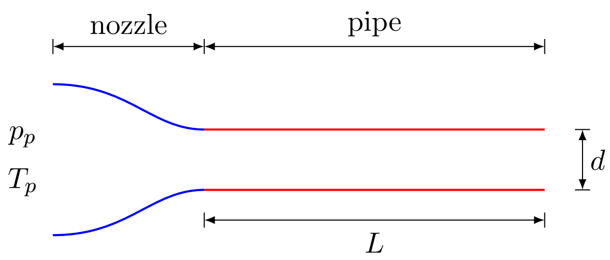
T13. (2 p.)
Prove, by using a suitable equation, that a steady-state irrotational flow with constant total enthalpy must also be isentropic.

T14. (1 p.)
How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - Friction (10 p.)

A pipe with constant diameter is attached to a convergent nozzle. Air in a tank upstream of the nozzle is accelerated through the nozzle and flows through the pipe. The pressure and temperature of the air in the plenum chamber (the gas chamber upstream of the nozzle) is $p_p = 1.0$ MPa and $T_p = 400.0$ K, respectively. The nozzle flow can be assumed to be isentropic.



Calculate:

(a) The maximum achievable mass flow through the construction if the pipe friction factor, f, is 0.032 and the length and diameter of of the pipe is 0.2 m and 0.02 m, respectively

(b) The maximum mass flow through the nozzle if the pipe is removed

Problem 2 - Shock Waves (10 p.)

The picture below shows a Schlieren image of a bullet fired from a rifle. The Schlieren technique shows variation in refractive index in a gas and thus it visaulizes density gradients in a flow field. Since density gradients are essential in compressible flows and aeroacoustics, the Schlieren technique is often used as a visaulization tool for such flows. Since the picture shows a person holding the rifle, I think that we can assume that the bullet is fired into air at standard atmosphere conditions. Furthermore, we can assume that the front of the bullet has a sharp conical shape with a half angle of 10.0°.

Estimate:

(a) The Mach number at which the bullet propagates at the instant of the photo

(b) The pressure ratio over the leading shock wave

(c) The Mach number range for which we would get a detached shock in front of the bullet

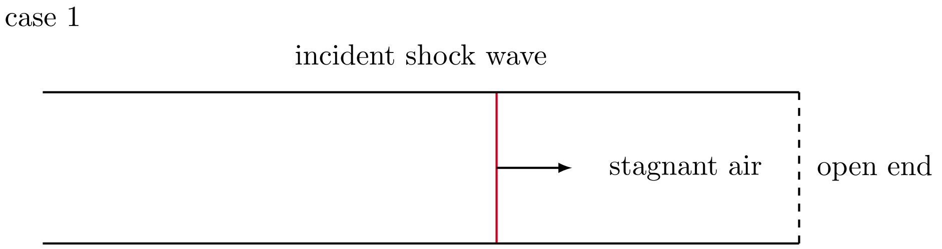
Problem 3 - Moving Shock (10 p.)

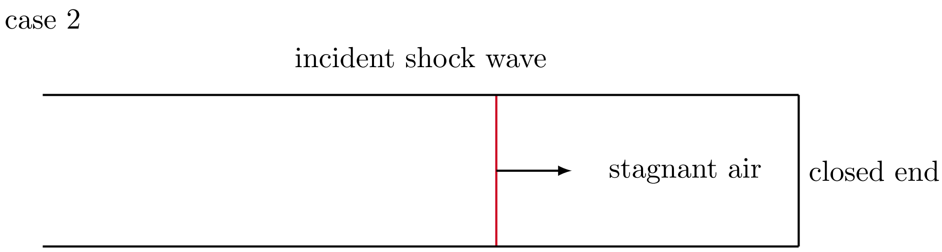
The figure below shows two different shock tube configurations. In the upper figure, the right end of the tube is open and in lower figure, the right end is closed. In both cases a shock moves downstream in the tube with the velocity (relative to the stagnant air into which it propagates) of 415 m/s. The pressure and temperature in the air in front of the shock is 101 kPa and 288 K, respectively. When the incident shock reaches the right end, we will in case 1 get an expansion wave moving to the left and in case 2 we will get a reflected shock wave.

Calculate:

(a) The temperature and pressure behind the reflected wave in case 1

(b) The temperature and pressure behind the reflected wave in case 2

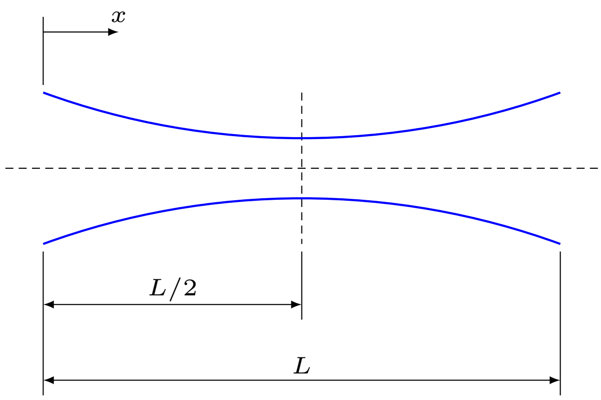




Problem 4 - Convergent-Divergent Nozzle (10 p.)

A converging-diverging nozzle with an exit to throat area ratio, A_e/A_t, of 1.40, is designed to operate with atmospheric pressure at the exit plane, $p_e = p_{atm}$. The converging-diverging nozzle area, A, varies with position, x, as:

$$\frac{A(x)}{A_t} = \left(\frac{A_e}{A_t} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1$$



(a) Determine the range(s) of pressure ratios (p_o/p_{atm}) for which the nozzle will be free from normal shocks

(b) Will there be a normal shock in the nozzle if nozzle pressure ratio is p_o/p_{atm}=1.5? If so, at what position (x/L) will the normal shock occur?
Hint: calculating the exit Mach number in case of existence of an internal normal shock is a good starting point

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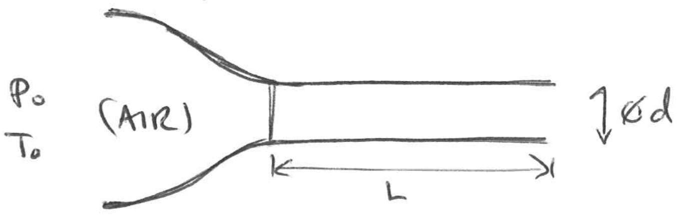
P1 (FRICTION)

$p_o = 1.0\,MPa$
$T_o = 400.\,K$



ISENTROPIC FLOW THROUGH NOZZLE ⇒ p_o AND T_o ARE CONSTANT THROUGH THE NOZZLE.

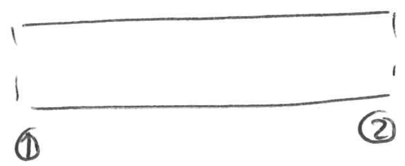
a) CALCULATE THE MAXIMUM MASSFLOW IF

$\bar{f} = 0.032$ (rather high)

$d = 0.02\,m$

$L = 0.2\,m$

MAX MASSFLOW OBTAINED FOR FRICTION CHOKING ($M = 1.0$ @ EXIT)



$M_2 = 1.0$

$M_1 = ?$

$L_1^* = L \,,\; L_2^* = 0.0$

$$(3.107) \quad \frac{4\bar{f}L_1^*}{d} = \frac{1 - M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2 + (\gamma-1)M_1^2}\right)$$

$$\Rightarrow M_1 = 0{,}48$$

$$(3{,}28) \quad \frac{T_o}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2$$

$$(3{,}30) \quad \frac{p_o}{p_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow \quad p_1 = 856{,}1\,kPa, \quad T_1 = 382{,}6\,K$$

$$\left.\begin{aligned} a_1 &= \sqrt{\gamma R T_1} \\ u_1 &= M_1 a_1 \end{aligned}\right\} \Rightarrow u_1 = 186.8\,m/s$$

$$\dot{m} = u_1 \rho_1 A_1 = u_1 \frac{p_1}{RT_1} \frac{\pi d^2}{4} = \underline{0.46\ kg/s}$$

b) CALCULATE MAXIMUM MASSFLOW WITHOUT THE PIPE

MAX MASSFLOW FOR CHOKED NOZZLE EXIT ($M_e = 1.0$)

$$(5.21) \qquad \dot{m} = \frac{p_0 A_t}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} = \underline{0.63\ kg/s}$$

P2 (SHOCK WAVES)

$P_{amb} = 101325\,Pa \quad (P_1)$

$T_{amb} = 293.0\,K \quad (T_1)$

$\theta = 10°$

WE CAN ESTIMATE THE SHOCK ANGLE FROM THE PROVIDED FIGURE

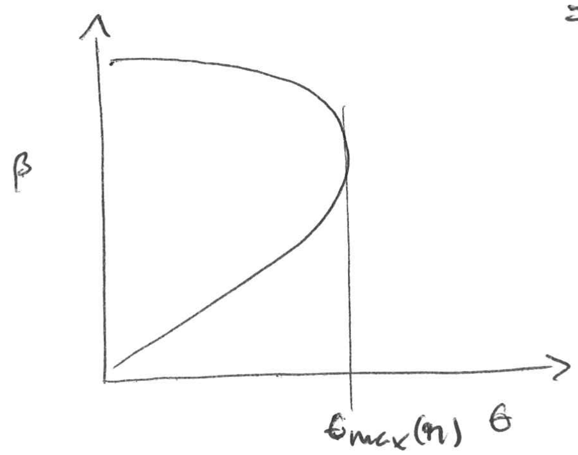
$$\tan(\beta) \simeq \frac{1.0}{1.5} \Rightarrow \beta \simeq 33.7°$$

($\theta-\beta-M$ WITH $\beta = 33.7°$ AND $\theta = 10°$) $\Rightarrow$ $M_1 = 2.35$

$$\left.\begin{array}{ll} (4.7) & M_{n_1} = M_1 \sin\beta \\ (4.9) & \dfrac{P_2}{P_1} = 1 + \dfrac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right) \end{array}\right\} \Rightarrow \frac{P_2}{P_1} \simeq 1.8$$

WE WILL GET A DETACHED SHOCK FOR MACH NUMBERS THAT CAN NOT SUPPORT A FLOW DEFLECTION OF 10° (MACH NUMBERS TO THE LEFT OF 10° IN THE $\theta-\beta-M$ RELATION)

$\Rightarrow$ DETACHED SHOCK FOR $M < 1.72$ (APPROXIMATELY)



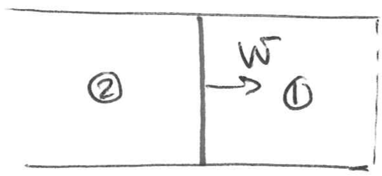
P3 (MOVING SHOCK)

SHOCK WAVE PROPAGATING AT THE VELOCITY $W = 415$ m/s TO THE RIGHT IN A TUBE

FLOW CONDITION AHEAD OF THE MOVING SHOCK

$u_1 = 0$, $P_1 = 101$ kPa, $T_1 = 288$ K

a) OPEN END:



WHEN THE SHOCK REACHES THE OPEN END, AN EXPANSION TRAVELING TO THE LEFT WILL BE FORMED SUCH THAT THE PRESSURE MATCHES THE AMBIENT PRESSURE (IF THE INDUCED FLOW VELOCITY BEHIND THE SHOCK IS SUBSONIC)

SHOCK MACH NUMBER

$$M_s = W / \sqrt{\gamma R T_1} = 1.22$$

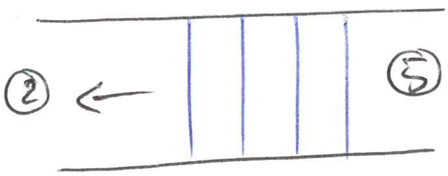
$$(7.13) \quad M_s = \sqrt{\frac{\gamma+1}{2}\left(\frac{P_2}{P_1} - 1\right) + 1} \Rightarrow \frac{P_2}{P_1} = 1{,}57$$

$$(7.16) \quad u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2} = 113.5 \text{ m/s}$$

$$(7.10) \quad \frac{T_2}{T_1} = \frac{P_2}{P_1}\left(\frac{\frac{\gamma+1}{\gamma-1} + \frac{P_2}{P_1}}{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{P_2}{P_1}\right)}\right) \Rightarrow T_2 = 328{,}5 \text{ K}$$

INDUCED FLOW MACH NUMBER:

$$M_2 = u_p / \sqrt{\gamma R T_2} = 0.31 \quad \text{(SUBSONIC)}$$



$$P_5 = P_1$$

THE EXPANSION IS ISENTROPIC:

$$\left(\frac{P_5}{P_2}\right) = \left(\frac{T_5}{T_2}\right)^{\gamma/(\gamma-1)}$$

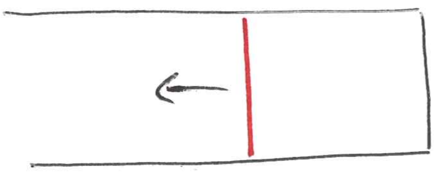
$$P_5 = P_1$$

$$\frac{T_5}{T_2} = \left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma}$$

$\Rightarrow$ $\underline{P_5 = 100\ kPa}$

$\underline{T_5 = 288.8\ K}$

b)



REFLECTED SHOCK

$$(7.23) \quad \frac{M_R}{M_R^2-1} = \frac{M_s}{M_s^2-1}\sqrt{1+\frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2-1)\left(\gamma+\frac{1}{M_s^2}\right)}$$

$$(3.57) \quad \frac{P_5}{P_2} = 1 + \frac{2\gamma}{\gamma+1}(M_R^2-1)$$

$$\frac{P_5}{P_1} = \frac{P_5}{P_2}\frac{P_2}{P_1} \Rightarrow \underline{P_5 = 242.1\ kPa}$$

$$(3.59) \quad \frac{T_5}{T_2} = \frac{P_5}{P_2}\left[\frac{2+(\gamma-1)M_R^2}{(\gamma+1)M_R^2}\right], \quad \frac{T_5}{T_1} = \frac{T_5}{T_2}\frac{T_2}{T_1} \Rightarrow \underline{T_5 = 371.5\ K}$$

P4 (CONVERGENT-DIVERGENT NOZZLE)

$A_e/A_t = 1.4$

(5.20) $$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

SUBSONIC SOLUTION: $M_{e_c} = 0.47$

SUPERSONIC SOLUTION: $M_{e_{sc}} = 1.76$

(3.30)
$$\frac{p_o}{p_{e_c}} = \left(1+\frac{\gamma-1}{2}M_{e_c}^2\right)^{\gamma/(\gamma-1)} = 1.16$$

$$\frac{p_o}{p_{e_{sc}}} = \left(1+\frac{\gamma-1}{2}M_{e_{sc}}^2\right)^{\gamma/(\gamma-1)} = 5.43$$

NORMAL SHOCK AT EXIT:

(3.57) $$\left.\begin{aligned}\frac{p_{b_{nse}}}{p_{e_{sc}}} &= 1+\frac{2\gamma}{\gamma+1}\left(M_{e_{sc}}^2-1\right)\\ \frac{p_o}{p_{b_{nse}}} &= \frac{p_o}{p_{e_{sc}}}\frac{p_{e_{sc}}}{p_{b_{nse}}}\end{aligned}\right\} \Rightarrow \frac{p_o}{p_{b_{nse}}} = 1.57$$

THE NOZZLE INTERIOR WILL BE FREE OF NORMAL SHOCKS FOR SUBSONIC FLOW AND FOR NOZZLE PRESSURE RATIOS HIGHER THAN THE SHOCK-AT-EXIT PRESSURE RATIO

SUBSONIC FLOW: $1.0 < NPR < NPR_c = 1.16$

SUPERSONIC SHOCK-FREE FLOW: $NPR > NPR_{nse} = 1.57$

b) $\frac{P_0}{P_{ah}} = 1.5 \Rightarrow \quad NPR_c < NPR < NPR_{nse}$

$\Rightarrow$ THERE WILL BE A NORMAL SHOCK INSIDE THE NOZZLE

(5.28)

$$M_e^2 = \frac{-1}{\gamma - 1} + \sqrt{\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)} \left(\frac{P_{01} A_t}{P_e A_e}\right)^2}$$

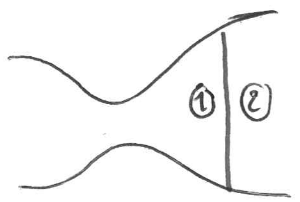
$P_{01} / P_e = 1.5$

$A_t / A_e = 1 / 1.4$

$\Rightarrow M_e = 0.60$

(3.30) $$\frac{P_{02}}{P_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}}{P_e} \frac{P_e}{P_{01}}$$ (DECREASE OF P_0 OVER THE SHOCK)



THE MASS FLOW THROUGH THE NOZZLE IS NOT AFFECTED BY THE SHOCK (CHOKED MASS FLOW)

(5.21) $$\dot{m} = \frac{P_{01} A_1^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$\dot{m} = \frac{P_{02} A_2^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$\Rightarrow P_{01} A_1^* = P_{02} A_2^* \Rightarrow \frac{P_{02}}{P_{01}} = \frac{A_1^*}{A_2^*}$$

AT THE SHOCK:

(5,20) $$\left(\frac{A}{A_1^*}\right)^2 = \frac{1}{M_1^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_1^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\left(\frac{A}{A_2^*}\right)^2 = \frac{1}{M_2^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_2^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

(3,51) $$M_2^2 = \frac{1+((\gamma-1)/2)M_1^2}{\gamma M_1^2-(\gamma-1)/2}$$

FROM BEFORE: $\frac{p_{02}}{p_{01}} = \frac{A_1^*}{A_2^*} = \frac{p_{02}}{p_e}\frac{p_e}{p_{01}}$

ITERATE $\Rightarrow M_1 = 1,71$

$$\frac{A}{A_1^*} = 1,35 = \frac{A}{A_t}$$

$$\frac{A}{A_t} = \left(\frac{A_e}{A_t}-1\right)\left(2\frac{x}{L}-1\right)^2+1 \Rightarrow x/L = 0,97$$