

# Compressible Flow - TME085

## Lecture 16

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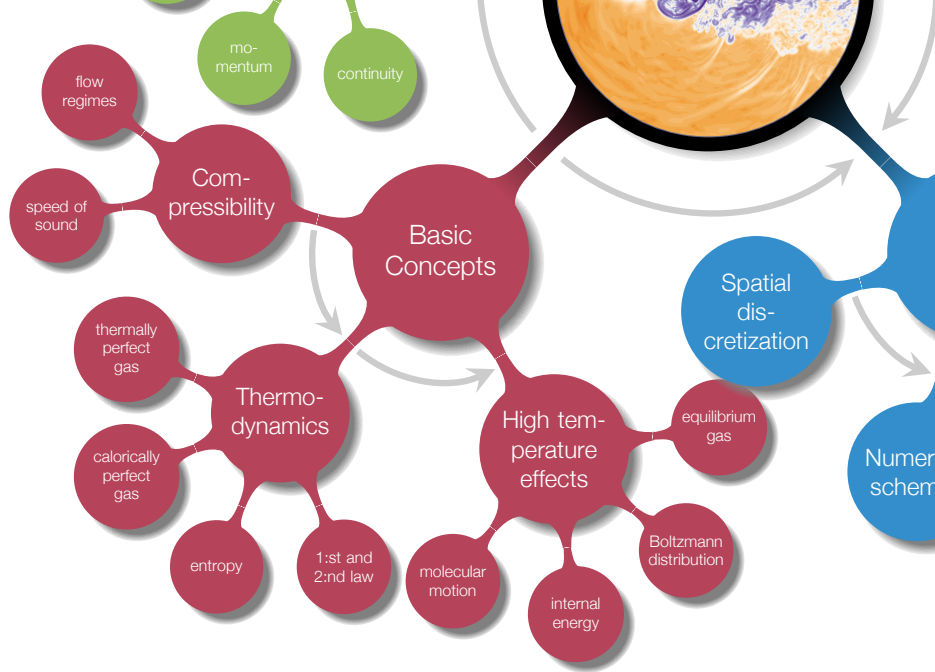
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## Chapter 17 - High-Temperature Flows: Basic Examples

# Overview



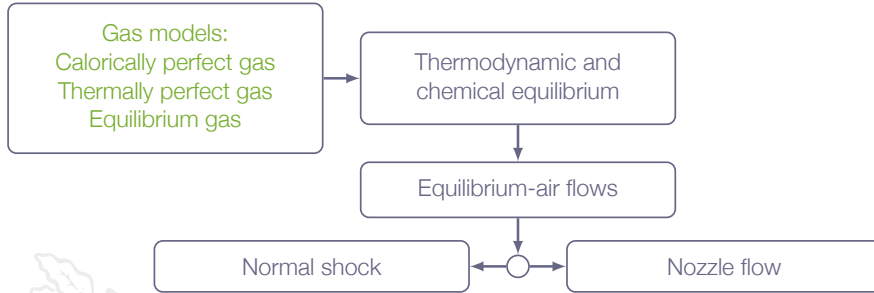
# Learning Outcomes

- 6 **Define** the special cases of calorically perfect gas, thermally perfect gas and real gas and **explain** the implication of each of these special cases
- 8 **Derive** (marked) and **apply** (all) of the presented mathematical formulae for classical gas dynamics
  - b normal shocks\*
  - i detached blunt body shocks, nozzle flows

*How does increased temperature affect a compressible flow?*



# Roadmap - High Temperature Effects



# Motivation

High-temperature effects can be rather dramatic

We will examine a couple of flow situations where the temperature is high enough to effect the flow properties significantly in order to get e feeling for high-temperature flows



# Properties of High - Temperature Gases

## Applications:

Rocket nozzle flows

Reentry vehicles

Shock tubes / Shock tunnels

Internal combustion engines

Gasturbines



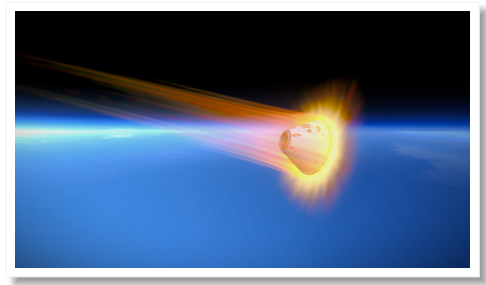
# Properties of High - Temperature Gases

Example: Reentry vehicle

Mach number: 32.5

Gas: air

Temperature:  $T_{\infty} = 283$



# Properties of High - Temperature Gases

Example: Reentry vehicle

Assume calorically perfect gas

Normal shock relations gives  
 $T/T_{\infty} = 206$

$$T_{\infty} = 283 \Rightarrow T = 58\,300 \text{ K}$$



# Properties of High - Temperature Gases

Example: Reentry vehicle

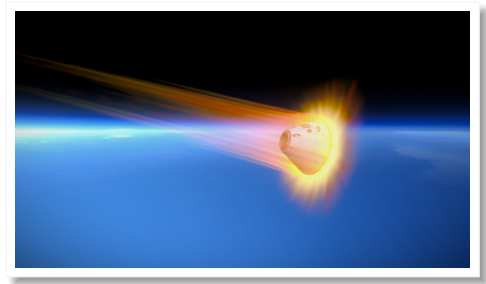
Assume calorically perfect gas

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 $T/T_{\infty} = 206$

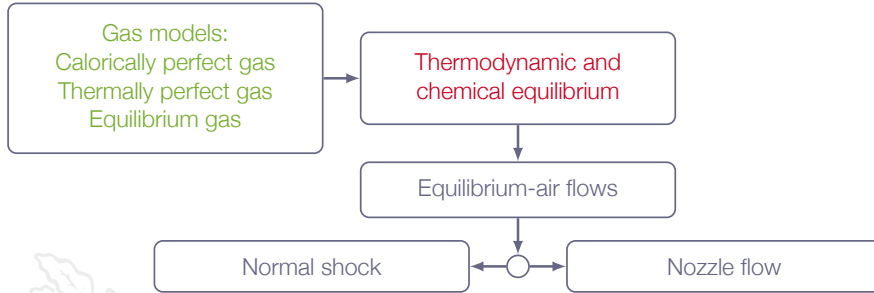
$$T_{\infty} = 283 \Rightarrow T = 58\,300 \text{ K}$$

A more correct value is  $T = 11\,600 \text{ K}$

**Something is fishy here!**



# Roadmap - High Temperature Effects



# Chapter 17.1

## Thermodynamic and Chemical Equilibrium





# Thermodynamic Equilibrium

Molecules are distributed among their possible energy states according to the **Boltzmann distribution** (which is a **statistical equilibrium**) for the given temperature of the gas

extremely fast process (time and length scales of the molecular processes)

much faster than flow time scales in general (not true inside shocks)



# Thermodynamic Equilibrium

Global thermodynamic equilibrium:

*"true thermodynamic equilibrium"*

there are no gradients of  $p$ ,  $T$ ,  $\rho$  (or flow velocity, species concentrations, ... )

Local thermodynamic equilibrium:

gradients can be neglected locally

this requirement is fulfilled in most cases (hard not to get)

# Chemical Equilibrium

**Composition** of gas (species concentrations) is **fixed in time**

forward and backward rates of all chemical reactions are equal

zero net reaction rates

chemical reactions may be either slow or fast in comparison to flow time scale depending on the case studied



# Chemical Equilibrium

## Global chemical equilibrium:

there are no gradients of species concentrations

together with global thermodynamic equilibrium  $\Rightarrow$   
all gradients are zero

## Local chemical equilibrium

gradients of species concentrations can be neglected locally

not always true - depends on reaction rates and flow time scales

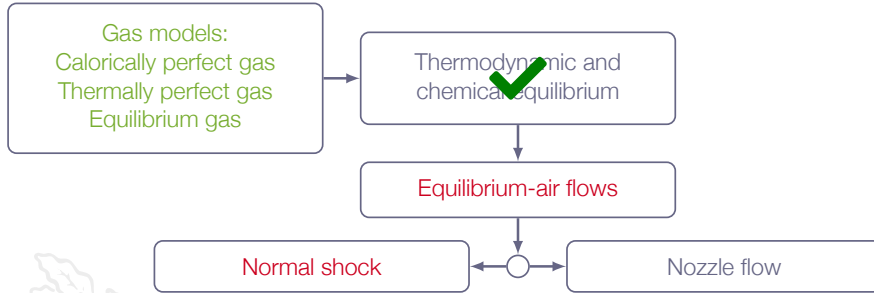
# Thermodynamic and Chemical Equilibrium

Most common cases:

	Thermodynamic Equilibrium	Chemical Equilibrium	Gas Model
1	local thermodynamic equilibrium	local chemical equilibrium	equilibrium gas
2	local thermodynamic equilibrium	chemical non-equilibrium	finite rate chemistry
3	local thermodynamic equilibrium	frozen composition	frozen flow
4	thermodynamic non-equilibrium	frozen composition	vibrationally frozen flow

length and time scales of flow decreases from 1 to 4

# Roadmap - High Temperature Effects



# Chapter 17.2

## Equilibrium Normal Shock Wave Flows



# Equilibrium Normal Shock Wave Flows

Question:

Is the **equilibrium gas** assumption OK for normal shocks?

Answer:

for **hypersonic** flows with very **little ionization** in the shock region, it is a fair approximation

not perfect, since the assumption of **local thermodynamic** and **chemical equilibrium** is not really true around the shock

however, it gives a significant improvement compared to the calorically perfect gas assumption



# Equilibrium Normal Shock Wave Flows

Basic relations (for all gases), stationary normal shock:

$$\left\{ \begin{array}{l} \rho_1 u_1 = \rho_2 u_2 \\ \rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2 \\ h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2} u_2^2 \end{array} \right.$$

For equilibrium gas we have:

$$\left\{ \begin{array}{l} \rho = \rho(p, h) \\ T = T(\rho, h) \end{array} \right.$$

(we are free to choose any two states as independent variables)

# Equilibrium Normal Shock Wave Flows

Assume that  $\rho_1$ ,  $u_1$ ,  $p_1$ ,  $T_1$ , and  $h_1$  are known

$$u_2 = \frac{\rho_1 u_1}{\rho_2} \Rightarrow \rho_1 u_1^2 + p_1 = \rho_2 \left( \frac{\rho_1}{\rho_2} u_1 \right)^2 + p_2 \Rightarrow$$

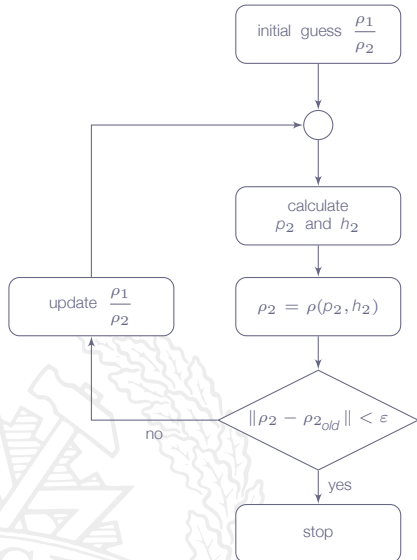
$$p_2 = p_1 + \rho_1 u_1^2 \left( 1 - \frac{\rho_1}{\rho_2} \right)$$

Also

$$h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2} \left( \frac{\rho_1}{\rho_2} u_1 \right)^2 \Rightarrow$$

$$h_2 = h_1 + \frac{1}{2} u_1^2 \left( 1 - \left( \frac{\rho_1}{\rho_2} \right)^2 \right)$$

# Equilibrium Normal Shock Wave Flows



when converged:

$$\left. \begin{aligned} \rho_2 &= \rho(p_2, h_2) \\ T_2 &= T(\rho_2, h_2) \end{aligned} \right\} \Rightarrow$$

$\rho_2, u_2, p_2, T_2, h_2$  known

# Equilibrium Air - Normal Shock

Tables of thermodynamic properties for different conditions are available

For a very strong shock case ( $M_1 = 32$ ), the table below shows results for equilibrium air

	calorically perfect gas ( $\gamma = 1.4$ )	equilibrium air
$p_2/p_1$	1233	1387
$\rho_2/\rho_1$	5.97	15.19
$h_2/h_1$	206.35	212.80
$T_2/T_1$	206.35	41.64

# Equilibrium Air - Normal Shock

## Analysis:

Pressure ratio is comparable

Density ratio differs by factor of 2.5

Temperature ratio differs by factor of 5



# Equilibrium Air - Normal Shock

## Explanation:

Using equilibrium gas means that vibration, dissociation and chemical reactions are accounted for

The chemical reactions taking place in the shock region lead to an **absorption of energy** into chemical energy

drastically reducing the temperature downstream of the shock

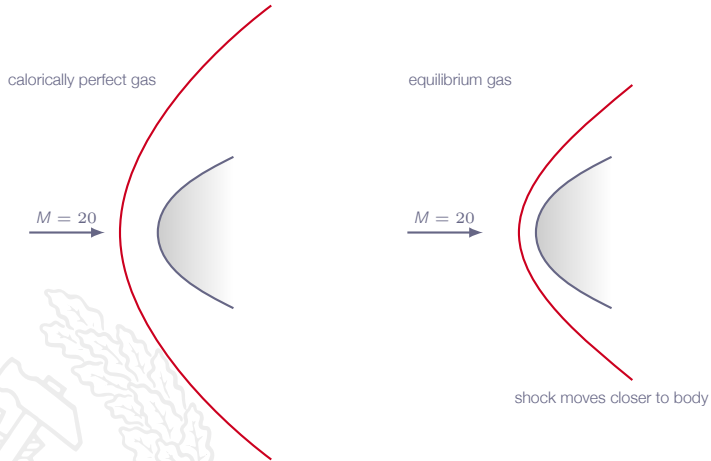
this also explains the difference in density after the shock

# Equilibrium Air - Normal Shock

Additional notes:

1. For a normal shock in an **equilibrium gas**, the pressure ratio, density ratio, enthalpy ratio, temperature ratio, etc all depend on **three upstream variables**, e.g.  $u_1, p_1, T_1$
2. For a normal shock in a **thermally perfect gas**, the pressure ratio, density ratio, enthalpy ratio, temperature ratio, etc all depend on **two upstream variables**, e.g.  $M_1, T_1$
3. For a normal shock in a **calorically perfect gas**, the pressure ratio, density ratio, enthalpy ratio, temperature ratio, etc all depend on **one upstream variable**, e.g.  $M_1$

# Equilibrium Gas - Detached Shock



What's the reason for the difference in predicted shock position?



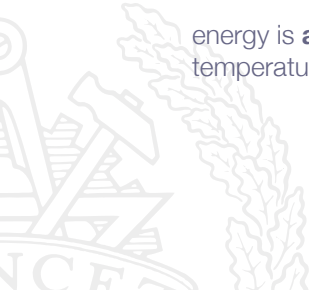
# Equilibrium Gas - Detached Shock

Calorically perfect gas:

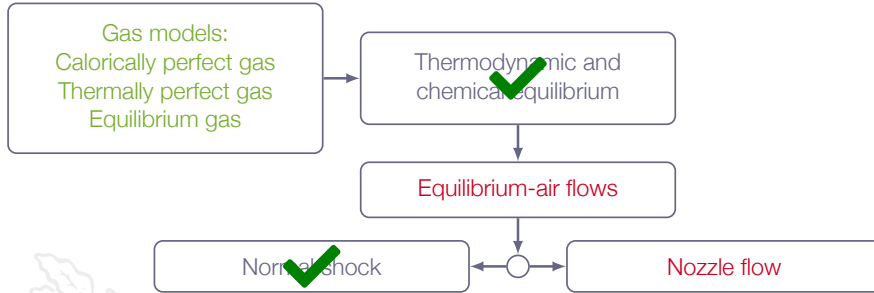
all energy ends up in translation and rotation  $\Rightarrow$  increased temperature

Equilibrium gas:

energy is **absorbed by reactions**  $\Rightarrow$  does not contribute to the increase of gas temperature

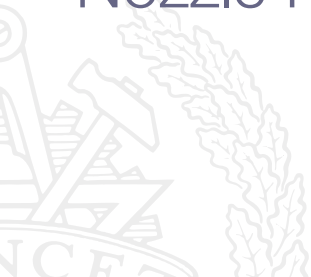


# Roadmap - High Temperature Effects



# Chapter 17.3

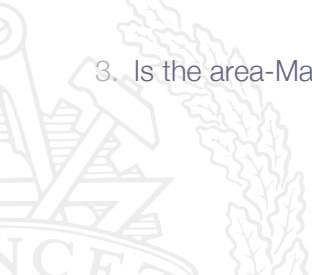
## Equilibrium Quasi-One-Dimensional Nozzle Flows



# Equilibrium Quasi-1D Nozzle Flows

For a chemically reacting gas at high temperature:

1. Assuming inviscid and adiabatic flow, is the flow isentropic?
2. Can we use the area-velocity relation?
3. Is the area-Mach-number relation valid?



# Equilibrium Quasi-1D Nozzle Flows - Isentropic Flow

First question:

Is a flow of a chemically reacting gas isentropic (*assuming inviscid and adiabatic flow*)?

entropy equation:  $Tds = dh - \nu dp$

momentum equation:  $dp = -\rho u du$

energy equation:  $dh + u du = 0$

**Note!** The momentum and energy equations are the inviscid adiabatic quasi-1D equations on differential form (*valid for all gases*).

# Equilibrium Quasi-1D Nozzle Flows - Isentropic Flow

momentum equation:  $dp = -\rho u du \Rightarrow u du = -\frac{dp}{\rho} = -v dp$

energy equation:  $dh + u du = 0 \Rightarrow dh = -u du$

entropy equation:  $T ds = dh - v dp = -u du + u du = 0 \Rightarrow ds = 0$

**Isentropic flow!**

# Equilibrium Quasi-1D Nozzle Flows - Area-Velocity Relation

Second question:

Can we use the area-velocity relation for a chemically reacting gas?

The area-velocity relation was derived from the quasi-1D formulation of the governing equations assuming isentropic flow

continuity equation:  $d(\rho u A) = 0$

momentum equation:  $dp = -\rho u du$

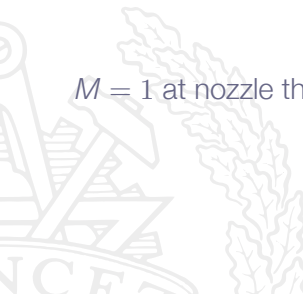
energy equation:  $dh + u du = 0$

# Equilibrium Quasi-1D Nozzle Flows - The Area-Velocity Relation

No assumption about the gas is made in the derivation, which means that we can use the area-velocity relation for a flow of chemically reacting gas

$$\frac{dA}{A} = (M^2 - 1) \frac{du}{u}$$

$M = 1$  at nozzle throat still holds





# Equilibrium Quasi-1D Nozzle Flows - The Area-Mach Relation

Third question:

Is the area-Mach number relation valid for a chemically reacting gas?

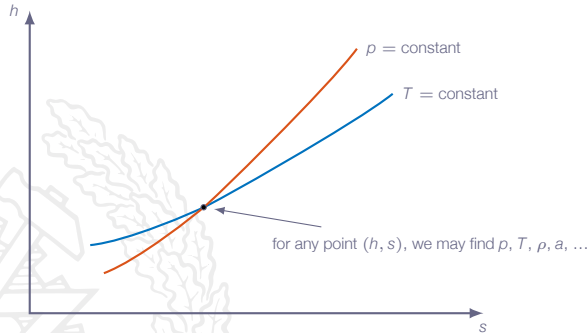
In the derivation of the **area-Mach number relation**, calorically perfect gas is assumed and thus the relation is **not valid for a chemically reacting gas**



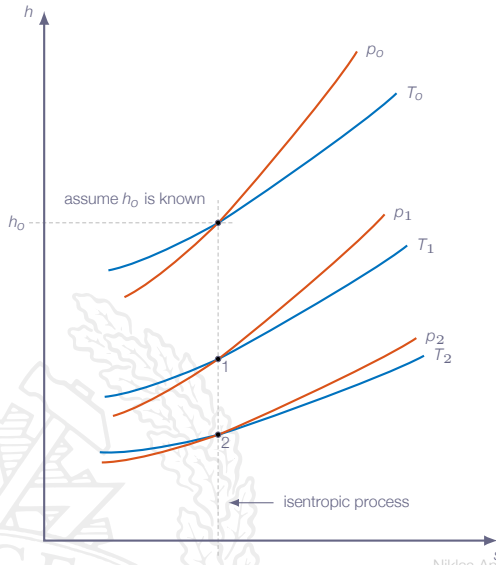
# Equilibrium Quasi-1D Nozzle Flows

For general gas mixture in thermodynamic and chemical equilibrium, we may find tables or graphs describing relations between state variables.

Example: Mollier diagram



# Equilibrium Quasi-1D Nozzle Flows



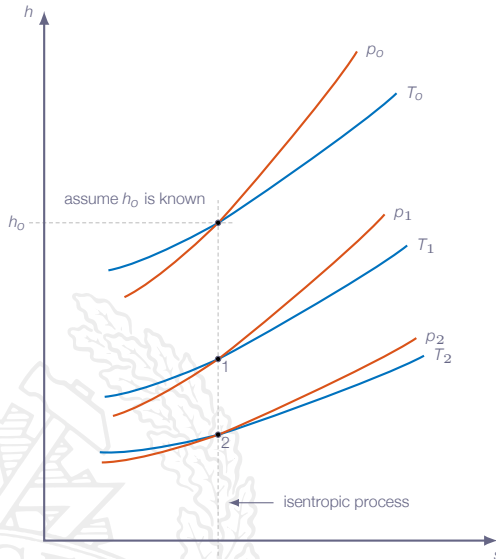
The energy equation for steady-state inviscid adiabatic nozzle flow:

$$dh_o = 0 \Rightarrow$$

$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2 = h_o$$

where  $h_o$  is the reservoir enthalpy.

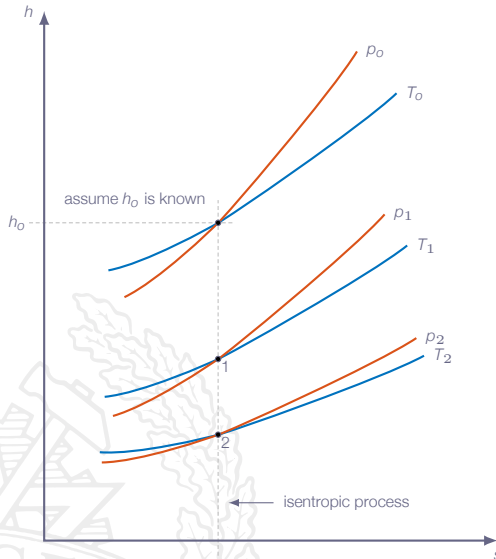
# Equilibrium Quasi-1D Nozzle Flows



The velocity at point 1 can be obtained as:

$$\frac{1}{2}u_1^2 = h_o - h_1 \Rightarrow u_1 = \sqrt{2(h_o - h_1)}$$

# Equilibrium Quasi-1D Nozzle Flows

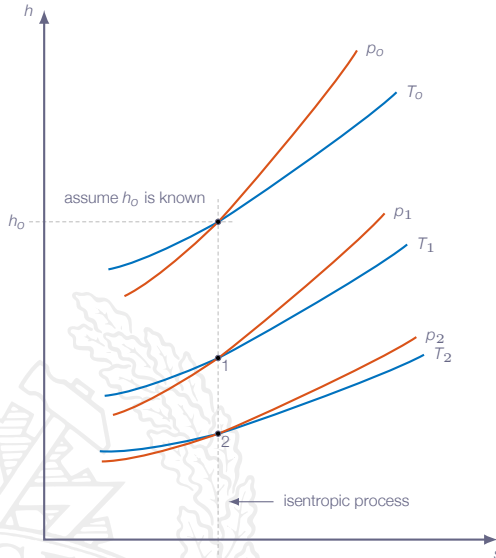


At any point along the isentropic line

$$\frac{1}{2}u^2 = h_o - h \Rightarrow u = \sqrt{2(h_o - h)}$$

$T, \rho, \rho, a$  are given by the diagram

# Equilibrium Quasi-1D Nozzle Flows



The continuity equation gives  $\rho u A = \text{const}$

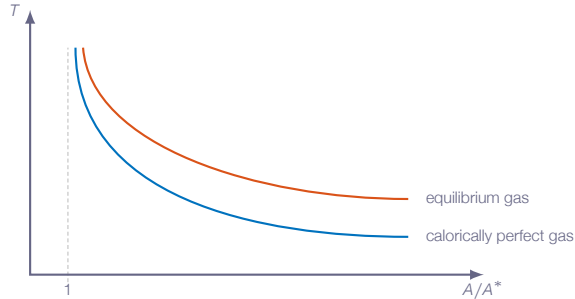
$$\rho u A = \rho^* a^* A^* \Rightarrow \frac{A}{A^*} = \frac{\rho^* a^*}{\rho u}$$

Thus,  $A/A^*$  may be computed for any point along isentropic line

# Equilibrium Quasi-1D Nozzle Flows

Equilibrium gas gives higher  $T$  and more thrust than calorically perfect gas

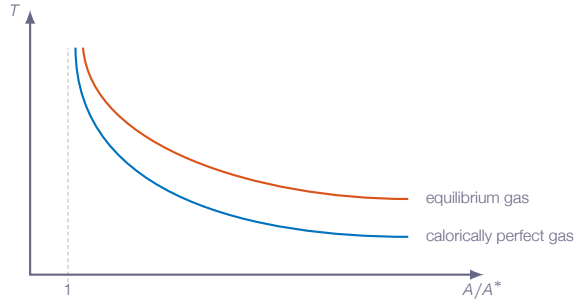
During the expansion chemical **energy is released** due to shifts in the equilibrium composition



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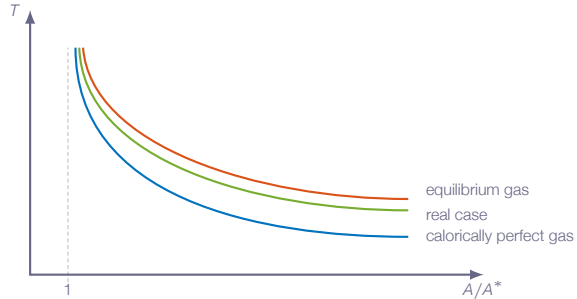


Chemical and vibrational energy transferred to **translation** and **rotation**  $\Rightarrow$  increased temperature



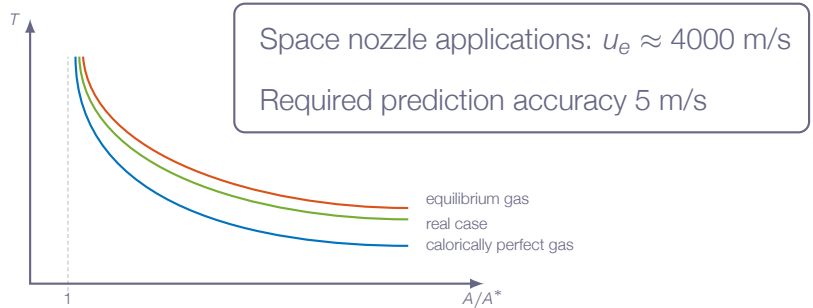
# Equilibrium Quasi-1D Nozzle Flows - Reacting Mixture

Real nozzle flow with reacting gas mixture:



# Equilibrium Quasi-1D Nozzle Flows - Reacting Mixture

Real nozzle flow with reacting gas mixture:



# Large Nozzles

High  $T_o$ , high  $p_o$ , high reactivity

very **fast chemical reactions**

local thermodynamic and chemical equilibrium



# Large Nozzles

Real case is close to **equilibrium gas** results

Example: Ariane 5 launcher, main engine (Vulcain 2)

Chemical reactions:  $H_2 + O_2 \rightarrow H_2O$  (in principle),  
but many different radicals and reactions involved  
(at least 10 species and 20 reactions)

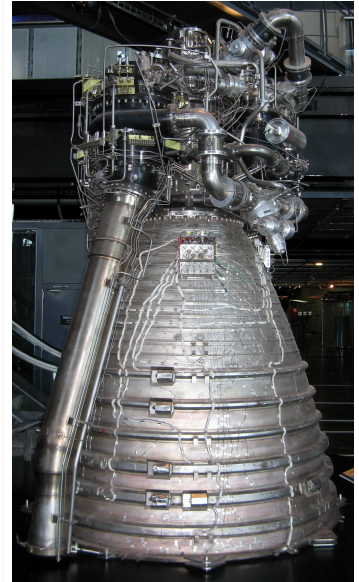
Nozzle inlet conditions:

$$T_o \sim 3600 \text{ K}$$

$$p_o \sim 120 \text{ bar}$$

Length scale  $\sim$  a few meters

Gas mixture is quite close to equilibrium  
conditions all the way through the expansion



# Small Nozzles

Real case is close to **frozen flow** results

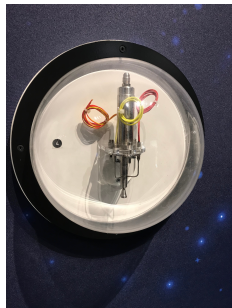
Example: nozzles used on satellites for maneuvering, orbital adjustments, etc

Low  $T_o$ , low  $p_o$

Low reactivity (compared to the flow in the previous example)



# Small Nozzles



# Small Nozzle With High-Speed Flow

High-speed flows (short flow time scales)  $\Rightarrow$  **thermodynamic non-equilibrium**

Very slow (or no) chemical reactions  $\Rightarrow$  **frozen composition**

The residence time is too short for the vibrational energy of the molecules to change  $\Rightarrow$  **Vibrationally frozen flow**

Only translational and rotational energy  $\Rightarrow$  **Calorically perfect gas!**

# Roadmap - High Temperature Effects

