Fluid Mechanics MTF053

Dimensional Analysis and Similarity

Complementary Material

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Forces on a Solid Body

A solid body immersed into a flow is exposed to forces normal to its surface caused by pressure and tangential forces (forces along the surface of the body) caused by friction. The resulting net force is denoted **F**. The net force **F** can be divided into two components

Drag force: F_D (the component of **F** aligned with the freestream direction)

Lift force: F_L (the component of **F** normal to the freestream direction)

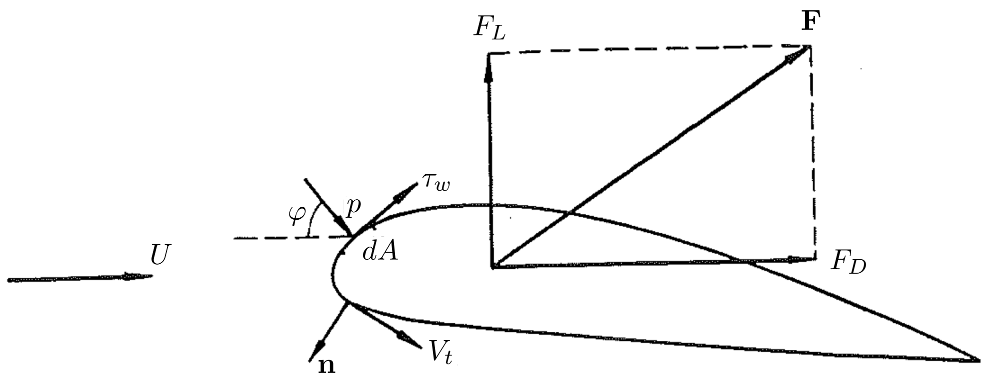

Figure 1: Forces on a wing immersed into a fluid flow

Drag Force

The contribution to the drag force caused by normal forces is called *pressure drag* and the contribution from tangential forces is called *friction drag*. For steady-state flow, the pressure drag, F_{D_n} can be calculated as

$$
\begin{aligned}
F_{D_n} &= \int_A (p + \rho g z)\, dA \cos\varphi = \int_{A^*} \rho U^2 p^* \left(x^*, y^*, z^*, Re\right) L^2 dA^* \cos\varphi = \\
&\rho U^2 L^2 \int_{A^*} p^* dA^* \cos\varphi = \rho U^2 L^2 f(Re)
\end{aligned}
\tag{1}
$$

Further, if the surface of the body projection normal to the freestream is A_p

$$A_p \propto L^2$$

and thus

$$F_{D_n} = C_{D_n}(Re)A_p\frac{\rho U^2}{2} \tag{2}$$

Note: it is of course possible to select other surfaces related to the dimensions of the body

In the same way we can get a relation for the friction drag

$$\begin{aligned} F_{D_t} &= \int \tau dA \sin\varphi = \int \mu\frac{\partial V_t}{\partial n}dA\sin\varphi = \mu\frac{U}{L}L^2\int\frac{\partial V_t^*}{\partial n^*}dA^*\sin\varphi = \\ &\mu U L f(Re) = \rho U^2 L^2 \underbrace{\frac{\mu}{\rho U L}}_{1/Re} f(Re) \end{aligned} \tag{3}$$

which gives

$$F_{D_t} = C_{D_t}(Re)A_p\frac{\rho U^2}{2} \tag{4}$$

With the pressure drag and friction drag now expressed in the same form, the net drag force is obtained as

$$F_D = C_D(Re)A_p\frac{\rho U^2}{2} \tag{5}$$

where $C_D = C_{D_n} + C_{D_t}$

Lift Force

With the same methodology as was used for the drag force it is possible to show that the lift force F_L can be expressed as

$$F_L = C_L(Re)A_p\frac{\rho U^2}{2} \tag{6}$$

Similarity

With the relations given above, it is possible to calculate the forces in prototype scale if the model-scale forces are known and given that the prototype and model are scaled properly (geometric similarity) and that the Reynolds number is the same.