Compressible Flow - TME085

Lecture 10

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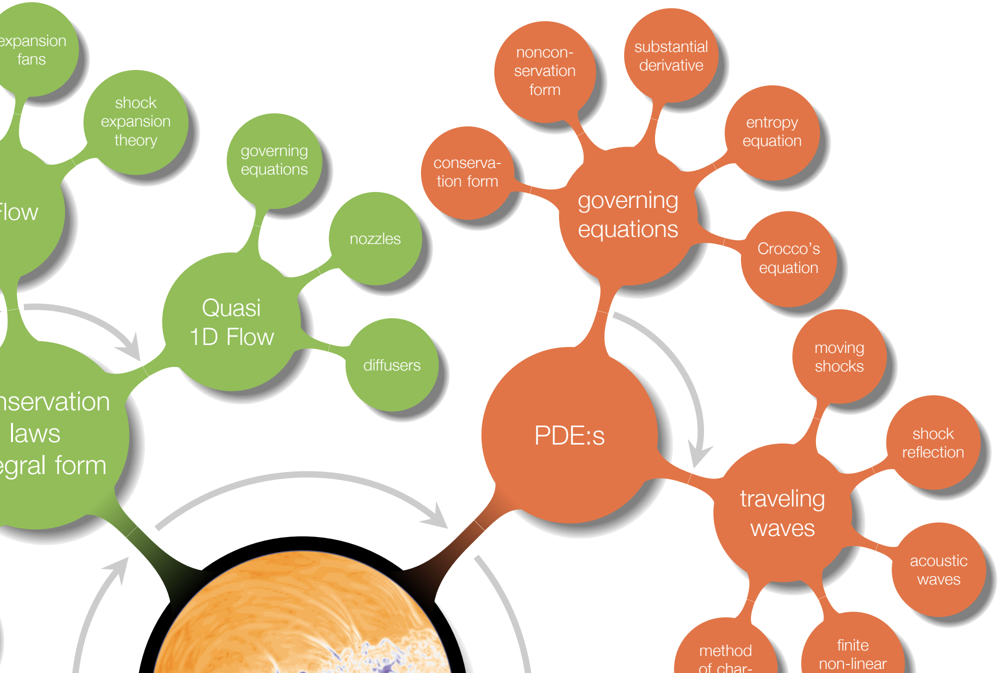
Chapter 7
Unsteady Wave Motion

Overview



expansion fans
shock expansion theory
Flow
governing equations
nozzles
Quasi 1D Flow
diffusers
nservation laws egral form
noncon-servation form
substantial derivative
entropy equation
conserva-tion form
governing equations
Crocco’s equation
PDE:s
moving shocks
shock reflection
traveling waves
acoustic waves
method of char-
finite non-linear

Learning Outcomes

3 Describe typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)

4 Present at least two different formulations of the governing equations for compressible flows and explain what basic conservation principles they are based on

8 Derive (marked) and apply (all) of the presented mathematical formulae for classical gas dynamics

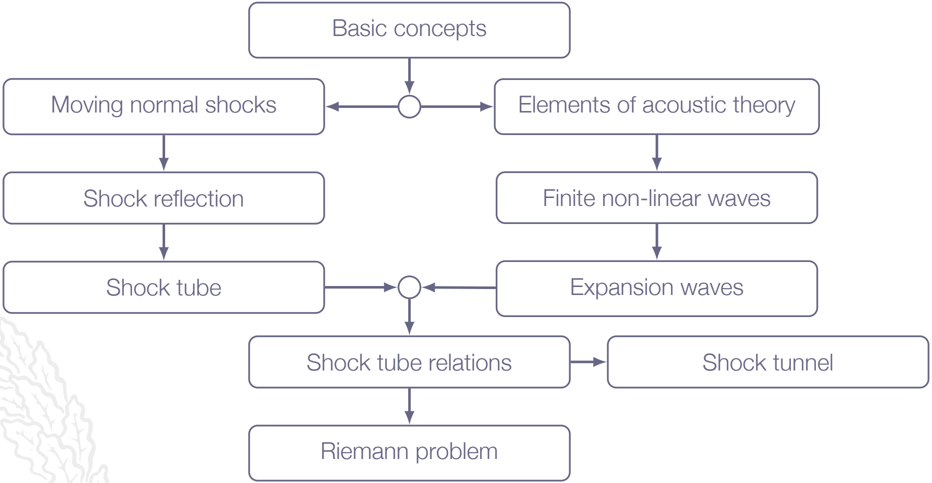
  a 1D isentropic flow*
  b normal shocks*
  j unsteady waves and discontinuities in 1D
  k basic acoustics

9 Solve engineering problems involving the above-mentioned phenomena (8a-8k)

11 Explain how the equations for aero-acoustics and classical acoustics are derived as limiting cases of the compressible flow equations

moving normal shocks - frame of reference seems to be the key here?!

Roadmap - Unsteady Wave Motion



Motivation

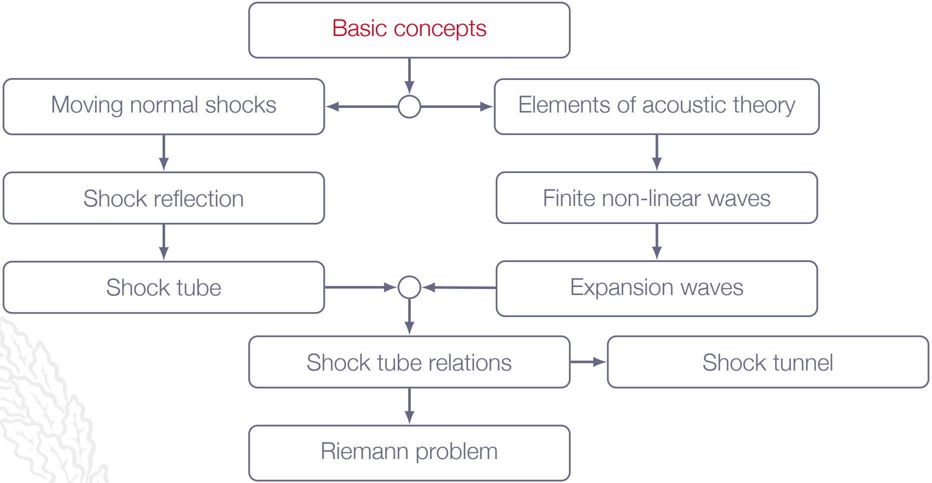
Most practical flows are unsteady

Traveling waves appears in many real-life situations and is an important topic within compressible flows

We will study unsteady flows in one dimension in order to reduce complexity and focus on the physical effects introduced by the unsteadiness

Throughout this section, we will study an application called the shock tube, which is a rather rare application but it lets us study unsteady waves in one dimension and it includes all physical principles introduced in chapter 7

Roadmap - Unsteady Wave Motion



Unsteady Wave Motion

inertial frames!

Physical laws are the same for both frame of references

Shock characteristics are the same for both observers (shape, strength, etc)

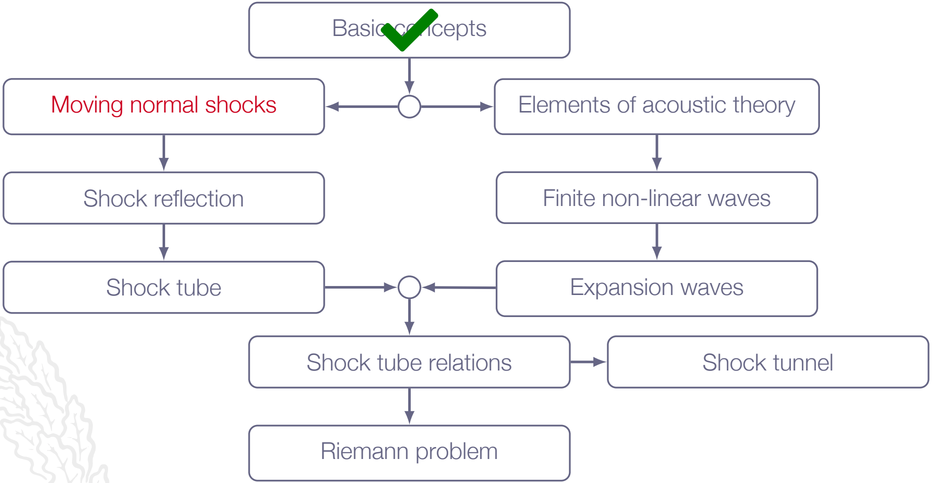
Unsteady Wave Motion

Is there a connection with stationary shock waves?

Answer: Yes!

Locally, in a moving frame of reference, the shock may be viewed as a stationary normal shock

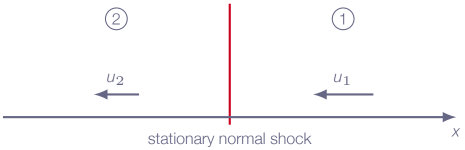
Roadmap - Unsteady Wave Motion



Chapter 7.2
Moving Normal Shock Waves

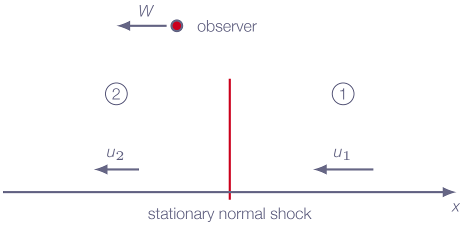
Moving Normal Shock Waves

Chapter 3: stationary normal shock



$u_1 > a_1$	(supersonic flow)
$u_2 < a_2$	(subsonic flow)
$p_2 > p_1$	(sudden compression)
$s_2 > s_1$	(shock loss)

Moving Normal Shock Waves



- Introduce observer moving to the left with speed W
  - if W is constant the observer is still in an inertial system
  - all physical laws are unchanged

- The observer sees a normal shock moving to the right with speed W
  - gas velocity ahead of shock: $u_1' = W - u_1$
  - gas velocity behind shock: $u_2' = W - u_2$

Moving Normal Shock Waves

Now, let $W = u_1 \Rightarrow$

$$u_1' = 0$$

$$u_2' = u_1 - u_2 > 0$$

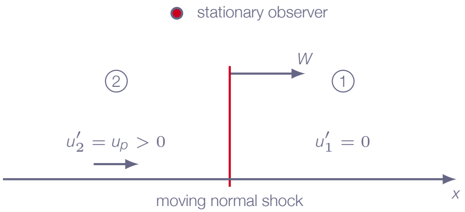
The observer now sees the shock traveling to the right with speed $W = u_1$ into a stagnant gas, leaving a compressed gas ($p_2 > p_1$) with velocity $u_2' > 0$ behind it

Introducing u_p:

$$u_p = u_2' = u_1 - u_2$$

Moving Normal Shock Waves

Analogy:



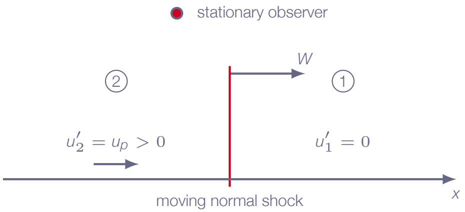
Case 1

- stationary normal shock
- observer moving with velocity W

Case 2

- normal shock moving with velocity W
- stationary observer

Moving Normal Shock Waves - Governing Equations



For stationary normal shocks we have:

$$\rho_1 u_1 = \rho_2 u_2$$
$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$$
$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2$$

With $(u_1 = W)$ and $(u_2 = W - u_p)$ we get:

$$\rho_1 W = \rho_2 (W - u_p)$$
$$\rho_1 W^2 + p_1 = \rho_2 (W - u_p)^2 + p_2$$
$$h_1 + \frac{1}{2}W^2 = h_2 + \frac{1}{2}(W - u_p)^2$$

Moving Normal Shock Waves - Relations

Starting from the governing equations

$$\rho_1 W = \rho_2 (W - u_p)$$

$$\rho_1 W^2 + p_1 = \rho_2 (W - u_p)^2 + p_2$$

$$h_1 + \frac{1}{2} W^2 = h_2 + \frac{1}{2} (W - u_p)^2$$

and using $h = e + \dfrac{p}{\rho}$

it is possible to show that

$$e_2 - e_1 = \frac{p_1 + p_2}{2} \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right)$$

Moving Normal Shock Waves - Relations

$$e_2 - e_1 = \frac{p_1 + p_2}{2}\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

same Hugoniot equation as for stationary normal shock

This means that we will have same shock strength, *i.e.* same jumps in density, velocity, pressure, etc

Moving Normal Shock Waves - Relations

Starting from the Hugoniot equation one can show that

$$\frac{\rho_2}{\rho_1} = \frac{1 + \dfrac{\gamma+1}{\gamma-1}\left(\dfrac{p_2}{p_1}\right)}{\dfrac{\gamma+1}{\gamma-1} + \dfrac{p_2}{p_1}}$$

and

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left[\frac{\dfrac{\gamma+1}{\gamma-1} + \dfrac{p_2}{p_1}}{1 + \dfrac{\gamma+1}{\gamma-1}\left(\dfrac{p_2}{p_1}\right)}\right]$$

Moving Normal Shock Waves - Relations

For calorically perfect gas and stationary normal shock:

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_s^2 - 1)$$

same as eq. (3.57) in Anderson with $M_1 = M_s$

where

$$M_s = \frac{W}{a_1}$$

- M_s is simply the speed of the shock (W), traveling into the stagnant gas, normalized by the speed of sound in this stagnant gas (a_1)
  - $M_s > 1$, otherwise there is no shock!
  - shocks always moves faster than sound - no warning before it hits you ☺

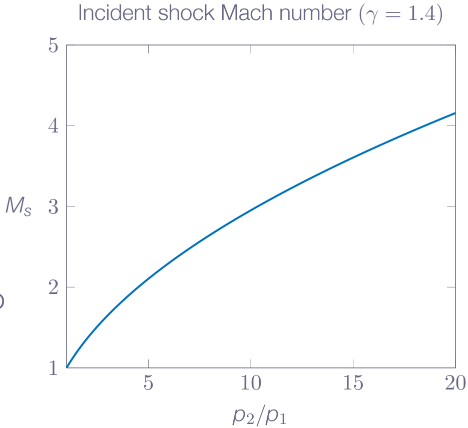
Moving Normal Shock Waves - Relations

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_s^2 - 1)$$

Re-arrange $\Rightarrow$

$$M_s = \sqrt{\frac{\gamma + 1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1}$$

shock speed directly linked to pressure ratio



$$M_s = \frac{W}{a_1} \Rightarrow W = a_1 M_s = a_1\sqrt{\frac{\gamma + 1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1}$$

Moving Normal Shock Waves - Relations

From the continuity equation we get:

$$u_p = W\left(1 - \frac{\rho_1}{\rho_2}\right) > 0$$

After some derivation we obtain:

$$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right]^{1/2}$$

Moving Normal Shock Waves - Relations

Induced Mach number:

$$M_p = \frac{u_p}{a_2} = \frac{u_p}{a_1}\frac{a_1}{a_2} = \frac{u_p}{a_1}\sqrt{\frac{T_1}{T_2}}$$

inserting u_p/a_1 and T_1/T_2 from relations on previous slides we get:

$$M_p = \frac{1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{\gamma-1}{\gamma+1} + \frac{p_2}{p_1}}\right]^{1/2}\left[\frac{1 + \left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right)}{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right) + \left(\frac{p_2}{p_1}\right)^2}\right]^{1/2}$$

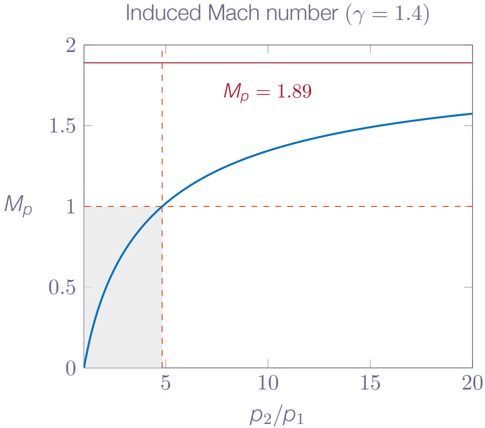
Moving Normal Shock Waves - Relations

Note!

$$\lim_{\frac{p_2}{p_1}\to\infty} M_p \to \sqrt{\frac{2}{\gamma(\gamma-1)}}$$

for air ($\gamma = 1.4$)

$$\lim_{\frac{p_2}{p_1}\to\infty} M_p \to 1.89$$



Moving Normal Shock Waves - Relations

Moving normal shock with $p_2/p_1 = 10$

($p_1 = 1.0\ bar$, $T_1 = 300\ K$, $\gamma = 1.4$)

$\Rightarrow M_s = 2.95$ and $W = 1024.2\ m/s$

The shock is advancing with almost three times the speed of sound!

Behind the shock the induced velocity is $u_p = 756.2\ m/s \Rightarrow$ supersonic flow ($a_2 = 562.1\ m/s$)

May be calculated by formulas 7.13, 7.16, 7.10, 7.11 or by using Table A.2 for stationary normal shock ($u_1 = W$, $u_2 = W - u_p$)

Moving Normal Shock Waves - Relations

Note! $h_{o_1} \neq h_{o_2}$

constant total enthalpy is only valid for stationary shocks!

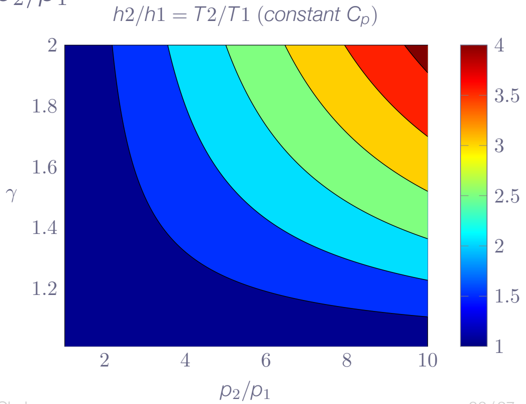
shock is uniquely defined by pressure ratio p_2/p_1

$$u_1 = 0$$

$$h_{o_1} = h_1 + \frac{1}{2}u_1^2 = h_1$$

$$h_{o_2} = h_2 + \frac{1}{2}u_2^2$$

$$h_2 > h_1 \Rightarrow h_{o_2} > h_{o_1}$$



Moving Normal Shock Waves - Relations

Gas/Vapor	**Ratio of specific heats** (γ)	**Gas constant** R
Acetylene	1.23	319
Air (standard)	1.40	287
Ammonia	1.31	530
Argon	1.67	208
Benzene	1.12	100
Butane	1.09	143
Carbon Dioxide	1.29	189
Carbon Disulphide	1.21	120
Carbon Monoxide	1.40	297
Chlorine	1.34	120
Ethane	1.19	276
Ethylene	1.24	296
Helium	1.67	2080
Hydrogen	1.41	4120
Hydrogen chloride	1.41	230
Methane	1.30	518
Natural Gas (Methane)	1.27	500
Nitric oxide	1.39	277
Nitrogen	1.40	297
Nitrous oxide	1.27	180
Oxygen	1.40	260
Propane	1.13	189
Steam (water)	1.32	462
Sulphur dioxide	1.29	130