TME085 - Compressible Flow

2016-03-17, 08.30-13.30, M-building

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- Chalmers approved hand calculator

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (1 p.)
For a stationary normal shock, describe how the entropy, velocity, pressure and total temperature of a fluid particle is affected as it passes through the shock.

T2. (1 p.)
What is the general definition (valid for any gas) of the "*total*" conditions p_0, T_0, ρ_0,... etc at some location in the flow?

T3. (1 p.)
Derive the relation

$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

for calorically perfect gas from the energy equation form

$$h_0 = h + \frac{1}{2}V^2.$$

T4. (1 p.)
What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T5. (1 p.)
An engineer wants to apply a numerical solution scheme for compressible flow. The flow he is interested in contains shocks. He has to choose between two different solution methods – one which is based on the conservation form of the governing equations and one which is based on the non-conservation form of the governing equations. Which method should he choose and why?

T6. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T7. (1 p.)
Assume a steady-state flow through a pipe with varying cross-section area. If the pipe has negligible heat transfer and wall friction and there are no shocks, then the flow is

(a) adiabatic
(b) isentropic
(c) isobaric (constant pressure)
(d) isenthalpic (constant enthalpy)

Which statements are true and which are false?

T8. (1 p.)
In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

T9. (2 p.)

(a) When applying a time-marching flow solution scheme the so-called CFL number is an important parameter. Define the CFL number and describe its significance. What is a typical maximum CFL number for an explicit time stepping scheme.

(b) What is meant by the terms "*density-based*" and "*fully coupled*" when discussing CFD codes for compressible flow?

T10. (2 p.)

(a) What is it meant by choking the flow in a nozzle? Describe it.

(b) How does the absolute Mach number change after a *weak* and a *strong* stationary oblique shock, repectively?

T11. (2 p.)
Derive the continuity equation in non-conservation form from the corresponding conservation form.

T12. (2 p.)
Prove, by using one of the non-conservation forms of the energy equation, that for steady-state, adiabatic flow with no body forces the total enthalpy is preserved along stream-lines.

T13. (2 p.)
Describe in words the significance of Crocco's equation.

Prove that a steady-state irrotational flow with constant total enthalpy must also be isentropic.

T14. (1 p.)
How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - Lift and Drag (10 p.)

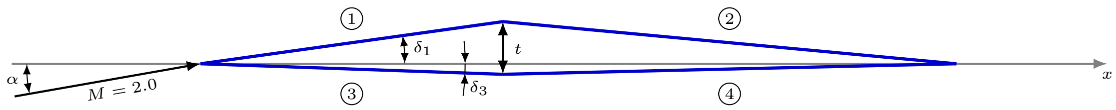
A "cambered" diamond-shaped airfoil is exposed to a Mach 2.0 air stream in a wind tunnel. The total pressure and total temperature in the free stream flow are $p_o = 862.0$ kPa and $T_o = 88.0$ °C, respectively. The angles defining the leading edge of the wing section are $\delta_1 = 8.0°$ and $\delta_3 = 2.0°$. The maximum wing thickness is $t = 0.07c$ (where c is the wing coord length) and is located at $x = 0.40c$. The flow meets the wing section at an angle of attack equal to $\alpha = 10°$.

Calculate:

(a) The pressures in regions 1-4 (see figure below)

(b) The net lift of the airfoil for the described conditions (calculate lift per unit span and use coord length $c = 1.0$ m)

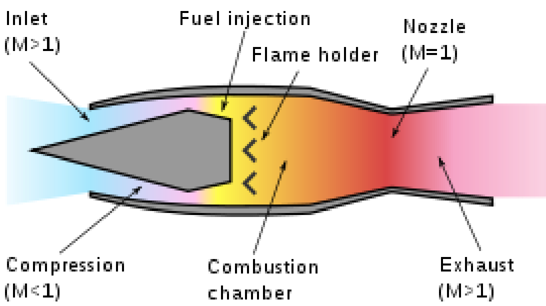
(c) The net drag of the airfoil for the described conditions (calculate drag per unit span and use coord length $c = 1.0$ m)



Problem 2 - Ramjet Engine (10 p.)

The ramjet engine is an air-breathing engine concept with very few moving parts. The absence of moving parts makes this concept much less complex than a corresponding gas turbine construction and therefore it was an appealing option in the early days of supersonic aviation. For a ramjet engine, the speed of the aircraft itself is sufficient to compress the air in the engine intake, which eliminates the need for compressors. As the air enters the ingine intake, a system of shocks decelerates the flow to subsonic speeds and raises the pressure. After the engine intake, the air passes through a subsonic combustor in which fuel is sprayed into the the airstream and and burned. After the combustor the air passes through a nozzle where the flow is again accelerated to supersonic speed generating a propelling jet flow.

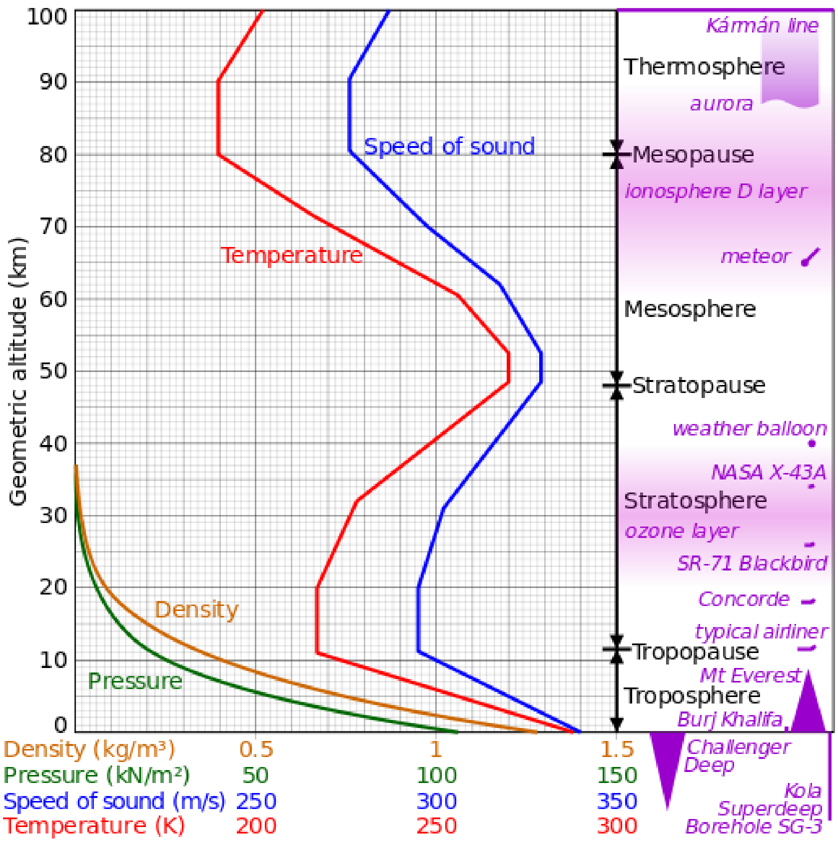
Suppose that it is possible to represent the ramjet engine by a pipe where first supersonic air flow is decelerated by a normal shock (*i.e. the system of oblique shocks in the engine inlet diffusor is replaced by a single normal shock*). After the normal shock, heat is added to the flow in the combustor section after which the flow leaves the engine through passing the exit section. Assume that the airplane, for which the ramjet engine is the propelling unit, operates at an altitude of 6000 m and moves through the air at a speed which gives a Mach number of 2.5 (*thermodynamic data as a function of altitude is given in the figure below*)



(a) The normal shock at in the engine intake slows down the flow and increases the pressure. Calculate the pressure and temperature in the burner inlet section.

(b) Calculate the the highest possible temperature that can be obtained in the combustor section of the engine. The mass of the added fuel and changes in the gas composition can be neglected.

(c) Calculate the amount of heat added per unit mass when the maximum temperature is reached



Atmospheric variations of thermodynamic quantities (Wikipedia)

Problem 3 - Moving Shock (10 p.)

A shock moves in an open-ended tube with the velocity (relative to the stagnant air into which it propagates) of 415 m/s. The pressure and temperature in the air in front of the shock is 101 kPa and 288 K, respectively. When the incident shock reaches the open end, a reflected expansion fan is generated in order to maintain a constant pressure at the boundary.

Calculate:

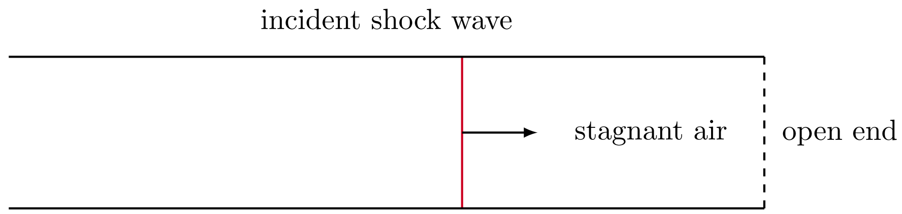
(a) The pressure increase over the incident shock wave

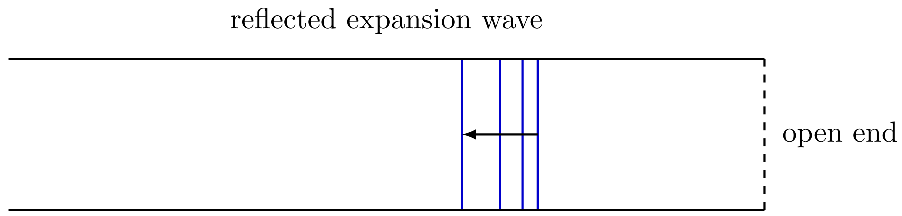
(b) The induced velocity behind the incident shock wave

(c) The propagation velocity of the leading part of the expansion wave (the left-most part)

(d) The propagation velocity of the trailing part of the expansion wave (the right-most part)

hint: The expansion waves are left-running characteristics (C^-) with $\frac{dx}{dt} = u - a$ and the Riemann invariant J^+ is constant over the expansion region.





Problem 4 - Convergent-Divergent Nozzle (10 p.)

A convergent divergent nozzle with a throat area of $A_t = 0.5 \text{ m}^2$ is designed to produce an exit Mach number of 2.5. When the reservoir pressure is $p_o = 2.645$ bar, normal-shock-at-exit conditions are reached. If the back-pressure is reduced, we get an overexpanded nozzle flow with an oblique shock formed at the nozzle exit plane. The angle of the oblique shock is $\beta = 30°$. The oblique shock makes the jet flow deflect towards the jet centerline.
Calculate:

(a) The back pressure when we have a normal shock at the exit

(b) The back pressure for the overexpanded conditions as described above

(c) The jet deflection angle (θ) for the overexpanded nozzle flow

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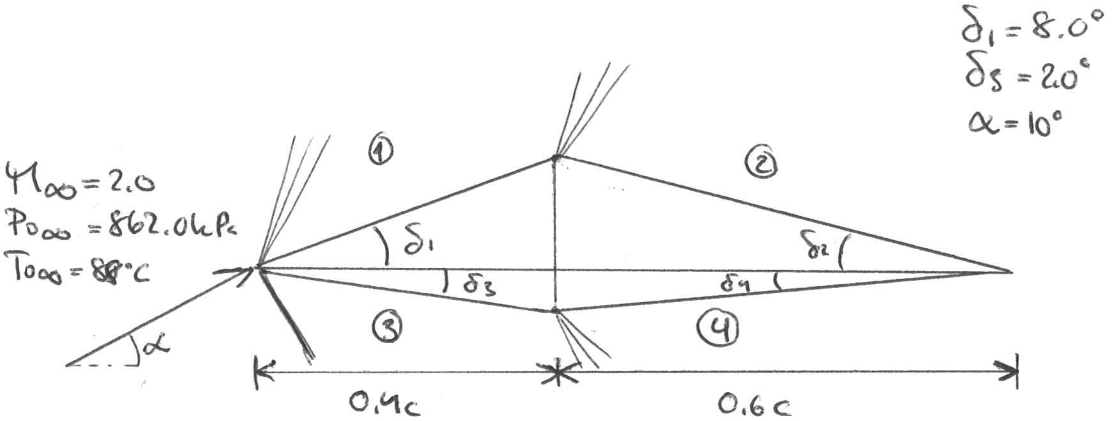
P1 (LIFT & DRAG)



a) CALCULATE THE PRESSURE IN REGIONS 1–4

REGION 1

$\alpha > \delta_1 \Rightarrow$ EXPANSION AT LEADING EDGE (UPPER SIDE)

$\Delta\theta = \alpha - \delta_1 = 2.0°$

(4.44) $\nu(M_\infty) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_\infty^2-1)} - \tan^{-1}\sqrt{M_\infty^2-1} = \nu_1$

$\nu_2 = \nu_1 + \Delta\theta$

(4.44) $\Rightarrow M_1 = 2.07$

(3.30) $\frac{p_{0\infty}}{p_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \Rightarrow p_1 = 98.27\,kPa$

REGION 2

EXPANSION BETWEEN REGION 1 AND REGION 2

$\Delta\theta = \delta_1 + \delta_2$

δ_2 NOT GIVEN..

$$\left.\begin{aligned} \tan(\delta_1) &= \frac{h}{0.4c} \\ \tan(\delta_2) &= \frac{h}{0.6c} \end{aligned}\right\} \Rightarrow \tan(\delta_2) = \frac{2}{3}\tan(\delta_1)$$

$$(4.44) \Rightarrow \nu_1 = \nu(M_1)$$

$$\nu_2 = (\nu_1 + \Delta\theta)$$

$$(4.44) \Rightarrow M_2 = 2.6$$

$$(3.30) \quad \frac{p_{0\infty}}{p_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/(\gamma-1)} \Rightarrow p_2 = 42.26 \text{ kPa}$$

REGION 3

OBLIQUE SHOCK AT LEADING EDGE (LOWER SIDE)

$$\theta = \alpha + \delta_3 = 12^\circ$$

$$(\theta\text{-}\beta\text{-}M,\ \theta = 12^\circ \ \&\ M = 2.0) \Rightarrow \beta = 41.6^\circ$$

$$\left.\begin{aligned} &(4.7) \quad M_{n_1} = M_\infty \sin\beta \\ &(4.9) \quad \frac{p_3}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1) \end{aligned}\right\} \Rightarrow p_3 = 208.0 \text{ kPa}$$

$$\left.\begin{aligned} &(4.10) \quad M_{n_2}^2 = \frac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1} \\ &(4.12) \quad M_2 = M_{n_2} / \sin(\beta - \theta) \end{aligned}\right\} \Rightarrow M_2 = 1.6$$

REGION 4

EXPANSION BETWEEN REGION 3 AND REGION 4

$$\Delta\theta = \delta_3 + \delta_4$$

δ_4 NOT GIVEN

$$\left.\begin{aligned} &\tan(\delta_3) = \frac{h}{0.4c} \\ &\tan(\delta_4) = \frac{h}{0.6c} \end{aligned}\right\} \Rightarrow \tan(\delta_4) = \frac{2}{3}\tan(\delta_3)$$

$$(4.44) \Rightarrow \nu_1 = \nu(M_3)$$

$$\nu_2 = (\nu_1 + \Delta\epsilon)$$

$$(4.44) \Rightarrow M_4 = 1.7$$

$$(3.30) \quad \left.\begin{aligned} \frac{P_{03}}{P_3} &= \left(1 + \frac{\gamma-1}{2} M_3^2\right)^{\gamma/(\gamma-1)} \\ \frac{P_{03}}{P_4} &= \left(1 + \frac{\gamma-1}{2} M_4^2\right)^{\gamma/(\gamma-1)} \end{aligned}\right\} \Rightarrow P_4 = 175.89\,kPa$$

c) CALCULATE LIFT PER UNIT SPAN

d) CALCULATE DRAG PER UNIT SPAN

$$F_1 = P_1 l_1 b = P_1 (0.4c/\cos(\delta_1))b$$
$$F_2 = P_2 l_2 b = P_2 (0.6c/\cos(\delta_2))b$$
$$F_3 = P_3 l_3 b = P_3 (0.4c/\cos(\delta_3))b$$
$$F_4 = P_4 l_4 b = P_4 (0.6c/\cos(\delta_4))b$$

WITH X ALIGNED WITH THE CCORD OF THE WING:

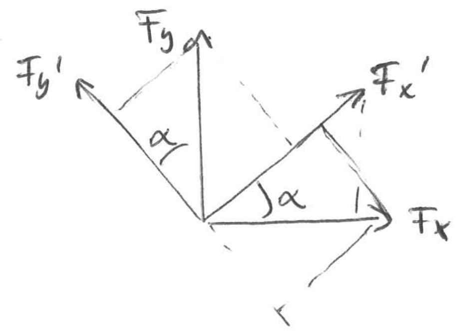
$$F_{1x} = F_1 \sin(\delta_1) = P_1\, 0.4c \tan(\delta_1)\, b$$
$$F_{2x} = -F_2 \sin(\delta_2) = -P_2\, 0.6c \tan(\delta_2)\, b$$
$$F_{3x} = F_3 \sin(\delta_3) = P_3\, 0.4c \tan(\delta_3)\, b$$
$$F_{4x} = -F_4 \sin(\delta_4) = -P_4\, 0.6c \tan(\delta_4)\, b$$

$$F_{1y} = -F_1 \cos(\delta_1) = -P_1\, 0.4c\, b$$
$$F_{2y} = -F_2 \cos(\delta_2) = -P_2\, 0.6c\, b$$
$$F_{3y} = F_3 \cos(\delta_3) = P_3\, 0.4c\, b$$
$$F_{4y} = F_4 \cos(\delta_7) = P_4\, 0.6c\, b$$

$$F_L = F_{1y} + F_{2y} + F_{3y} + F_{4y} = 124.1 \cdot c \ [kN/m]$$

$$F_D = F_{1x} + F_{2x} + F_{3x} + F_{4x} = 3.6c \ [kN/m]$$

IN THE DIRECTION OF THE FLOW



$F_x' = F_x \cos\alpha + F_y \sin\alpha$

$F_y' = -F_x \sin\alpha + F_y \cos\alpha$

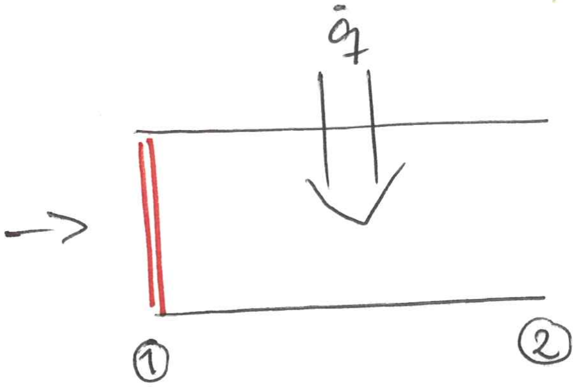
$F_x' = 25.10 \ (kN/m)$

$F_y' = 121.60 \ (kN/m)$

P2 (RAMJET ENGINE)

A RAMJET ENGINE OPERATES AT $M = 2.5$ AT AN ALTITUDE OF 6000 m

$$H = 6000\,m \Rightarrow \begin{cases} T_\infty = 250K \\ p_\infty = 50kPa \end{cases}$$



(NORMAL SHOCK AT INLET)

$$(3.51) \quad M_1^2 = \frac{1 + ((\gamma-1)/2)M_\infty^2}{\gamma M_\infty^2 - (\gamma-1)/2} \Rightarrow M_1 = 0.51$$

a) CALCULATE PRESSURE AND TEMPERATURE AT THE INLET OF THE BURNER (1)

NORMAL SHOCK:

$$\frac{p_1}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1}(M_\infty^2 - 1) \quad (3.57)$$

$$\frac{T_1}{T_\infty} = \left(1 + \frac{2\gamma}{\gamma+1}(M_\infty^2 - 1)\right)\left(\frac{2 + (\gamma-1)M_\infty^2}{(\gamma+1)M_\infty^2}\right) \quad (3.59)$$

$\Rightarrow$ 356.3 kPa (p_1)

537.4 K (T_1)

b) THE HIGHEST TEMPERATURE POSSIBLE IS T^*
(THERMAL CHOKING)

$$(3.86) \quad \frac{T_1}{T^*} = M_1^2 \left(\frac{1+\gamma}{1+\gamma M_1^2}\right)^2 \Rightarrow T^* = 660{,}2\,K$$

c) CALCULATE THE AMOUNT OF HEAT ADDED PER UNIT MASS TO GET THE MAXIMUM TEMPERATURE.

$$(3{,}28) \quad \frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \Rightarrow T_{01} = 562{,}5\,K$$

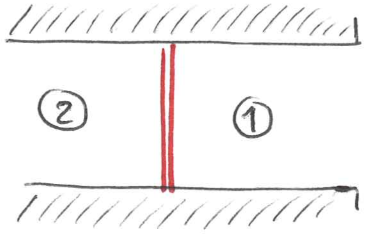
$$(3{,}89) \quad \frac{T_{01}}{T_0^*} = \frac{(\gamma+1)M_1^2}{(1+\gamma M_1^2)^2}\left(2 + (\gamma-1)M_1^2\right) \Rightarrow T_0^* = 792.2\,K$$

$$q^* = C_p \left(T_0^* - T_{01}\right) = 230.7\,kJ/kg$$

P3 (MOVING SHOCK)

A SHOCK MOVES AT A VELOCITY OF 415 m/s THROUGH AN OPEN-ENDED TUBE. THE PRESSURE AND TEMPERATURE AHEAD OF THE SHOCK IS 101 kPa AND 288 K, RESPECTIVELY.

a) CALCULATE THE PRESSURE INCREASE OVER THE MOVING SHOCK.



THE SHOCK MACH NUMBER $M_s = \frac{W}{a_1} = \frac{W}{\sqrt{\gamma R T_1}} = 1.22$

(7.13) $M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \Rightarrow \frac{p_2}{p_1} = 1.57$

b) CALCULATE THE INDUCED FLOW VELOCITY BEHIND THE MOVING SHOCK.

(7.16) $$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

WHERE $a_1 = \sqrt{\gamma R T_1}$

$\Rightarrow \quad u_p = 113.5 \text{ m/s}$

CHECK INDUCED FLOW MACH NUMBER:

$$\left.\begin{aligned} M_2 &= \frac{u_p}{a_2} = \frac{u_p}{\sqrt{\gamma R T_2}} \\ \frac{T_2}{T_1} &= \frac{p_2}{p_1}\left(\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}\right) \end{aligned}\right\} \Rightarrow M_2 = 0.31 \text{ (SUBSONIC)}$$

$(T_2 = 328.5 \text{ K})$

C) AT THE PIPE OPENING, THE SHOCK WILL BE TERMINATED AND AN EXPANSION MOVING BACK INTO THE TUBE WILL BE FORMED.

CALCULATE THE PROPAGATION VELOCITY OF THE LEADING PART OF THE EXPANSION

THE EXPANSION PROPAGATES INTO A REGION WITH THE FLOW VELOCITY $u_2 = u_p$ AND THE SPEED OF SOUND $a_2 = \sqrt{\gamma R T_2}$

THUS THE EXPANSION HEAD MOVES TO THE LEFT AT THE VELOCITY

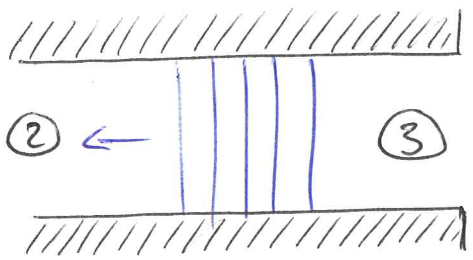
$$u_{HEAD} = u_2 - a_2 = \underline{-249.8 \text{ m/s}}$$

d) CALCULATE THE PROPAGATION VELOCITY OF THE TAIL OF THE EXPANSION.

THE J_+-INVARIANT WILL BE CONSTANT OVER THE EXPANSION

$$J_+ = u_2 + \frac{2a_2}{\gamma - 1} = u_3 + \frac{2a_3}{\gamma - 1}$$

$$a_3 = \sqrt{\gamma R T_3}$$



THE EXPANSION IS ISENTROPIC

$$\left(\frac{P_2}{P_3}\right) = \left(\frac{T_2}{T_3}\right)^{\gamma/(\gamma-1)}$$

$P_3 = P_1$ (THAT IS WHY THE EXPANSION IS FORMED..)

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{T_2}{T_3}\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow T_3 = T_2 \Big/ \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

NOW, LET'S GO BACK TO THE $J+$ INVARIANT

$$u_2 + \frac{2a_2}{\gamma-1} = u_3 + \frac{2a_3}{\gamma-1}$$

$$u_2 + \frac{2}{\gamma-1}(a_2 - a_3) = u_3 \Rightarrow u_3 = 226.8 \text{ m/s}$$

$$u_{TAIL} = u_3 - a_3 = \underline{-113.8 \text{ m/s}}$$

P4 (CONVERGENT – DIVERGENT NOZZLE)

THROAT AREA: $A_t = 0{,}5\,m^2$

DESIGN EXIT MACH NUMBER: $M_e = 2.5$

RESERVOIR PRESSURE: $P_0 = 2{,}645$ bar $= 264.5\,kPa \Rightarrow$ NORMAL SHOCK AT EXIT.

a) CALCULATE THE BACK-PRESSURE THAT GIVES NORMAL SHOCK AT THE EXIT.

$M_{e_{sc}} = 2.5 \Rightarrow$

(5.20) $$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_{e_{sc}}^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_{e_s}^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$\Rightarrow A_e = 1.32\,m^2$

NORMAL-SHOCK AT EXIT $\Rightarrow$ EXIT MACH NUMBER EQUALS THE DESIGN MACH NUMBER

$\Rightarrow$

(3.30) $$\frac{P_0}{P_e} = \left(1+\frac{\gamma-1}{2}M_{e_{sc}}^2\right)^{\gamma/(\gamma-1)} \Rightarrow \underline{P_e = 15{,}48\,kPa}$$

NORMAL SHOCK AT EXIT $\Rightarrow$

(3.57) $$\frac{P_b}{P_e} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{e_{sc}}^2 - 1\right) \Rightarrow \underline{P_b = 110{,}3\,kPa}$$

b) (and c) REDUCING THE BACK PRESSURE GIVES OVEREXPANDED NOZZLE FLOW. AN OBLIQUE SHOCK WITH THE SHOCK ANGLE $\beta = 30°$ IS FORMED DOWNSTREAM OF THE NOZZLE EXIT.

$(\theta - \beta - M\ ;\ \beta = 30° \ \&\ M = M_{e_{sc}}) \Rightarrow \theta = 8°$

(4.7) $M_{n1} = M_{esc} \sin(\beta)$

(4.9) $\frac{p_b}{p_e} = 1 + \frac{2\gamma}{\gamma+1}(M_{n1}^2 - 1)$

$\Rightarrow \underline{P_b = 25{,}6\,kPa}$