TME085 - Compressible Flow

2015-04-13, 08.30-13.30, M-building

Approved aids:

- *TME085 Compressible Flow - Formulas, tables and graphs* (provided with exam)
- *Beta - Mathematics Handbook for Science and Engineering*
- Optional calculator/Valfri miniräknare (graph drawing calculators with cleared memory allowed)

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Note that at least 6 points needs to be in the theoretical part (Part I) and at least 10 points in the problem part (Part II)

Part I - Theory Questions (20 p.)

T1. (1 p.)
For a stationary normal shock, describe how the entropy, velocity, pressure and total temperature of a fluid particle is affected as it passes through the shock.

T2. (1 p.)
An unsteady expansion wave is traveling inside a tube in which viscous effects are found to be negligible. Which of the following variables are constant throughout the expansion wave?

(a) pressure
(b) temperature
(c) entropy
(d) density

T3. (1 p.)
What is the general definition (valid for any gas) of the "*total*" conditions p_0, T_0, ρ_0,... etc at some location in the flow?

T4. (1 p.)
What types of waves or discontinuities are generated in a shock tube with two initially stagnant regions at different pressure (separated by a thin membrane which is removed very quickly)?

T5. (1 p.)
An engineer wants to apply a numerical solution scheme for compressible flow. The flow he is interested in contains shocks. He has to choose between two different solution methods – one which is based on the conservation form of the governing equations and one which is based on the non-conservation form of the governing equations. Which method should he choose and why?

T6. (2 p.)

(a) High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by equilibrium gas? What kind of thermodynamic relations are needed to compute the flow of equilibrium gas?
(b) What is the difference between a calorically perfect gas and a thermally perfect gas?

T7. (1 p.)
A mixture of chemically reacting perfect gases, where the reactions are always in equilibrium, may be thermodynamically described as a single-species gas. How does this thermodynamic description differ from that of a calorically perfect or thermally perfect gas?

T8. (1 p.)
Assume a steady-state flow in a convergent-divergent nozzle. Describe what characterizes the following operating conditions:

(a) Sub-critical nozzle flow
(b) Over-expanded nozzle flow
(c) Under-expanded nozzle flow

T9. (2 p.)
Derive the area-velocity relation in quasi-1D flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

What are the implications of the area-velocity relation for quasi-one-dimensional flow?

T10. (2 p.)

(a) Describe the choking of flow that occurs for pipe flow with friction. What happens if the real length L of a pipe is longer than L^* (for either subsonic flow or supersonic flow)?

(b) What is it meant by choking the flow in a nozzle? Describe it.

T11. (1 p.)
How does the absolute Mach number change after a weak/strong stationary oblique shock?

T12. (2 p.)

(a) How can you derive (describe in words only) the PDE:s in conservation form from the control volume formulations for the conservation of mass, momentum and energy?

(b) What is the criteria for classifying a PDE as being in "*conservation form*"?

T13. (2 p.)
Derive the continuity equation in non-conservation form from the corresponding conservation form.

T14. (2 p.)
Describe in words the significance of Crocco's equation.

Prove that a steady-state irrotational flow with constant total enthalpy must also be isentropic.

Part II - Problems (40 p.)

Problem 1 - Combustion Chamber (10 p.)

A fuel-air mixture (approximated as air with γ=1.4) enters a duct combustion chamber at $u_1 = 75$ [m/s], $p_1 = 150$ [kPa], and $T_1 = 300$ [K]. The heat addition from the combustion is 900 kJ per kg of mixture.

Compute:

(a) the exit properties u_2, p_2, and T_2

(b) the total heat addition which would have caused a sonic exit flow

Solution:

First we calculate the inlet conditions

$$\frac{T_{o1}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$M_1 = \frac{u_1}{\sqrt{\gamma R T_1}}$$

$$T_{o1} = T_1 + \frac{\gamma - 1}{2\gamma R} u_1^2 = T_1 + \frac{u_1^2}{2C_p}$$

The calculated inlet total temperature and the speciifed added heat gives us the outlet total temperature according to

$$T_{o2} = T_{o1} + q/C_p$$

Now, calculate T_o^* using eqn 3.89

$$\frac{T_o}{T_o^*} = \frac{(\gamma + 1)M_1^2}{(1 + \gamma M_1^2)^2} \left(2 + (\gamma - 1)M_1^2\right)$$

At the outlet we get T_{o2}/T_o^*=0.7886

Table A.3
T_o/T_o^*=7.96478e^{-01} $\Rightarrow$ M=0.58
T_o/T_o^*=8.18923e^{-01} $\Rightarrow$ M=0.6

Interpolation gives M_2=0.573

$$\frac{T_{o2}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2$$

gives T_2=1124.9 K

$$a_2 = \sqrt{\gamma R T_2}$$

$$u_2 = M_2 a_2$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

gives: u_2=385.2 m/s, p_2=109.5 kPa

The amount needed in order to choke the flow is calculated using

$$q^* = C_p(T_o^* - T_o) = C_p T_o \left(\frac{T_o^*}{T_o} - 1\right)$$

which gives q^*=1.2 [MJ/kg]

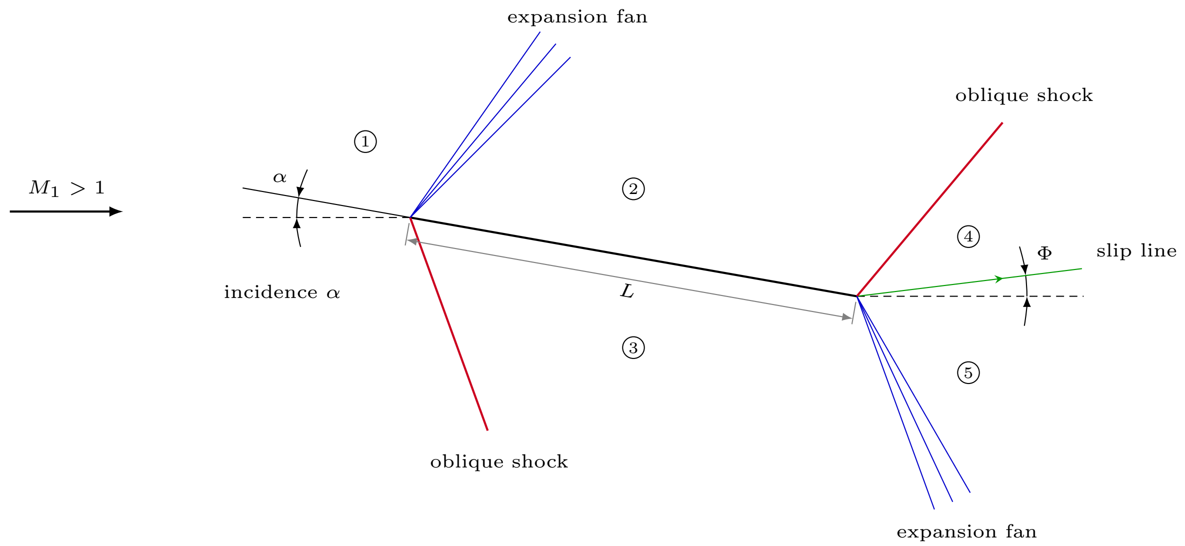
Problem 2 - Shock-Expansion Theory (10 p.)

A thin plate is placed in a supersonic flow of air. The plate has an angle relative to the free stream of 20° (angle of attack) and the freestream Mach number is 3.0.

Calculate:

(a) the lift and drag per unit length of the plate if the width (L) is 0.5 [m]

(b) the direction of the flow leaving the trailing edge of the plate (Φ)
Hint: in the figure below, the angle Φ is largely exaggerated. In reality it is very small. Calculating the angle will require an iterative solution process. In order to not spend to much time on that, start by assuming that Φ is zero (or at least close to zero).

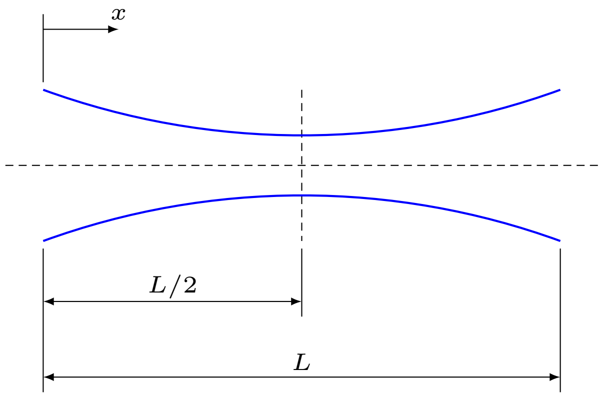


Solution: Follow example 4.16 in course book (pp. 178-180)

Problem 3 - Convergent-Divergent Nozzle (10 p.)

A converging-diverging nozzle with an exit to throat area ratio, A_e/A_t, of 1.633, is designed to operate with atmospheric pressure at the exit plane, $p_e = p_{atm}$. The converging-diverging nozzle area, A, varies with position, x, as:

$$\frac{A(x)}{A_t} = \left(\frac{A_e}{A_t} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1$$



(a) Determine the range(s) of pressure ratios (p_o/p_{atm}) for which the nozzle will be free from normal shocks

(b) Will there be a normal shock in the nozzle if nozzle pressure ratio is p_o/p_{atm}=1.5? If so, at what position (x/L) will the normal shock occur?
Hint: calculating the exit Mach number in case of existence of an internal normal shock is a good starting point

Solution:

3a. find pressure ranges for which the nozzle is free from normal shocks

There are two ranges of pressure ratios for which there will be no interior normal shocks.

- the subsonic branch
- pressures higher than the one generating a normal shock at the nozzle exit (flow in nozzle follows supersonic branch)

At the limit of choked nozzle flow, the exit Mach number is found to be M_{2sub}=0.39 using the Area-Mach-number relation. This corresponds to a pressure ratio given by

$$\frac{p_o}{p_{amb}} = \left(1 + \frac{\gamma - 1}{2} M_{2sub}^2\right)^{\gamma/(\gamma-1)} = 1.11$$

In order to calculate the limits of the second range, we first need to calculate the exit Mach number following the supersonic branch. The Area-Mach-number relation gives us M_{2sup}=1.96. With a normal shock at the nozzle exit, we will get isentropic flow throughout the nozzle

$$\frac{p_o}{p_e} = \left(1 + \frac{\gamma - 1}{2} M_{2_{sup}}^2\right)^{\gamma/(\gamma-1)} = 7.36$$

the normal shock relations gives us:

$$\frac{p_{amb}}{p_e} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{2_{sup}}^2 - 1\right) = 4.32$$

$$\frac{p_o}{p_{amb}} = \frac{p_o}{p_e}\frac{p_e}{p_{amb}} = 1.70$$

Summarized:

subsonic branch:

$1.0 < \frac{p_o}{p_{amb}} < 1.11$

supersonic branch:

$1.70 < \frac{p_o}{p_{amb}}$

3b. Will there be a normal shock in the nozzle if nozzle pressure ratio is p_o/p_{atm}=1.5? If so, at what position (x/L) will the normal shock occur?

Form 3a we have that pressure ratios larger than 1.11 and less than 1.70 will result in interior normal shocks *i.e.* for p_o/p_{atm}=1.5 we will get a normal shock in the nozzle.

We use eqn 5.28 to calculate the exit Mach number

$$M_e^2 = \frac{-1}{\gamma - 1} + \sqrt{\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma - 1}\right)\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{p_{o_1} A_t}{p_e A_e}\right)^2}$$

which gives M_e=0.52

$$\frac{p_{o2}}{p_{amb}} = \left(1 + \frac{\gamma - 1}{2} {M_e}^2\right)^{\gamma/(\gamma-1)} = 1.2$$

$$\frac{p_{o1}}{p_{amb}} = \frac{p_{o2}}{p_{amb}}\frac{p_{o1}}{p_{o2}} \Rightarrow \frac{p_{o2}}{p_{o1}} = \frac{p_{o1}}{p_{amb}} \Big/ \frac{p_{o2}}{p_{amb}} = 1.2/1.5 = 0.8$$

from Table A.2 we get:
p_{o2}/p_{o1}=7.94761e^{-01} $\Rightarrow$ M=1.84
p_{o2}/p_{o1}=8.03763e^{-01} $\Rightarrow$ M=1.82
Interpolation gives us the Mach number ahead of the shock: M_1=1.83

The Area-Mach-number relation gives us the area-ratio for the calculated Mach number $A(x)/A_t$=1.47

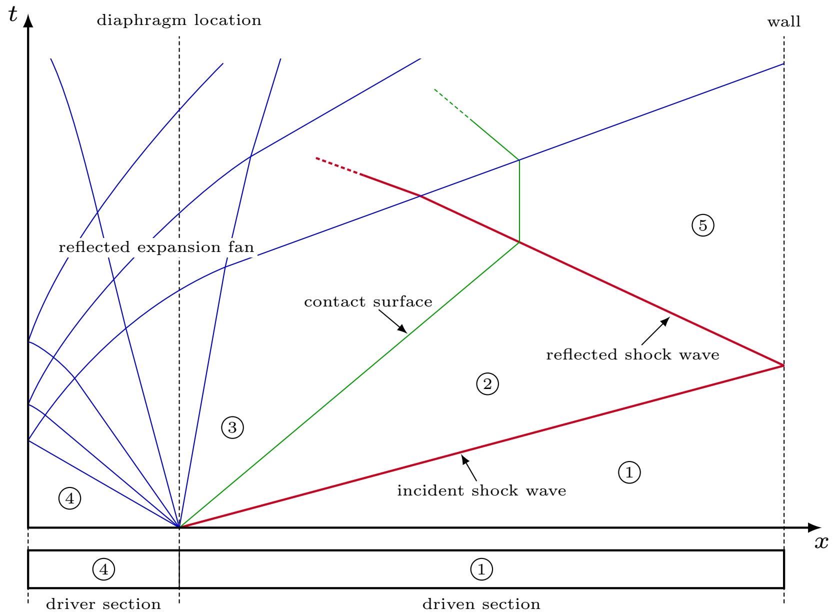
From the given area distribtion

$$\frac{A(x)}{A_t} = \left(\frac{A_e}{A_t} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1$$

we get x/L=0.93

Problem 4 - Shock-Tube (10 p.)

In a shock tube, the desired conditions at the right end of the tube after the incident shock wave has reflected at the right end wall are T_5=1500 [K] and p_5=56 [bar]. In order to get this result, the pressure ratio over the incident shock wave must be p_2/p_1=11. Before the diaphragm (the membrane separating conditions 1 and 4) breaks the gas in the driver section is heated to T_4=600 [K] and the pressure and temperature in the driven section (region 1) are p_1=1.0 [bar] and T_1=300 [K], respectively. The driven gas is air assumed to be calorically perfect. The maximum allowed pressure in the driver section is 35 [bar]. Make a suggestion (based on calculations and the tabulated gas data below) of a suitable gas for the driver section (region 4). The chosen gas should meet with the above-mentioned criteria, i.e. produce a pressure ratio over the incident shock of p_2/p_1=11 without exceeding the pressure limit for the driver section tube, (in an authentic situation, environmental consequences and working environment regulations would of course also have to be considered).



Gas/Vapor	**Ratio of specific heats** (γ)	**Gas constant** R
Acetylene	1.23	319
Air (standard)	1.40	287
Ammonia	1.31	530
Argon	1.67	208
Benzene	1.12	100
Butane	1.09	143
Carbon Dioxide	1.29	189
Carbon Disulphide	1.21	120
Carbon Monoxide	1.40	297
Chlorine	1.34	120
Ethane	1.19	276
Ethylene	1.24	296
Helium	1.67	2080
Hydrogen	1.41	4120
Hydrogen chloride	1.41	230
Methane	1.30	518
Natural Gas (Methane)	1.27	500
Nitric oxide	1.39	277
Nitrogen	1.40	297
Nitrous oxide	1.27	180
Oxygen	1.40	260
Propane	1.13	189
Steam (water)	1.32	462
Sulphur dioxide	1.29	130

http://www.engineeringtoolbox.com

Solution:

This problem is solved using one of the shock-tube relations (eqn 7.94)

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left\{1 - \frac{(\gamma_4 - 1)(a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma_1\left[2\gamma_1 + (\gamma_1 + 1)(p_2/p_1 - 1)\right]}}\right\}^{\frac{-2\gamma_4}{\gamma_4 - 1}}$$

given:

- $p2/p1$=11
- p_1=1 bar
- gas in driven section: air $\Rightarrow$ γ_1=1.4, R_1=287
- T_1=300 K $\Rightarrow a_1 = \sqrt{\gamma_1 R_1 T_1}$=347 m/s
- T_4=600 K
- p_{4max}=35 bar

solution:

- chose gas for driver section $\Rightarrow \gamma_4$ and $R_4 \Rightarrow a_4$
- use equation 7.94 to calculate p_4
- verify that $p_4/p_1 < 35$

example:

- driver section gas: Helium
- γ_4=1.67, R_4=2080
- p_4=30.7 bar