Compressible Flow - TME085

Lecture 11

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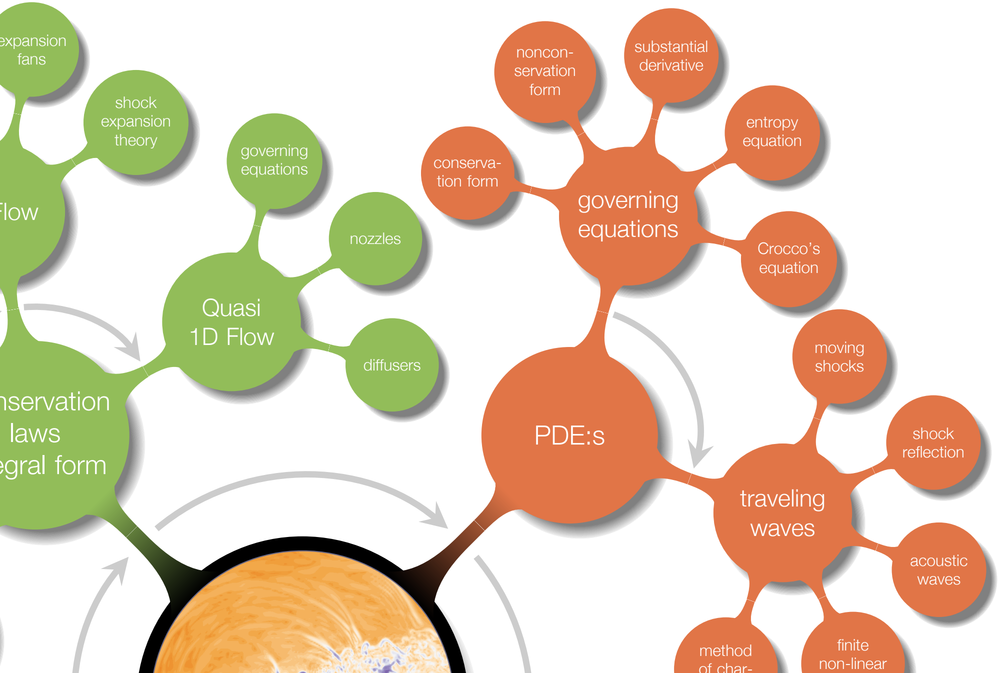
Chapter 7
Unsteady Wave Motion

Overview



expansion fans
shock expansion theory
Flow
governing equations
nozzles
Quasi 1D Flow
diffusers
nservation laws
egral form
noncon-servation form
substantial derivative
entropy equation
conserva-tion form
governing equations
Crocco's equation
PDE:s
moving shocks
shock reflection
traveling waves
acoustic waves
method of char-
finite non-linear

Learning Outcomes

3 Describe typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)

4 Present at least two different formulations of the governing equations for compressible flows and explain what basic conservation principles they are based on

8 Derive (marked) and apply (all) of the presented mathematical formulae for classical gas dynamics

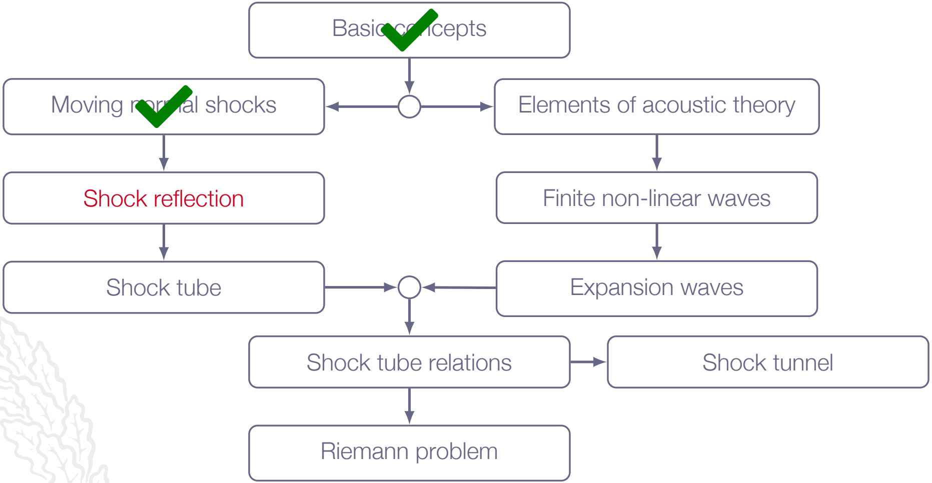
  a 1D isentropic flow*
  b normal shocks*
  j unsteady waves and discontinuities in 1D
  k basic acoustics

9 Solve engineering problems involving the above-mentioned phenomena (8a-8k)

11 Explain how the equations for aero-acoustics and classical acoustics are derived as limiting cases of the compressible flow equations

moving normal shocks - frame of reference seems to be the key here?!

Roadmap - Unsteady Wave Motion

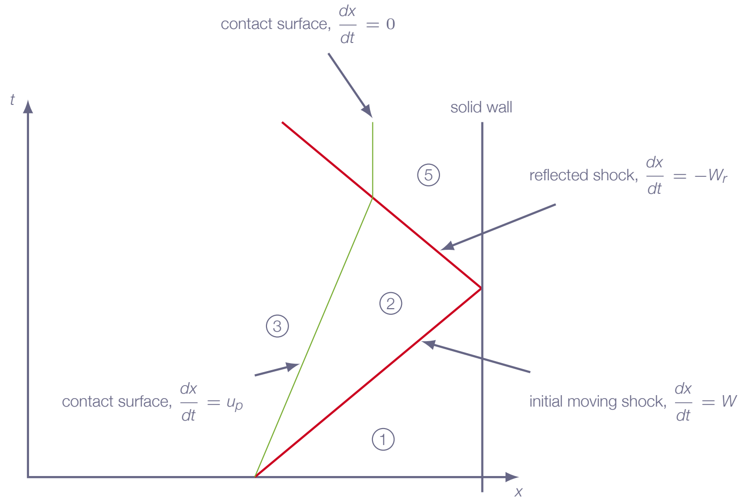


Chapter 7.3
Reflected Shock Wave

One-Dimensional Flow with Friction

what happens when a moving shock approaches a wall?

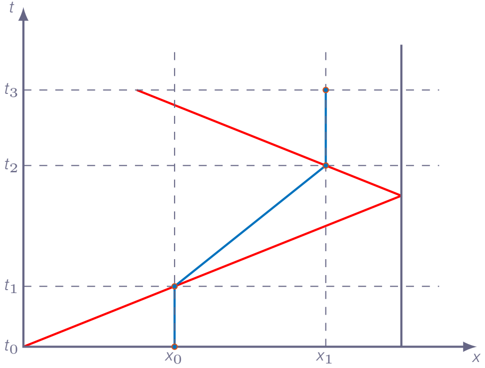
Shock Reflection



Shock Reflection - Particle Path

A fluid particle located at x_0 at time t_0 (a location ahead of the shock) will be affected by the moving shock and follow the blue path

time	location	velocity
t_0	x_0	0
t_1	x_0	u_p
t_2	x_1	u_p
t_3	x_1	0



Shock Reflection Relations

- velocity ahead of reflected shock: $W_r + u_p$
- velocity behind reflected shock: W_r

Continuity:

$$\rho_2(W_r + u_p) = \rho_5 W_r$$

Momentum:

$$p_2 + \rho_2(W_r + u_p)^2 = p_5 + \rho_5 W_r^2$$

Energy:

$$h_2 + \frac{1}{2}(W_r + u_p)^2 = h_5 + \frac{1}{2}W_r^2$$

Shock Reflection Relations

Reflected shock is determined such that $u_5 = 0$

$$\frac{M_r}{M_r^2 - 1} = \frac{M_s}{M_s^2 - 1}\sqrt{1 + \frac{2(\gamma - 1)}{(\gamma + 1)^2}(M_s^2 - 1)\left(\gamma + \frac{1}{M_s^2}\right)}$$

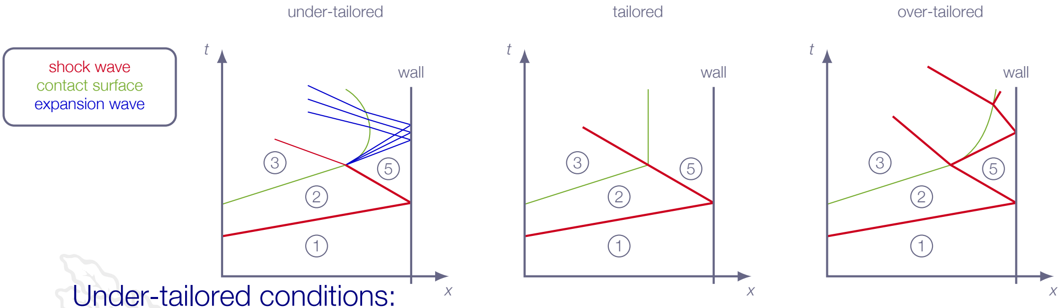
where

$$M_r = \frac{W_r + u_p}{a_2}$$

Tailored v.s. Non-Tailored Shock Reflection

- The time duration of condition 5 is determined by what happens after interaction between reflected shock and contact discontinuity
- For special choice of initial conditions (tailored case), this interaction is negligible, thus prolonging the duration of condition 5

Tailored v.s. Non-Tailored Shock Reflection



Under-tailored conditions:

Mach number of incident wave lower than in tailored conditions

Over-tailored conditions:

Mach number of incident wave higher than in tailored conditions

Shock Reflection - Example

Shock reflection in shock tube ($\gamma = 1.4$)
(Example 7.1 in Anderson)

Incident shock (given data)

p_2/p_1	10.0
M_s	2.95
T_2/T_1	2.623
p_1	1.0 [bar]
T_1	300.0 [K]

Calculated data

M_r	2.09
p_5/p_2	4.978
T_5/T_2	1.77

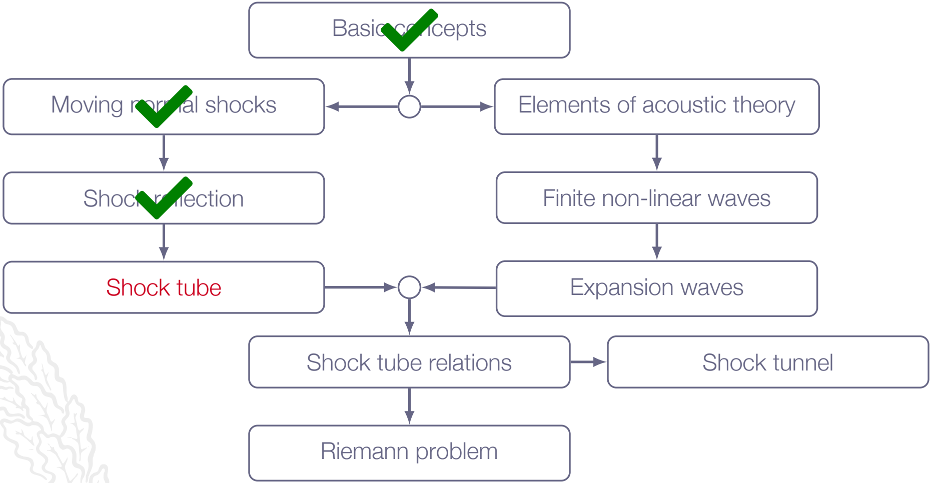
$$p_5 = \left(\frac{p_5}{p_2}\right)\left(\frac{p_2}{p_1}\right)p_1 = 49.78$$

$$T_5 = \left(\frac{T_5}{T_2}\right)\left(\frac{T_2}{T_1}\right)T_1 = 1393$$

Shock Reflection - Shock Tube

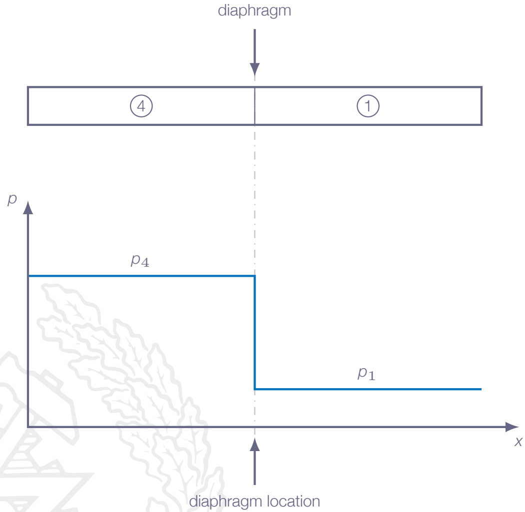
- Very high pressure and temperature conditions in a specified location with very high precision (p_5, T_5)
  - measurements of thermodynamic properties of various gases at extreme conditions, *e.g.* dissociation energies, molecular relaxation times, etc.
  - measurements of chemical reaction properties of various gas mixtures at extreme conditions

Roadmap - Unsteady Wave Motion



The Shock Tube

Shock Tube

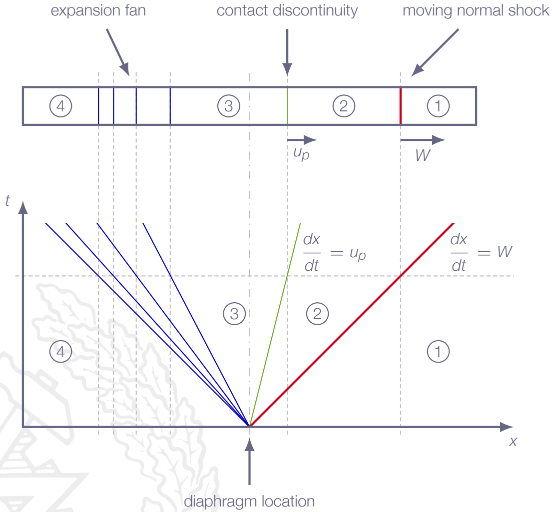


tube with closed ends
diaphragm inside, separating two different constant states
(could also be two different gases)

if diaphragm is removed suddenly (by inducing a breakdown) the two states come into contact and a flow develops

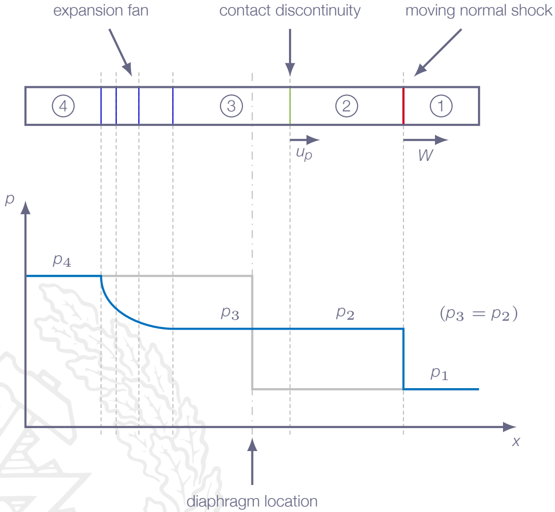
assume that $p_4 > p_1$:
state 4 is "driver" section
state 1 is "driven" section

Shock Tube



flow at some time after diaphragm breakdown

Shock Tube



flow at some time after diaphragm breakdown

Shock Tube

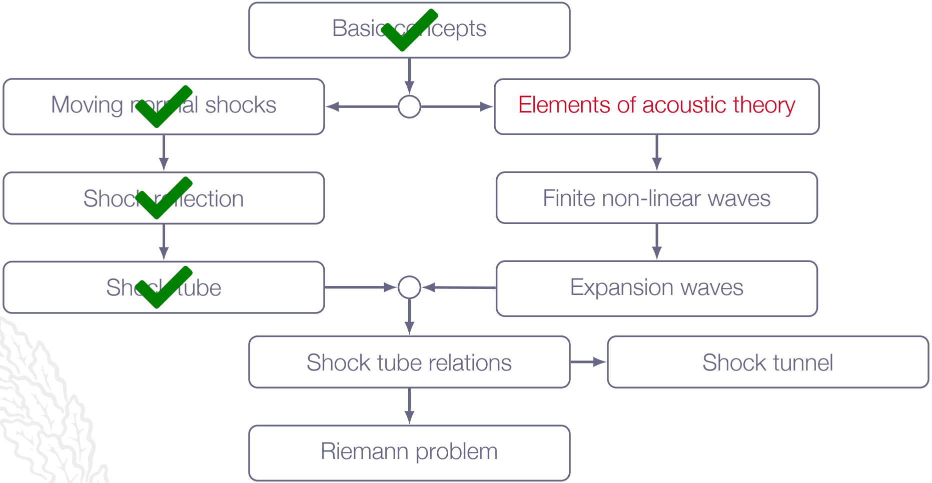
- As the diaphragm is removed, a pressure discontinuity is generated
- The only process that can generate a pressure difference in the gas is a shock
- The velocity upstream of the shock must be supersonic
- Since the gas is standing still when the shock tube is started, the shock must move in order to establish a relative velocity
- The shock must move in to the gas with the lower pressure

Shock Tube

- By using light gases for the driver section (*e.g.* He) and heavier gases for the driven section (*e.g.* air) the pressure p_4 required for a specific p_2/p_1 ratio is significantly reduced

- If T_4/T_1 is increased, the pressure p_4 required for a specific p_2/p_1 is also reduced

Roadmap - Unsteady Wave Motion



Chapter 7.5
Elements of Acoustic Theory

Sound Waves

- Weakest audible sound wave (0 dB): Δp ~0.00002 Pa
- Loud sound wave (94 dB): Δp ~1 Pa
- Threshold of pain (120 dB): Δp ~20 Pa
- Harmful sound wave (130 dB): Δp ~60 Pa

Example:

$\Delta p \sim 1$ Pa gives $\Delta \rho$ ~0.000009 kg/m^3 and Δu ~0.0025 m/s

Elements of Acoustic Theory

PDE:s for conservation of mass and momentum are derived in Chapter 6:

	conservation form	non-conservation form
mass	$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$	$\frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{v}) = 0$
momentum	$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathbf{v} + p\mathbf{I}) = 0$	$\rho \frac{D\mathbf{v}}{Dt} + \nabla p = 0$

Elements of Acoustic Theory

For adiabatic inviscid flow we also have the entropy equation as

$$\frac{Ds}{Dt} = 0$$

Assume one-dimensional flow

$$\left.\begin{aligned} \rho &= \rho(x,t) \\ \mathbf{v} &= u(x,t)\mathbf{e}_x \\ p &= p(x,t) \\ \dots \end{aligned}\right\} \Rightarrow$$

continuity	$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0$
momentum	$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = 0$
s=constant	

can $\frac{\partial p}{\partial x}$ be expressed in terms of density?

Elements of Acoustic Theory

From Chapter 1: any thermodynamic state variable is uniquely defined by any tow other state variables

$$p = p(\rho, s) \Rightarrow dp = \left(\frac{\partial p}{\partial \rho}\right)_s d\rho + \left(\frac{\partial p}{\partial s}\right)_\rho ds$$

s=constant gives

$$dp = \left(\frac{\partial p}{\partial \rho}\right)_s d\rho = a^2 d\rho$$

$$\Rightarrow \begin{cases} \dfrac{\partial \rho}{\partial t} + u\dfrac{\partial \rho}{\partial x} + \rho\dfrac{\partial u}{\partial x} = 0 \\ \rho\dfrac{\partial u}{\partial t} + \rho u\dfrac{\partial u}{\partial x} + a^2\dfrac{\partial \rho}{\partial x} = 0 \end{cases}$$

Elements of Acoustic Theory

Assume small perturbations around stagnant reference condition:

$$\rho = \rho_\infty + \Delta\rho \qquad p = p_\infty + \Delta p \qquad T = T_\infty + \Delta T \qquad u = u_\infty + \Delta u = \{u_\infty = 0\} = \Delta u$$

where ρ_∞, p_∞, and T_∞ are constant

Now, insert $\rho = (\rho_\infty + \Delta\rho)$ and $u = \Delta u$ in the continuity and momentum equations (derivatives of ρ_∞ are zero)

$$\Rightarrow \begin{cases} \dfrac{\partial}{\partial t}(\Delta\rho) + \Delta u \dfrac{\partial}{\partial x}(\Delta\rho) + (\rho_\infty + \Delta\rho)\dfrac{\partial}{\partial x}(\Delta u) = 0 \\ (\rho_\infty + \Delta\rho)\dfrac{\partial}{\partial t}(\Delta u) + (\rho_\infty + \Delta\rho)\Delta u \dfrac{\partial}{\partial x}(\Delta u) + a^2 \dfrac{\partial}{\partial x}(\Delta\rho) = 0 \end{cases}$$

Elements of Acoustic Theory

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Elements of Acoustic Theory

Speed of sound is a thermodynamic state variable $\Rightarrow a^2 = a^2(\rho, s)$. With entropy constant $\Rightarrow a^2 = a^2(\rho)$

Taylor expansion around a_∞ with $(\Delta\rho = \rho - \rho_\infty)$ gives

$$a^2 = a_\infty^2 + \left(\frac{\partial}{\partial\rho}(a^2)\right)_\infty \Delta\rho + \frac{1}{2}\left(\frac{\partial^2}{\partial\rho^2}(a^2)\right)_\infty (\Delta\rho)^2 + \ldots$$

$$\Rightarrow \begin{cases} \dfrac{\partial}{\partial t}(\Delta\rho) + \Delta u \dfrac{\partial}{\partial x}(\Delta\rho) + (\rho_\infty + \Delta\rho)\dfrac{\partial}{\partial x}(\Delta u) = 0 \\ (\rho_\infty + \Delta\rho)\dfrac{\partial}{\partial t}(\Delta u) + (\rho_\infty + \Delta\rho)\Delta u \dfrac{\partial}{\partial x}(\Delta u) + \left[a_\infty^2 + \left(\dfrac{\partial}{\partial\rho}(a^2)\right)_\infty \Delta\rho + \ldots\right]\dfrac{\partial}{\partial x}(\Delta\rho) = 0 \end{cases}$$

Elements of Acoustic Theory - Acoustic Equations

Since $\Delta\rho$ and Δu are assumed to be small ($\Delta\rho \ll \rho_\infty$, $\Delta u \ll a$)

- products of perturbations can be neglected
- higher-order terms in the Taylor expansion can be neglected

$$\Rightarrow \begin{cases} \dfrac{\partial}{\partial t}(\Delta\rho) + \rho_\infty \dfrac{\partial}{\partial x}(\Delta u) = 0 \\[2ex] \rho_\infty \dfrac{\partial}{\partial t}(\Delta u) + a_\infty^2 \dfrac{\partial}{\partial x}(\Delta\rho) = 0 \end{cases}$$

Note! Only valid for small perturbations (sound waves)

This type of derivation is based on linearization, *i.e.* the acoustic equations are linear

Elements of Acoustic Theory - Acoustic Equations

Acoustic equations:

"... describe the motion of gas induced by the passage of a sound wave ..."

Elements of Acoustic Theory - Wave Equation

Combining linearized continuity and the momentum equations we get

$$\frac{\partial^2}{\partial t^2}(\Delta\rho) = a_\infty^2 \frac{\partial^2}{\partial x^2}(\Delta\rho)$$

(combine the time derivative of the continuity eqn. and the divergence of the momentum eqn.)

General solution:

$$\Delta\rho(x,t) = F(x - a_\infty t) + G(x + a_\infty t)$$

wave traveling in positive x-direction with speed a_∞

wave traveling in negative x-direction with speed a_∞

F and G may be arbitrary functions

Wave shape is determined by functions F and G

Elements of Acoustic Theory - Wave Equation

Spatial and temporal derivatives of F are obtained according to

$$\begin{cases} \dfrac{\partial F}{\partial t} = \dfrac{\partial F}{\partial(x - a_\infty t)} \dfrac{\partial(x - a_\infty t)}{\partial t} = -a_\infty F' \\[2ex] \dfrac{\partial F}{\partial x} = \dfrac{\partial F}{\partial(x - a_\infty t)} \dfrac{\partial(x - a_\infty t)}{\partial x} = F' \end{cases}$$

spatial and temporal derivatives of G can of course be obtained in the same way...

Elements of Acoustic Theory - Wave Equation

with $\Delta\rho(x,t) = F(x - a_\infty t) + G(x + a_\infty t)$ and the derivatives of F and G we get

$$\frac{\partial^2}{\partial t^2}(\Delta\rho) = a_\infty^2 F'' + a_\infty^2 G''$$

and

$$\frac{\partial^2}{\partial x^2}(\Delta\rho) = F'' + G''$$

which gives

$$\frac{\partial^2}{\partial t^2}(\Delta\rho) - a_\infty^2 \frac{\partial^2}{\partial x^2}(\Delta\rho) = 0$$

i.e., the proposed solution fulfils the wave equation

Elements of Acoustic Theory - Wave Equation

F and G may be arbitrary functions, assume $G = 0$

$$\Delta\rho(x,t) = F(x - a_{\infty}t)$$

If $\Delta\rho$ is constant (constant wave amplitude), $(x - a_{\infty}t)$ must be a constant which implies

$$x = a_{\infty}t + c$$

where c is a constant

$$\frac{dx}{dt} = a_{\infty}$$

Elements of Acoustic Theory - Wave Equation

We want a relation between $\Delta\rho$ and Δu

$\Delta\rho(x,t) = F(x - a_\infty t)$ (wave in positive x direction) gives:

$$\frac{\partial}{\partial t}(\Delta\rho) = -a_\infty F' \quad \text{and} \quad \frac{\partial}{\partial x}(\Delta\rho) = F'$$

$$\underbrace{\frac{\partial}{\partial t}(\Delta\rho)}_{-a_\infty F'} + a_\infty \underbrace{\frac{\partial}{\partial x}(\Delta\rho)}_{F'} = 0$$

or

$$\frac{\partial}{\partial x}(\Delta\rho) = -\frac{1}{a_\infty}\frac{\partial}{\partial t}(\Delta\rho)$$

Elements of Acoustic Theory - Wave Equation

Linearized momentum equation:

$$\rho_\infty \frac{\partial}{\partial t}(\Delta u) = -a_\infty^2 \frac{\partial}{\partial x}(\Delta \rho) \Rightarrow$$

$$\frac{\partial}{\partial t}(\Delta u) = -\frac{a_\infty^2}{\rho_\infty}\frac{\partial}{\partial x}(\Delta \rho) = \left\{ \frac{\partial}{\partial x}(\Delta \rho) = -\frac{1}{a_\infty}\frac{\partial}{\partial t}(\Delta \rho) \right\} = \frac{a_\infty}{\rho_\infty}\frac{\partial}{\partial t}(\Delta \rho)$$

$$\frac{\partial}{\partial t}\left(\Delta u - \frac{a_\infty}{\rho_\infty}\Delta \rho\right) = 0 \Rightarrow \Delta u - \frac{a_\infty}{\rho_\infty}\Delta \rho = const$$

In undisturbed gas $\Delta u = \Delta \rho = 0$ which implies that the constant must be zero and thus

$$\boxed{\Delta u = \frac{a_\infty}{\rho_\infty}\Delta \rho}$$

Elements of Acoustic Theory - Wave Equation

Similarly, for $\Delta\rho(x,t) = G(x + a_\infty t)$ (wave in negative x direction) we obtain:

$$\boxed{\Delta u = -\frac{a_\infty}{\rho_\infty}\Delta\rho}$$

Also, since $\Delta p = a_\infty^2 \Delta\rho$ we get:

Right going wave (+x direction) $\Delta u = \frac{a_\infty}{\rho_\infty}\Delta\rho = \frac{1}{a_\infty \rho_\infty}\Delta p$

Left going wave (-x direction) $\Delta u = -\frac{a_\infty}{\rho_\infty}\Delta\rho = -\frac{1}{a_\infty \rho_\infty}\Delta p$

Elements of Acoustic Theory - Wave Equation

- Δu denotes induced mass motion and is positive in the positive x-direction

$$\Delta u = \pm \frac{a_\infty \Delta \rho}{\rho_\infty} = \pm \frac{\Delta p}{a_\infty \rho_\infty}$$

- condensation (the part of the sound wave where $\Delta \rho > 0$):
  Δu is always in the same direction as the wave motion
- rarefaction (the part of the sound wave where $\Delta \rho < 0$):
  Δu is always in the opposite direction as the wave motion

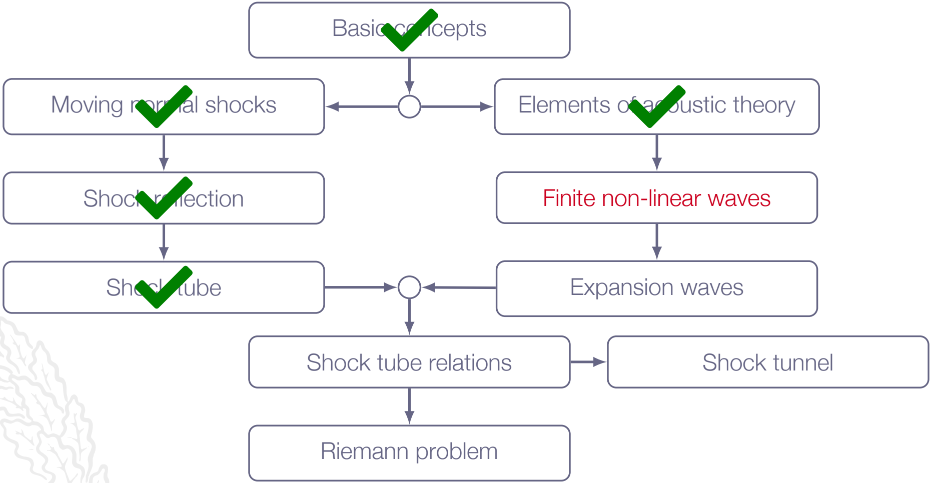
Elements of Acoustic Theory - Wave Equation *Summary*

Combining linearized continuity and the momentum equations we get

$$\frac{\partial^2}{\partial t^2}(\Delta\rho) = a_\infty^2 \frac{\partial^2}{\partial x^2}(\Delta\rho)$$

- Due to the assumptions made, the equation is not exact
- More and more accurate as the perturbations becomes smaller and smaller
- How should we describe waves with larger amplitudes?

Roadmap - Unsteady Wave Motion



Chapter 7.6
Finite (Non-Linear) Waves

Finite (Non-Linear) Waves

When $\Delta\rho$, Δu, Δp, ... Become large, the linearized acoustic equations become poor approximations

Non-linear equations must be used

One-dimensional non-linear continuity and momentum equations

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

Finite (Non-Linear) Waves

We still assume isentropic flow, $ds = 0$

$$\frac{\partial \rho}{\partial t} = \left(\frac{\partial \rho}{\partial p}\right)_s \frac{\partial p}{\partial t} = \frac{1}{a^2}\frac{\partial p}{\partial t}$$

$$\frac{\partial \rho}{\partial x} = \left(\frac{\partial \rho}{\partial p}\right)_s \frac{\partial p}{\partial x} = \frac{1}{a^2}\frac{\partial p}{\partial x}$$

Inserted in the continuity equation this gives:

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \rho a^2 \frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

Finite (Non-Linear) Waves

Add $1/(\rho a)$ times the continuity equation to the momentum equation:

$$\left[\frac{\partial u}{\partial t} + (u+a)\frac{\partial u}{\partial x}\right] + \frac{1}{\rho a}\left[\frac{\partial p}{\partial t} + (u+a)\frac{\partial p}{\partial x}\right] = 0$$

If we instead subtraction $1/(\rho a)$ times the continuity equation from the momentum equation, we get:

$$\left[\frac{\partial u}{\partial t} + (u-a)\frac{\partial u}{\partial x}\right] - \frac{1}{\rho a}\left[\frac{\partial p}{\partial t} + (u-a)\frac{\partial p}{\partial x}\right] = 0$$

Finite (Non-Linear) Waves

Since $u = u(x, t)$, we have:

$$du = \frac{\partial u}{\partial t} dt + \frac{\partial u}{\partial x} dx = \frac{\partial u}{\partial t} dt + \frac{\partial u}{\partial x} \frac{dx}{dt} dt$$

Let $\dfrac{dx}{dt} = u + a$ gives

$$du = \left[\frac{\partial u}{\partial t} + (u + a) \frac{\partial u}{\partial x} \right] dt$$

Interpretation: change of u in the direction of line $\dfrac{dx}{dt} = u + a$

Finite (Non-Linear) Waves

In the same way we get:

$$dp = \frac{\partial p}{\partial t}dt + \frac{\partial p}{\partial x}\frac{dx}{dt}dt$$

and thus

$$dp = \left[\frac{\partial p}{\partial t} + (u+a)\frac{\partial p}{\partial x}\right]dt$$

Finite (Non-Linear) Waves

Now, if we combine

$$\left[\frac{\partial u}{\partial t}+(u+a)\frac{\partial u}{\partial x}\right]+\frac{1}{\rho a}\left[\frac{\partial p}{\partial t}+(u+a)\frac{\partial p}{\partial x}\right]=0$$

$$du=\left[\frac{\partial u}{\partial t}+(u+a)\frac{\partial u}{\partial x}\right]dt$$

$$dp=\left[\frac{\partial p}{\partial t}+(u+a)\frac{\partial p}{\partial x}\right]dt$$

we get

$$\boxed{\frac{du}{dt}+\frac{1}{\rho a}\frac{dp}{dt}=0}$$

Characteristic Lines

Thus, along a line $dx = (u + a)dt$ we have

$$du + \frac{dp}{\rho a} = 0$$

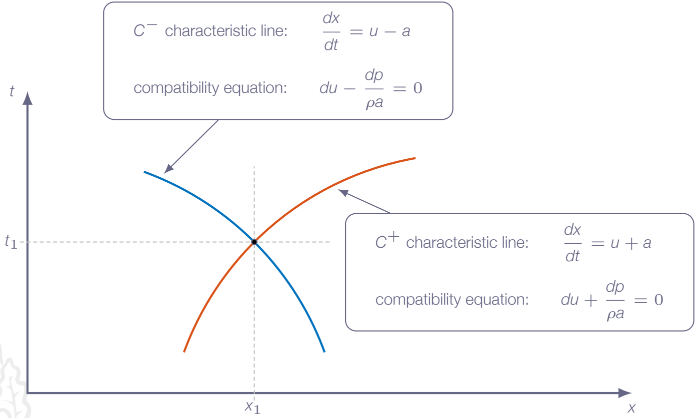
In the same way we get along a line where $dx = (u - a)dt$

$$du - \frac{dp}{\rho a} = 0$$

Characteristic Lines

- We have found a path through a point (x_1, t_1) along which the governing partial differential equations reduces to ordinary differential equations
- These paths or lines are called characteristic lines
- The C^+ and C^- characteristic lines are physically the paths of right- and left-running sound waves in the *xt*-plane

Characteristic Lines



Characteristic Lines - Summary

$$\frac{du}{dt} + \frac{1}{\rho a}\frac{dp}{dt} = 0 \quad \text{along } C^+ \text{ characteristic}$$

$$\frac{du}{dt} - \frac{1}{\rho a}\frac{dp}{dt} = 0 \quad \text{along } C^- \text{ characteristic}$$

$$du + \frac{dp}{\rho a} = 0 \quad \text{along } C^+ \text{ characteristic}$$

$$du - \frac{dp}{\rho a} = 0 \quad \text{along } C^- \text{ characteristic}$$

Riemann Invariants

Integration gives:

$$J^+ = u + \int \frac{dp}{\rho a} = \text{constant along } C^+ \text{ characteristic}$$

$$J^- = u - \int \frac{dp}{\rho a} = \text{constant along } C^- \text{ characteristic}$$

We need to rewrite $\frac{dp}{\rho a}$ to be able to perform the integrations

Riemann Invariants

Let's consider an isentropic processes:

$$p = c_1 T^{\gamma/(\gamma-1)} = c_2 a^{2\gamma/(\gamma-1)}$$

where c_1 and c_2 are constants and thus

$$dp = c_2 \left(\frac{2\gamma}{\gamma-1}\right) a^{[2\gamma/(\gamma-1)-1]} da$$

Assume calorically perfect gas: $a^2 = \dfrac{\gamma p}{\rho} \Rightarrow \rho = \dfrac{\gamma p}{a^2}$

with $p = c_2 a^{2\gamma/(\gamma-1)}$ we get $\rho = c_2 \gamma a^{[2\gamma/(\gamma-1)-2]}$

Riemann Invariants

$$J^+ = u + \int \frac{dp}{\rho a} = u + \int \frac{c_2 \left(\frac{2\gamma}{\gamma-1}\right) a^{[2\gamma/(\gamma-1)-1]}}{c_2 \gamma a^{[2\gamma/(\gamma-1)-1]}} da = u + \int \frac{2da}{\gamma - 1}$$

$$J^+ = u + \frac{2a}{\gamma - 1}$$

$$J^- = u - \frac{2a}{\gamma - 1}$$

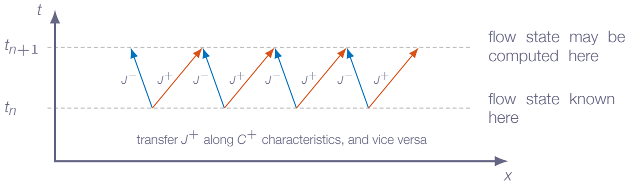
Riemann Invariants

If J^+ and J^- are known at some point (x, t), then

$$\begin{cases} J^+ + J^- = 2u \\ J^+ - J^- = \dfrac{4a}{\gamma - 1} \end{cases} \Rightarrow \begin{cases} u = \dfrac{1}{2}(J^+ + J^-) \\ a = \dfrac{\gamma - 1}{4}(J^+ - J^-) \end{cases}$$

Flow state is uniquely defined!

Method of Characteristics



Summary

Acoustic waves

- $\Delta\rho$, Δu, etc - very small
- All parts of the wave propagate with the same velocity a_∞
- The wave shape stays the same
- The flow is governed by linear relations

Finite (non-linear) waves

- $\Delta\rho$, Δu, etc - can be large
- Each local part of the wave propagates at the local velocity $(u + a)$
- The wave shape changes with time
- The flow is governed by non-linear relations

One-Dimensional Flow with Friction

the method of characteristics is a central element in classic compressible flow theory