TME085 - Compressible Flow

2015-08-19, 08.30-13.30, M-building

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (1 p.)
For a steady-state adiabatic compressible flow of calorically perfect gas, which of the variables p_0 (total pressure) and T_0 (total temperature) is/are constant along streamlines? Why?

T2. (1 p.)
What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T3. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?

(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T4. (2 p.)
In shock tubes, unsteady contact discontinuities are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables p, T, ρ, u, s is/are necessarily continuous across such a contact discontinuity?

T5. (1 p.)
A mixture of chemically reacting perfect gases, where the reactions are always in equilibrium, may be thermodynamically described as a single-species gas. How does this thermodynamic description differ from that of a calorically perfect or thermally perfect gas?

T6. (2 p.)
Derive the special formula for the speed of sound a for a calorically perfect gas:

$$a = \sqrt{\gamma R T}$$

starting from the general formula:

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

T7. (2 p.)
Assume a steady-state flow in a convergent-divergent nozzle. Describe what characterizes the following operating conditions:

(a) Sub-critical nozzle flow

(b) Over-expanded nozzle flow

(c) Under-expanded nozzle flow

What is it meant by choking the flow in a nozzle? Describe it.

T8. (1 p.)
Describe the choking of flow that occurs for pipe flow with friction. What happens if the real length L of a pipe is longer than L^* (for either subsonic flow or supersonic flow)?

T9. (1 p.)
How does the absolute Mach number change after a weak/strong stationary oblique shock?

T10. (1 p.)
How can you derive (describe in words only) the PDE:s in conservation form from the control volume formulations for the conservation of mass, momentum and energy?

T11. (2 p.)
Prove, by using one of the non-conservation forms of the energy equation, that for steady-state, adiabatic flow with no body forces the total enthalpy is preserved along stream-lines.

T12. (2 p.)
Prove, by using a suitable equation, that a steady-state irrotational flow with constant total enthalpy must also be isentropic.

T13. (1 p.)
Describe in words how a finite-volume spatial discretization can be achieved.

T14. (1 p.)
How can we use our knowledge of characteristics (and their speed of propagation) to guide us when determining suitable boundary conditions for compressible flows?

Part II - Problems (40 p.)

Problem 1 - Convergent-Divergent Nozzle (10 p.)

An 8.5 [m^3] vacuum tank is to be used to create a flow at an exit Mach number of M=2.0. A plug is put into the nozzle and the tank is evacuated until it contains 0.45 [kg] of air at a temperature of 296 [K]. When the plug is removed, air flows from the atmosphere into the tank through the convergent-divergent nozzle. The throat area of the nozzle is A_t=6.5 [cm^2].

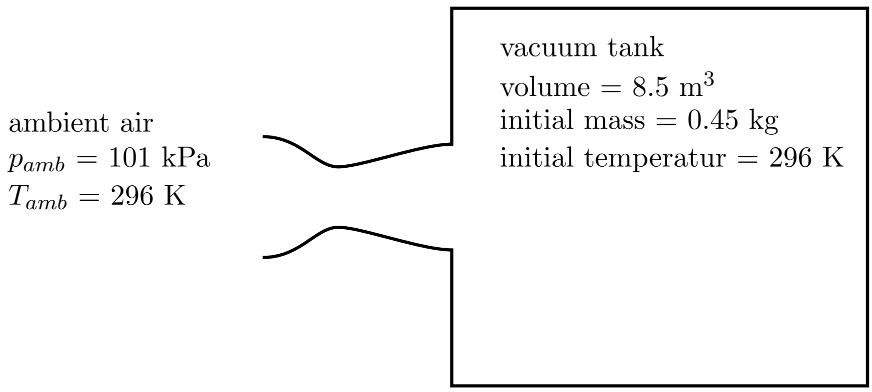
(a) Compute the design exit area

(b) Compute initial pressure in the tank

(c) Compute the initial mass flow rate through the nozzle

(d) Compute the nozzle exit pressure immediately after the flow starts

(e) Compute the tank pressure for which a normal shock will be formed at the nozzle exit plane



Problem 2 - Pipe flow with heat addition (10 p.)

A metal pipe with circular cross-section has a constant inner diameter D=1.0 [cm] and a length L=5.0 [dm]. The pipe is fitted with an external electric heater and the pipe plus heater assembly is surrounded with good thermal insulation such that all supplied heat is conducted into the gas stream. Viscous effects may be neglected due to the short length of the pipe. The inflow end of the pipe is fed with a constant massflow of air, with a constant total temperature T_{o_1}=300 [K]. The pipe flow exits into the ambient air, with p_2=1.0 [bar]. The heating power P of the electric heater is such that the measured static temperature T_2 and velocity u_2 at the pipe exit are found to be 658.7 [K] and 288.1 [m/s] respectively.

(a) Compute the Mach number M_2 at the pipe exit.

(b) Compute the total temperature T_{o_2} at the pipe exit.

(c) Compute the massflow of air through the pipe.

(d) Compute the heating power P.

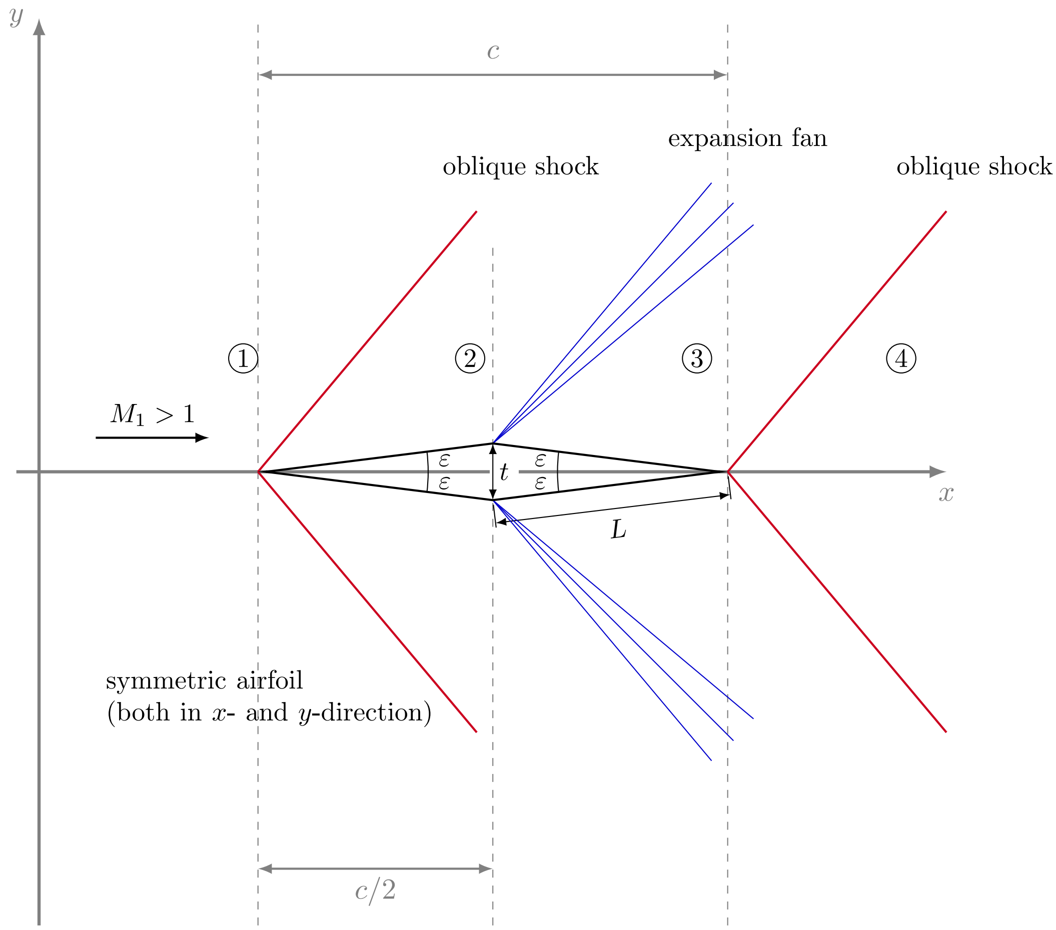
(e) Compute the total pressure p_{o_1} at the inflow end of the pipe.

The air may be treated as a calorically perfect gas with R=287 [J/kgK] and γ=1.4

Problem 3 - Shock-Expansion Theory (10 p.)

A symmetric diamond-shaped airfoil is placed in a supersonic flow of air. The airfoil is oriented with zero angle of attack relative to the freestream flow. The extent of the airfoil on the z-direction is long in comparison to the extent in the x- and y-directions, hence an assumption of two-dimensional flow can be justified. Calculate the drag (*wave drag*) per unit length if the freestream Mach number is M_1=2.0 and the upstream pressure is 101325 Pa.

airfoil parameters			
half angle	ε	5	[degrees]
coord length	c	1.0	[m]

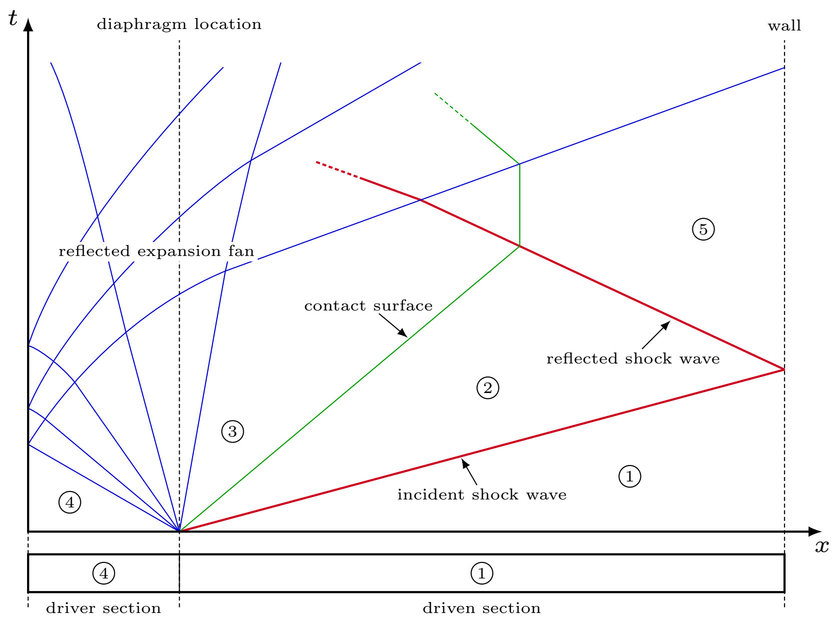


Problem 4 - Shock-Tube (10 p.)

A shock tube is to be constructed for experimental purposes. The desired conditions at the right end of the tube, after the incident shock wave has reflected at the right end wall, are T_5=1500 [K] and p_5=56 [bar]. In order to get this result, the pressure ratio over the incident shock wave must be at least p_2/p_1=11. Before the diaphragm (the membrane separating conditions 1 and 4) breaks the gas in the driver section (region 4) is heated to T_4=680 [K] and the pressure and temperature in the driven section (region 1) are p_1=1.0 [bar] and T_1=300 [K], respectively. The maximum allowed pressure in the driver section is 35 [bar] but to be on the safe side it it decided to not let the pressure be higher than 31.0 [bar]. The driven gas is air assumed to be calorically perfect.

(a) It is decided to use helium as the driver gas. Will it be possible to achieve the desired test conditions using helium? *justify your answer with calculations*

(b) In order to save money, the design team would like to replace the pre-heater in region 4 to one with lower power. What is the lowest temperature allowed in the driver section (region 4).



Gas/Vapor	Ratio of specific heats (γ)	Gas constant R
Acetylene	1.23	319
Air (standard)	1.40	287
Ammonia	1.31	530
Argon	1.67	208
Benzene	1.12	100
Butane	1.09	143
Carbon Dioxide	1.29	189
Carbon Disulphide	1.21	120
Carbon Monoxide	1.40	297
Chlorine	1.34	120
Ethane	1.19	276
Ethylene	1.24	296
Helium	1.67	2080
Hydrogen	1.41	4120
Hydrogen chloride	1.41	230
Methane	1.30	518
Natural Gas (Methane)	1.27	500
Nitric oxide	1.39	277
Nitrogen	1.40	297
Nitrous oxide	1.27	180
Oxygen	1.40	260
Propane	1.13	189
Steam (water)	1.32	462
Sulphur dioxide	1.29	130

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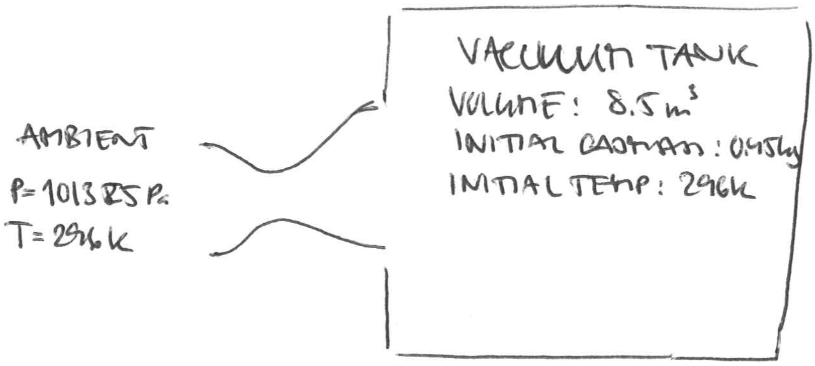
TME085 EXAM 2015-08-19

P1 (CONVERGENT - DIVERGENT NOZZLE)



(NOTE: IN THE TASK $P_{amb} = 101$ kPa WAS SPECIFIED, WHICH GIVES SLIGHTLY DIFFERENT RESULTS..)

$A_t = 6.5\ cm^2$

a) EXIT AREA

$$(5.20)\quad \left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow A_e \simeq 10.97\ cm^2$$

b) INITIAL PRESSURE IN TANK

$$\rho_{init} = m_{init}/V_{tank} = 0.45/8.5 = 0.053\ kg/m^3$$

$$P_{init} = \rho_{init} \cdot R \cdot T_{init} = 4497.5\ Pa$$

c) INITIAL MASSFLOW RATE

CHECK CHOKED FLOW BACKPRESSURE AND DESIGN FLOW BACK PRESSURE.

$M_{e\_sc} = 2.0 \Rightarrow$

$$(3.30)\quad \frac{P_0}{P_{e\_sc}} = \left(1+\frac{\gamma-1}{2}M_{e\_sc}^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_{e\_sc} = 12.95\ kPa$$

$$\frac{P_0}{P_{e_c}} = \left(1+\frac{\gamma-1}{2}M_{e_c}^2\right)^{\gamma/(\gamma-1)} \Rightarrow P_{e_c} = 92.1\ kPa$$

$P_{init} < P_{e_c} \Rightarrow$ choked flow

$P_{init} < P_{e_{sc}} \Rightarrow$ UNDEREXPANDED FLOW

$$\Rightarrow \dot m_{init} = \dot m_{choked} = \frac{P_0 A_t}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} = 0.15\ kg/s \quad (5.21)$$

d) NOZZLE EXIT PRESSURE WHEN THE FLOW STARTS.

SINCE THE FLOW IS UNDEREXPANDED, THE NOZZLE-EXIT-PRESSURE WILL BE THE DESIGN (SUPERCRITICAL) EXIT PRESSURE.

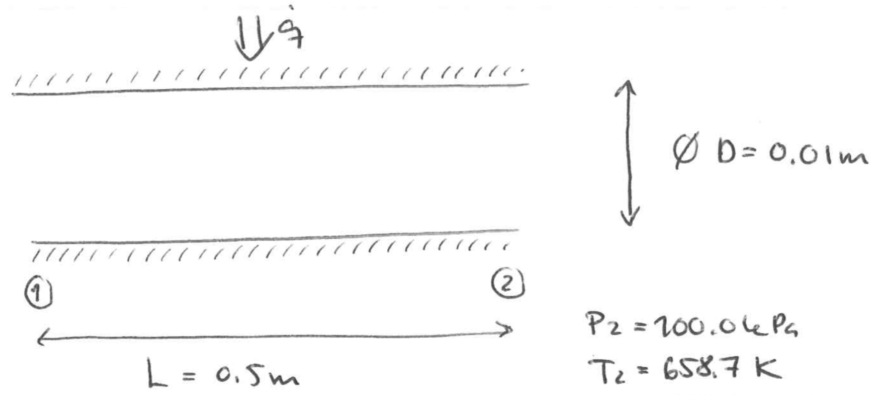
$$P_e = P_{e_{sc}} = 12.95\,kPa$$

e) FOR WHAT TANK PRESSURE WILL THERE BE A NORMAL SHOCK AT THE EXIT?

THE TANK PRESSURE WILL MATCH THE DOWNSTREAM PRESSURE OF A NORMAL SHOCK WITH THE UPSTREAM PRESSURE AND MACH NUMBER MATCHING THE DESIGN CONDITIONS.

$$(3.57) \quad \frac{P_{b\,nse}}{P_{e_{sc}}} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{e_{sc}}^2 - 1\right) \Rightarrow P_{b\,nse} = 58.27\,kPa$$

P2 (PIPE FLOW WITH HEAT ADDITION)



$T_{01} = 300\,K$

$P_2 = 100.06\,kPa$
$T_2 = 658.7\,K$
$u_2 = 288.1\,m/s$

a) CALCULATE EXIT MACHNUMBER: (STATION 2)

$$M_2 = \frac{u_2}{a_2} = \frac{u_2}{\sqrt{\gamma R T_2}} = 0.56$$

b) CALCULATE THE TOTAL TEMPERATURE AT THE EXIT

$$(3.28) \quad \frac{T_{02}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2 \Rightarrow T_{02} = 700\,K$$

c) CALCULATE THE MASSFLOW

$$\dot{m} = \rho_2 u_2 A = \frac{P_2}{R T_2} u_2 \frac{\pi D^2}{4} = 0.012\,kg/s$$

d) COMPUTE THE HEATING POWER

$$q = C_p (T_{02} - T_{01}) = 401.8\,kJ/kg$$

$$P = q \cdot \dot{m} = 4.81\,kW$$

e) COMPUTE INLET TOTAL PRESSURE

$$(3.89) \quad \frac{T_{02}}{T_0^*} = \frac{(\gamma+1) M_2^2}{(1+\gamma M_2^2)^2} \left(2 + (\gamma - 1) M_2^2\right) \Rightarrow T_0^* = 906.17\,K$$

$$\frac{T_{01}}{T_0^*} = \frac{(\gamma+1) M_1^2}{(1+\gamma M_1^2)^2} \left(2 + (\gamma - 1) M_1^2\right) \Rightarrow M_1 = 0.29$$

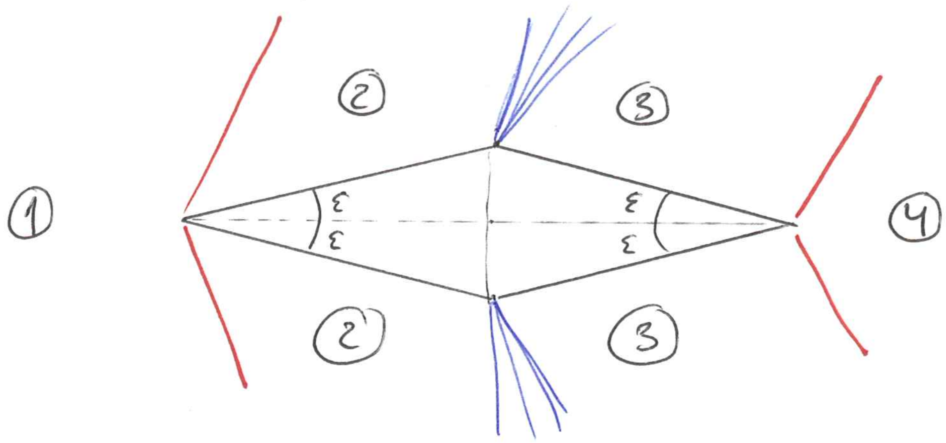
$$(3.30) \quad \frac{P_{01}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)}$$

WE NEED THE STATIC PRESSURE AT THE INLET

(3.85)
$$\left.\begin{aligned} \frac{p_2}{p^*} &= \frac{1+\gamma}{1+\gamma M_2^2} \\ \frac{p_1}{p^*} &= \frac{1+\gamma}{1+\gamma M_1^2} \end{aligned}\right\} \Rightarrow p_1 = 128.6 \text{ kPa}$$

(3.30) $\Rightarrow$ $\underline{p_{01} = 136.4 \text{ kPa}}$

P3 (SHOCK-EXPANSION THEORY)



$\varepsilon = 5°$

Symmetric wing chord $\pm c = 1.0$ m

$M_1 = 2.0$

$P_1 = 101385$ Pa

OBLIQUE SHOCK 1->2

$$\left.\begin{array}{l}\theta = \varepsilon \\ M = M_1\end{array}\right\} (\theta-\beta-M) \Rightarrow \beta = 34.3°$$

(4.7) $M_{n_1} = M_1 \sin\beta$

(4.9) $\dfrac{P_2}{P_1} = 1 + \dfrac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$

(4.10) $M_{n_2}^2 = \dfrac{M_{n_1}^2 + (2/(\gamma-1))}{(2\gamma/(\gamma-1))M_{n_1}^2 - 1}$

(4.12) $M_2 = \dfrac{M_{n_2}}{\sin(\beta-\theta)}$

$M_2 = 1.82$

$\dfrac{P_2}{P_1} = 1.32 \Rightarrow P_2 = 133.3$ kPa

EXPANSION $2 \rightarrow 3$

$$(4.44) \quad \nu_2 = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2-1)} - \tan^{-1}\sqrt{M_2^2-1}$$

FROM $2 \rightarrow 3$, THE FLOW MUST BE TURNED BACK ε DEGREES AND THEN ANOTHER ε DEGREES TO FOLLOW THE REAR PART OF THE WING, THUS $\Delta\theta = 2\varepsilon$

$$\Rightarrow \nu_3 = \nu_2 + \Delta\theta = 31.34^\circ$$

$$(4.44) \quad \nu_3 = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M_3^2-1)} - \tan^{-1}\sqrt{M_3^2-1}$$

$$\Rightarrow M_3 = 2.18$$

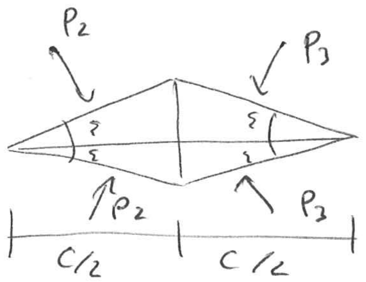
$$(3.30) \quad \frac{p_{02}}{p_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_{03}}{p_3} = \left(1 + \frac{\gamma-1}{2}M_3^2\right)^{\gamma/(\gamma-1)}$$

$p_{02} = p_{03}$ (EXPANSION IS ISENTROPIC)

$$\Rightarrow \frac{p_3}{p_2} = \left(\frac{1 + \frac{(\gamma-1)}{2}M_2^2}{1 + \frac{(\gamma-1)}{2}M_3^2}\right)^{\gamma/(\gamma-1)} = 0.57$$

$$\frac{p_3}{p_1} = \frac{p_3}{p_2}\frac{p_2}{p_1} = 0.75 \Rightarrow p_3 = 75.77 \text{ kPa}$$



$$F_D = 2\left((p_2 - p_3)\frac{c}{2}\tan(\varepsilon)\right)$$

$$= (p_2 - p_3)\, c \tan(\varepsilon)$$

$$= 5.0 \text{ kN}$$

P_4 (SHOCK TUBE)

$T_5 = 1500\,K$

$P_5 = 56.0\,bar = 5.6\,MPa$

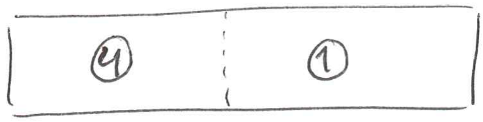
$P_2/P_1 \geq 11.0$

$T_4 = 680\,K$

$T_1 = 300\,K$

$P_1 = 1.0\,bar = 100.0\,kPa$

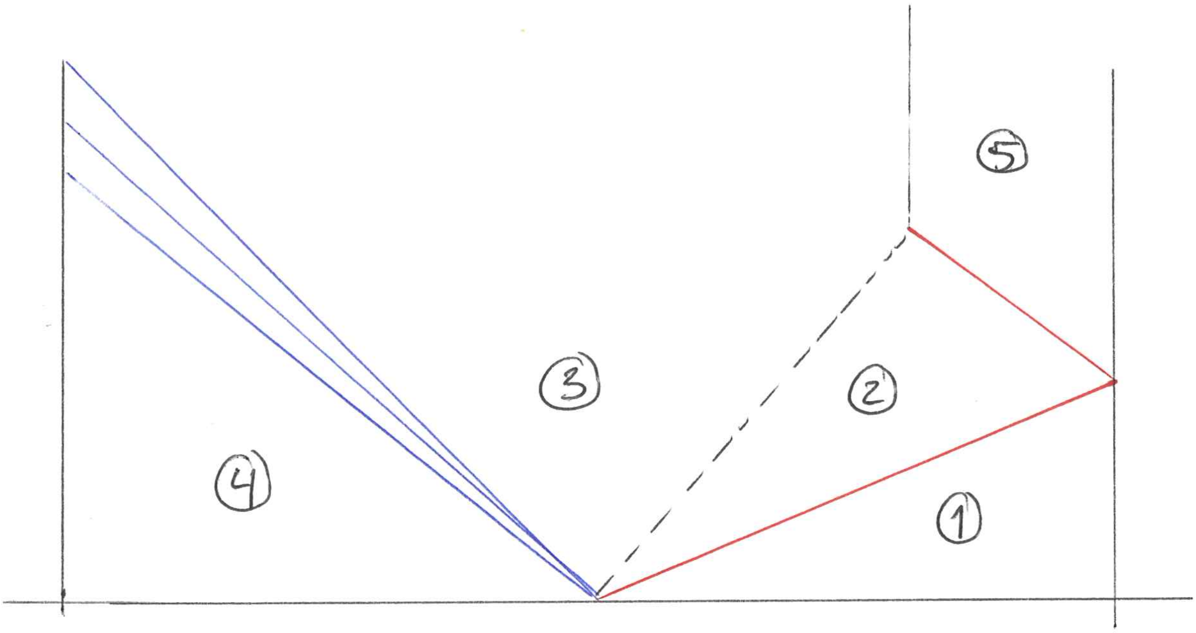
$P_4 \leq 31.0\,bar = 3.1\,MPa$



DRIVER GAS: HELIUM

DRIVEN GAS: AIR

$\left(\begin{array}{l} \gamma_{He} = 1.67 \\ R_{He} = 2080 \end{array}\right)$



a) (7.94) $$\frac{P_4}{P_1} = \frac{P_2}{P_1}\left(1 - \frac{(\gamma_4 - 1)(a_1/a_4)(P_2/P_1 - 1)}{\sqrt{2\gamma_1\left(2\gamma_1 + (\gamma_1 + 1)(P_2/P_1 - 1)\right)}}\right)^{-2\gamma_4/(\gamma_4-1)}$$

ITERATION GIVES (WITH $P_4 = 31.0\,bar$, $a_1 = \sqrt{\gamma_1 R_1 T_1}$, $a_4 = \sqrt{\gamma_4 R_4 T_4}$)

$\Rightarrow$ $P_2/P_1 = 11.54 > 11.0$ $\Rightarrow$

IT IS POSSIBLE TO FULFILL THE REQUIREMENTS WITHOUT EXCEEDING THE PRESSURE LIMITATION SET FOR THE DRIVER SECTION.

b) ASSUME $P_4 = 31.0\,bar$ AND $P_2/P_1 = 11.0$

$$\left(\frac{P_4}{P_1}\frac{P_1}{P_2}\right)^{-2\gamma_4/(\gamma_4-1)} = 1 - \frac{(\gamma_4 - 1)(a_1/a_4)(P_2/P_1 - 1)}{\sqrt{2\gamma_1\left(2\gamma_1 + (\gamma_1+1)(P_2/P_1 - 1)\right)}}$$

$$\frac{(\gamma_4 - 1)(a_1/a_4)(P_2/P_1 - 1)}{\sqrt{2\gamma_1(2\gamma_1 + (\gamma_1+1)(P_2/P_1 - 1))}} = 1 - \left(\frac{P_4}{P_1}\frac{P_1}{P_2}\right)^{-(\gamma_4-1)/(2\gamma_4)}$$

$$\frac{a_1}{a_4} = \left(1 - \left(\frac{P_4}{P_1}\frac{P_1}{P_2}\right)^{-(\gamma_4-1)/(2\gamma_4)}\right) \frac{\sqrt{2\gamma_1(2\gamma_1 + (\gamma_1+1)(P_2/P_1 - 1))}}{(\gamma_4 - 1)(P_2/P_1 - 1)}$$

$$\frac{a_1}{a_4} \simeq 0.24$$

$$a_1 = \sqrt{\gamma_1 R_1 T_1} \quad ; \quad a_4 = \sqrt{\gamma_4 R_4 T_4}$$

$$\Rightarrow T_4 \simeq \underline{589.5 \text{ K}}$$

WITHOUT MAKING ANY OTHER CHANGES THE DRIVER GAS TEMPERATURE MAY BE REDUCED FROM 680K TO 590K (90K).