Compressible Flow - TME085

Lecture 12

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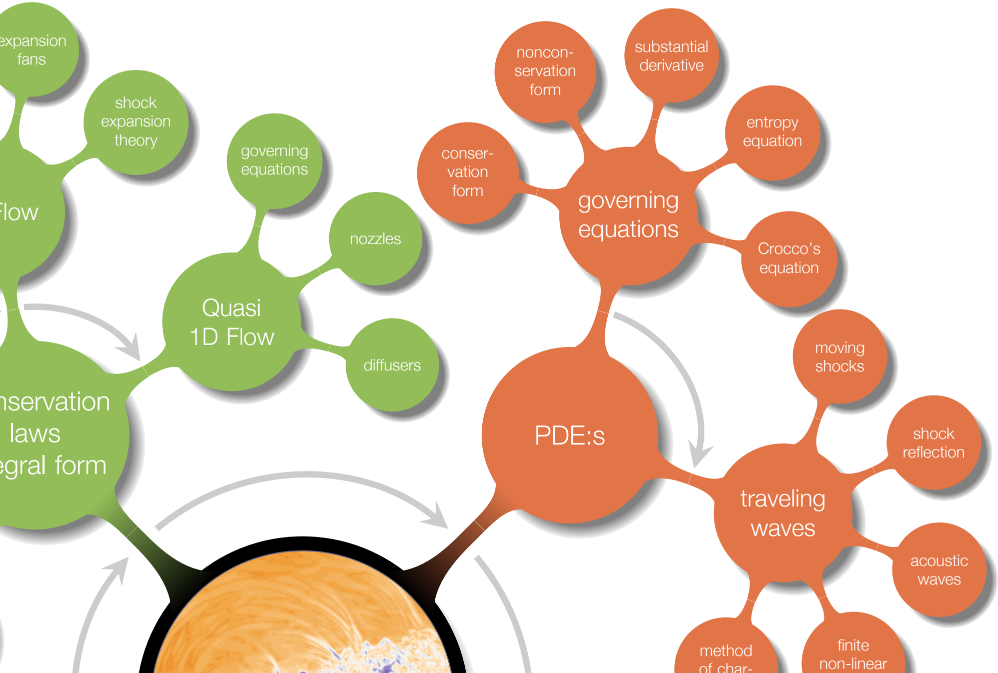
Chapter 7
Unsteady Wave Motion

Overview



expansion fans
shock expansion theory
Flow
nservation laws
egral form
governing equations
nozzles
Quasi 1D Flow
diffusers
noncon-servation form
substantial derivative
entropy equation
conser-vation form
governing equations
Crocco's equation
PDE:s
moving shocks
shock reflection
traveling waves
acoustic waves
method of char-
finite non-linear

Learning Outcomes

3 Describe typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)

4 Present at least two different formulations of the governing equations for compressible flows and explain what basic conservation principles they are based on

8 Derive (marked) and apply (all) of the presented mathematical formulae for classical gas dynamics

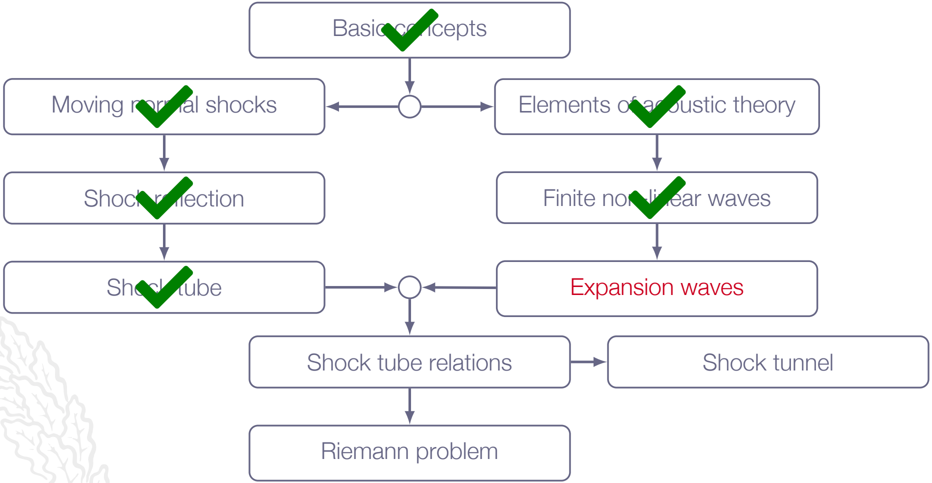
   a 1D isentropic flow*
   b normal shocks*
   j unsteady waves and discontinuities in 1D
   k basic acoustics

9 Solve engineering problems involving the above-mentioned phenomena (8a-8k)

11 Explain how the equations for aero-acoustics and classical acoustics are derived as limiting cases of the compressible flow equations

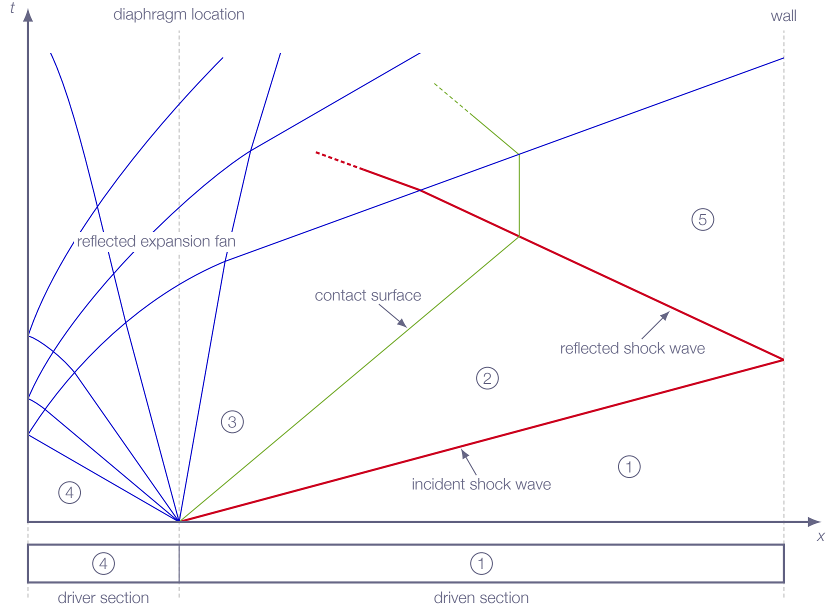
moving normal shocks - frame of reference seems to be the key here?!

Roadmap - Unsteady Wave Motion



Chapter 7.7
Incident and Reflected Expansion Waves

Expansion Waves



Expansion Waves

Properties of a left-running expansion wave

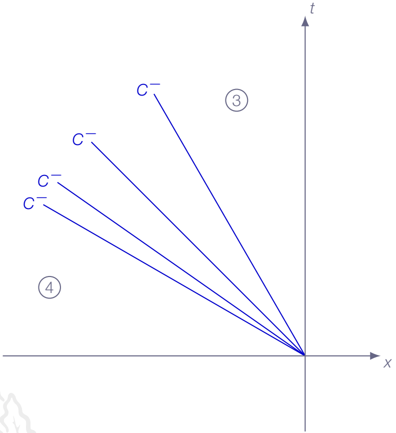
1. All flow properties are constant along C^- characteristics
2. The wave head is propagating into region 4 (high pressure)
3. The wave tail defines the limit of region 3 (lower pressure)
4. Regions 3 and 4 are assumed to be constant states

For calorically perfect gas:

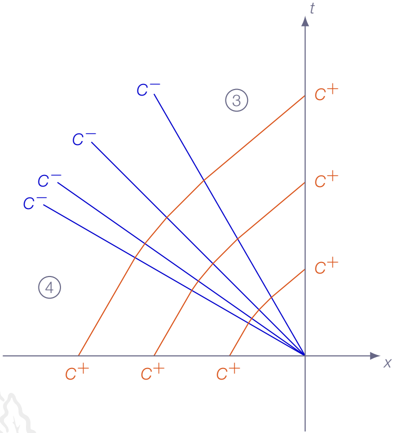
$$J^+ = u + \frac{2a}{\gamma - 1}$$ is constant along C^+ lines

$$J^- = u - \frac{2a}{\gamma - 1}$$ is constant along C^- lines

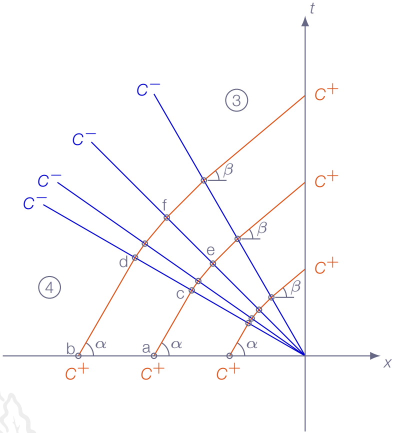
Expansion Waves



Expansion Waves



Expansion Waves



constant flow properties in region 4: $J_a^+ = J_b^+$

J^+ invariants constant along C^+ characteristics:

$$J_a^+ = J_c^+ = J_e^+$$

$$J_b^+ = J_d^+ = J_f^+$$

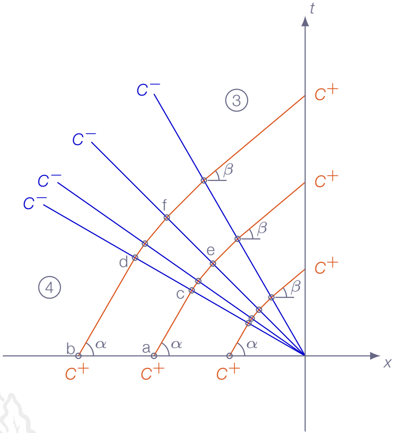
since $J_a^+ = J_b^+$ this also implies $J_e^+ = J_f^+$

J^- invariants constant along C^- characteristics:

$$J_c^- = J_d^-$$

$$J_e^- = J_f^-$$

Expansion Waves



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$$J_a^+ = J_c^+ = J_e^+$$

$$J_b^+ = J_d^+ = J_f^+$$

since $J_a^+ = J_b^+$ this also implies $J_e^+ = J_f^+$

J^- invariants constant along C^- characteristics:

$$J_c^- = J_d^-$$

$$J_e^- = J_f^-$$

$$u_e = \frac{1}{2}(J_e^+ + J_e^-),\; u_f = \frac{1}{2}(J_f^+ + J_f^-),\; \Rightarrow u_e = u_f$$

$$a_e = \frac{\gamma - 1}{4}(J_e^+ - J_e^-),\; a_f = \frac{\gamma - 1}{4}(J_f^+ - J_f^-),\; \Rightarrow a_e = a_f$$

Expansion Waves

Along each C^- line u and a are constants which means that

$$\frac{dx}{dt} = u - a = const$$

C^- characteristics are straight lines in xt-space

Expansion Waves

The start and end conditions are the same for all C^+ lines

J^+ invariants have the same value for all C^+ characteristics

C^- characteristics are straight lines in xt-space

Simple expansion waves centered at $(x, t) = (0, 0)$

Expansion Waves

In a left-running expansion fan:

- J^+ is constant throughout expansion fan, which implies:

$$u + \frac{2a}{\gamma - 1} = u_4 + \frac{2a_4}{\gamma - 1} = u_3 + \frac{2a_3}{\gamma - 1}$$

- J^- is constant along C^- lines, but varies from one line to the next, which means that

$$u - \frac{2a}{\gamma - 1}$$

is constant along each C^- line

Expansion Waves

Since $u_4 = 0$ we obtain:

$$u + \frac{2a}{\gamma - 1} = u_4 + \frac{2a_4}{\gamma - 1} = \frac{2a_4}{\gamma - 1} \Rightarrow$$

$$\frac{a}{a_4} = 1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}$$

with $a = \sqrt{\gamma RT}$ we get

$$\frac{T}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}\right]^2$$

Expansion Wave Relations

Isentropic flow $\Rightarrow$ we can use the isentropic relations

complete description in terms of u/a_4

$$\frac{T}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}\right]^2$$

$$\frac{p}{p_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}\right]^{\frac{2\gamma}{\gamma-1}}$$

$$\frac{\rho}{\rho_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}\right]^{\frac{2}{\gamma-1}}$$

Expansion Wave Relations

Since C^{-} characteristics are straight lines, we have:

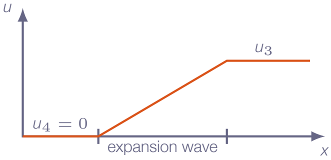
$$\frac{dx}{dt} = u - a \Rightarrow x = (u - a)t$$

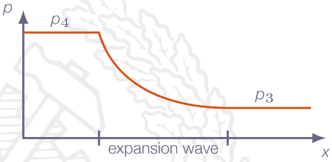
$$\frac{a}{a_4} = 1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4} \Rightarrow a = a_4 - \frac{1}{2}(\gamma - 1)u \Rightarrow$$

$$x = \left[u - a_4 + \frac{1}{2}(\gamma - 1)u\right] t = \left[\frac{1}{2}(\gamma - 1)u - a_4\right] t \Rightarrow$$

$$u = \frac{2}{\gamma + 1}\left[a_4 + \frac{x}{t}\right]$$

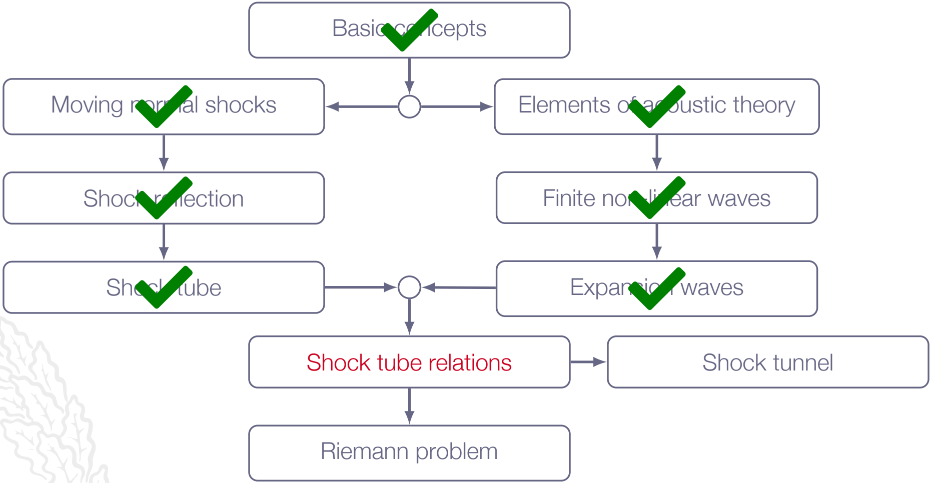
Expansion Wave Relations





- Expansion wave head is advancing to the left with speed a_4 into the stagnant gas
- Expansion wave tail is advancing with speed $u_3 - a_3$, which may be positive or negative, depending on the initial states

Roadmap - Unsteady Wave Motion



Chapter 7.8
Shock Tube Relations

Shock Tube Relations

$$u_p = u_2 = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\dfrac{2\gamma_1}{\gamma_1+1}}{\dfrac{p_2}{p_1} + \dfrac{\gamma_1-1}{\gamma_1+1}}\right]^{1/2}$$

$$\frac{p_3}{p_4} = \left[1 - \frac{\gamma_4-1}{2}\left(\frac{u_3}{a_4}\right)\right]^{2\gamma_4/(\gamma_4-1)}$$

solving for u_3 gives

$$u_3 = \frac{2a_4}{\gamma_4-1}\left[1 - \left(\frac{p_3}{p_4}\right)^{(\gamma_4-1)/(2\gamma_4)}\right]$$

Shock Tube Relations

But, $p_3 = p_2$ and $u_3 = u_2$ (no change in velocity and pressure over contact discontinuity)

$$\Rightarrow u_2 = \frac{2a_4}{\gamma_4 - 1}\left[1 - \left(\frac{p_2}{p_4}\right)^{(\gamma_4-1)/(2\gamma_4)}\right]$$

We have now two expressions for u_2 which gives us

$$\frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma_1}{\gamma_1+1}}{\frac{p_2}{p_1} + \frac{\gamma_1-1}{\gamma_1+1}}\right]^{1/2} = \frac{2a_4}{\gamma_4 - 1}\left[1 - \left(\frac{p_2}{p_4}\right)^{(\gamma_4-1)/(2\gamma_4)}\right]$$

Shock Tube Relations

Rearranging gives:

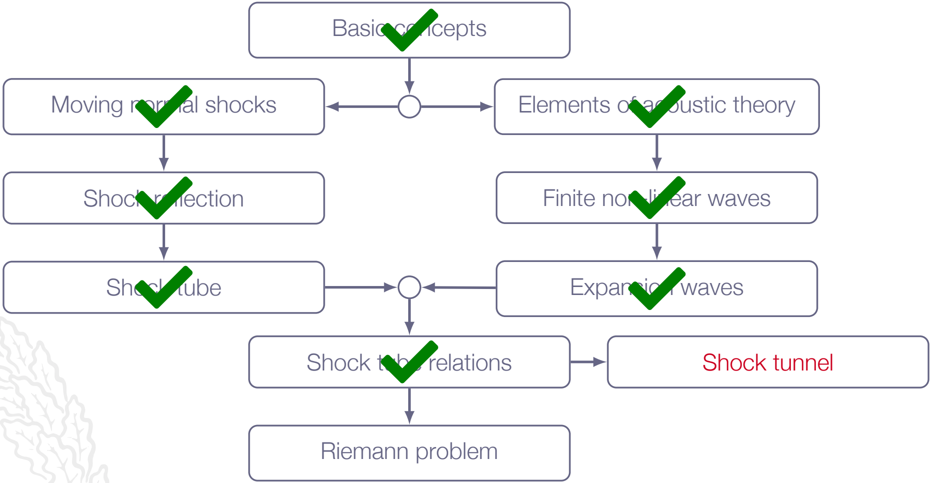
$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left\{1 - \frac{(\gamma_4 - 1)(a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma_1\left[2\gamma_1 + (\gamma_1 + 1)(p_2/p_1 - 1)\right]}}\right\}^{-2\gamma_4/(\gamma_4-1)}$$

- p_2/p_1 as implicit function of p_4/p_1
- for a given p_4/p_1, p_2/p_1 will increase with decreased a_1/a_4

$$a = \sqrt{\gamma R T} = \sqrt{\gamma (R_u/M) T}$$

- the speed of sound in a light gas is higher than in a heavy gas
  - driver gas: low molecular weight, high temperature
  - driven gas: high molecular weight, low temperature

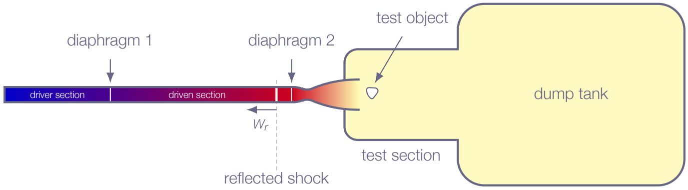
Roadmap - Unsteady Wave Motion



Shock Tunnel

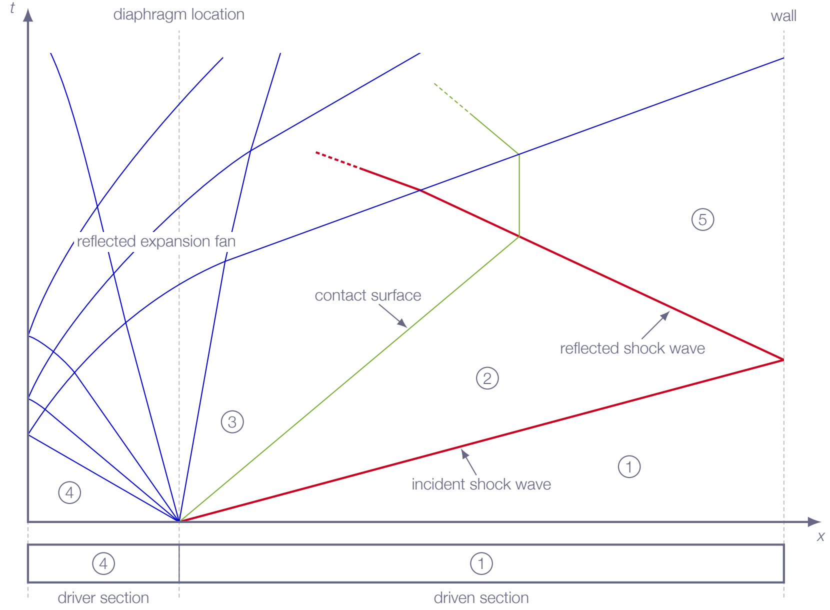
- Addition of a convergent-divergent nozzle to a shock tube configuration
- Capable of producing flow conditions which are close to those during the reentry of a space vehicles into the earth's atmosphere
  - high-enthalpy, hypersonic flows (short time)
  - real gas effects
- Example - Aachen TH2:
  - velocities up to 4 km/s
  - stagnation temperatures of several thousand degrees

Shock Tunnel



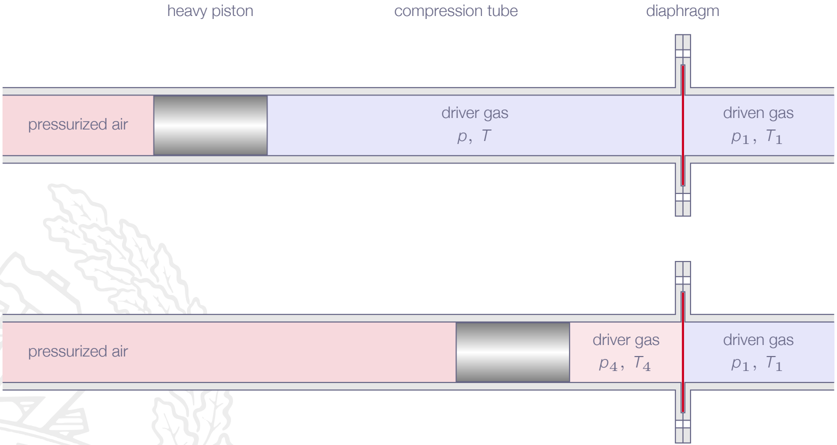
1. High pressure in region 4 (driver section)
   - diaphragm 1 burst
   - primary shock generated
2. Primary shock reaches end of shock tube
   - shock reflection
3. High pressure in region 5
   - diaphragm 2 burst
   - nozzle flow initiated
   - hypersonic flow in test section

Shock Tunnel

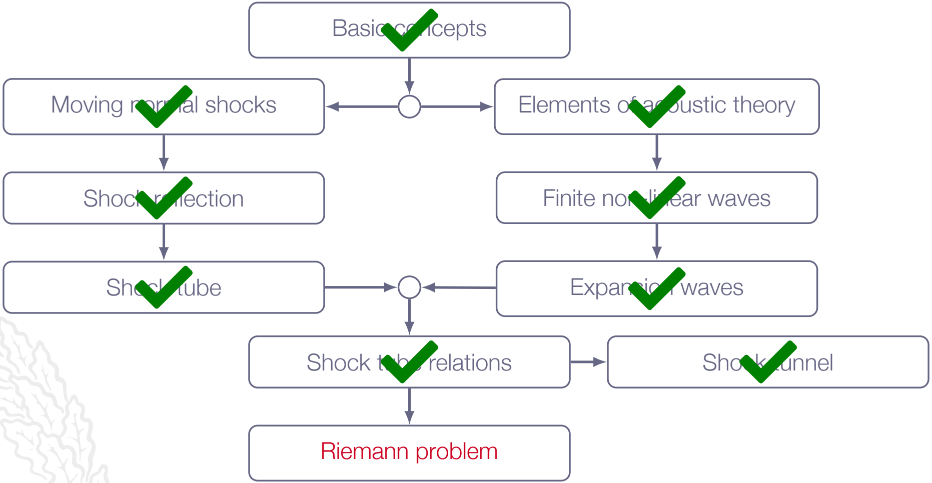


Shock Tunnel

By adding a compression tube to the shock tube a very high p_4 and T_4 may be achieved for any gas in a fairly simple manner



Roadmap - Unsteady Wave Motion



Riemann Problem

The shock tube problem is a special case of the general Riemann Problem

> *"... A Riemann problem, named after Bernhard Riemann, consists of an initial value problem composed by a conservation equation together with piece-wise constant data having a single discontinuity ..."*

Wikipedia

Riemann Problem

May show that solutions to the shock tube problem have the general form:

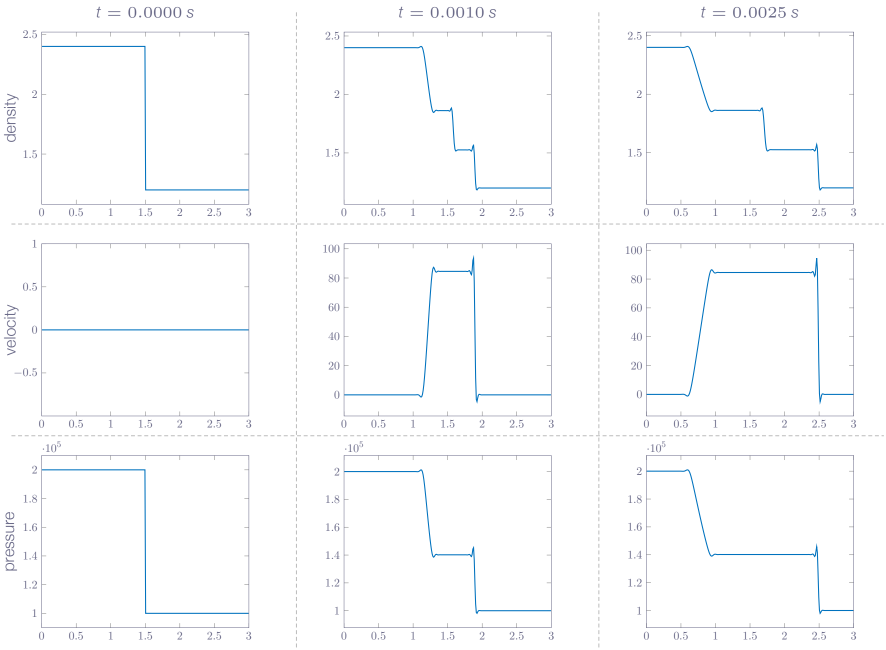
$$p = p(x/t)$$
$$\rho = \rho(x/t)$$
$$u = u(x/t)$$
$$T = T(x/t)$$
$$a = a(x/t)$$

where $x = 0$ denotes the position of the initial jump between states 1 and 4

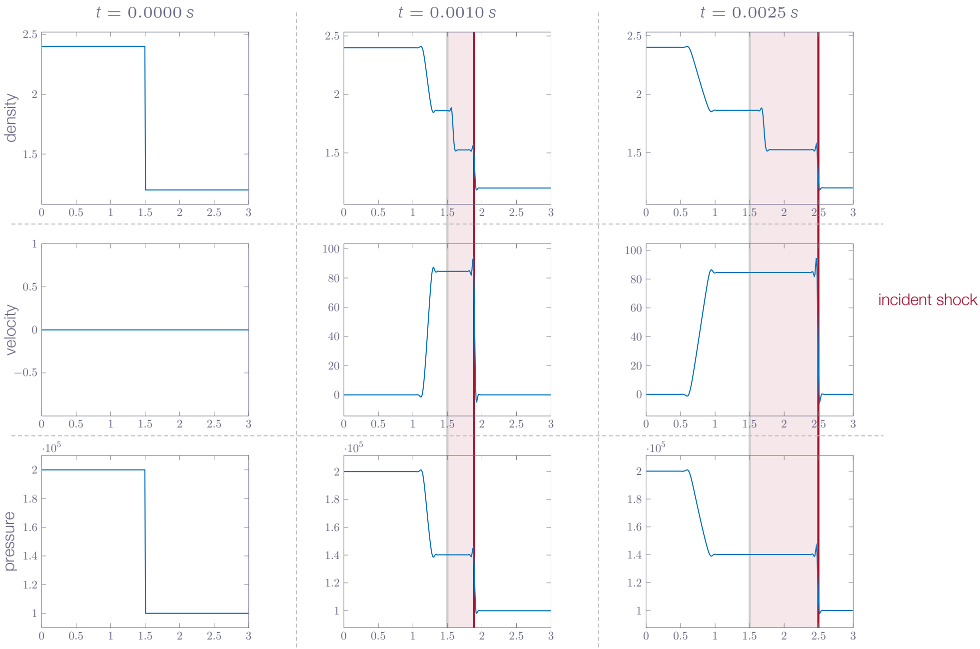
Riemann Problem - Shock Tube

Shock tube simulation:

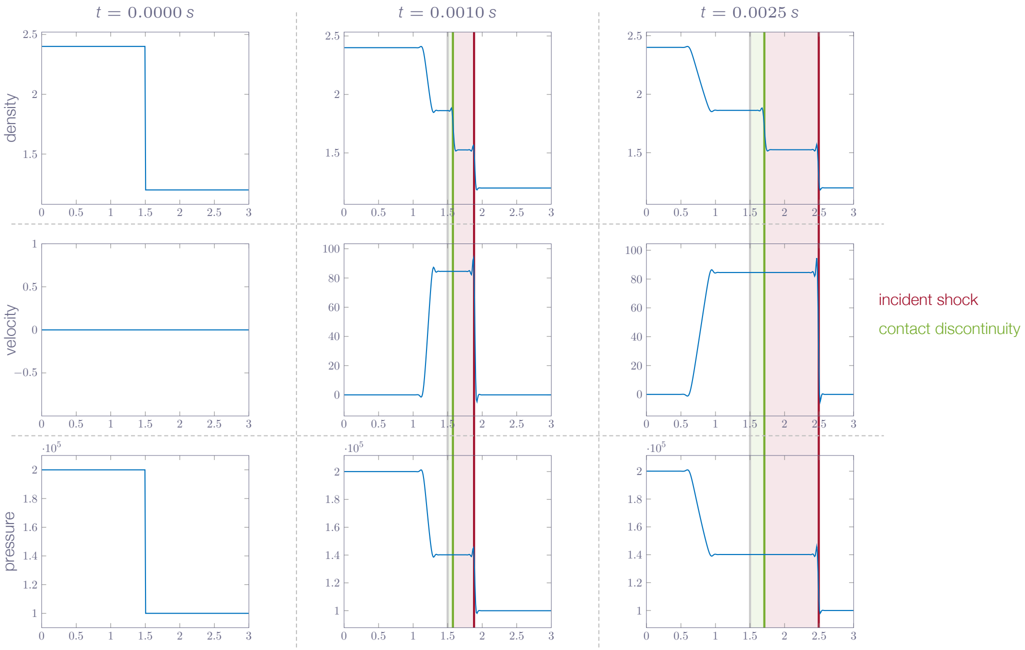
- left side conditions (state 4):
  - $\rho = 2.4\ kg/m^3$
  - $u = 0.0\ m/s$
  - $p = 2.0\ bar$
- right side conditions (state 1):
  - $\rho = 1.2\ kg/m^3$
  - $u = 0.0\ m/s$
  - $p = 1.0\ bar$
- Numerical method
  - Finite-Volume Method (FVM) solver
  - three-stage Runge-Kutta time stepping
  - third-order characteristic upwinding scheme
  - local artificial damping



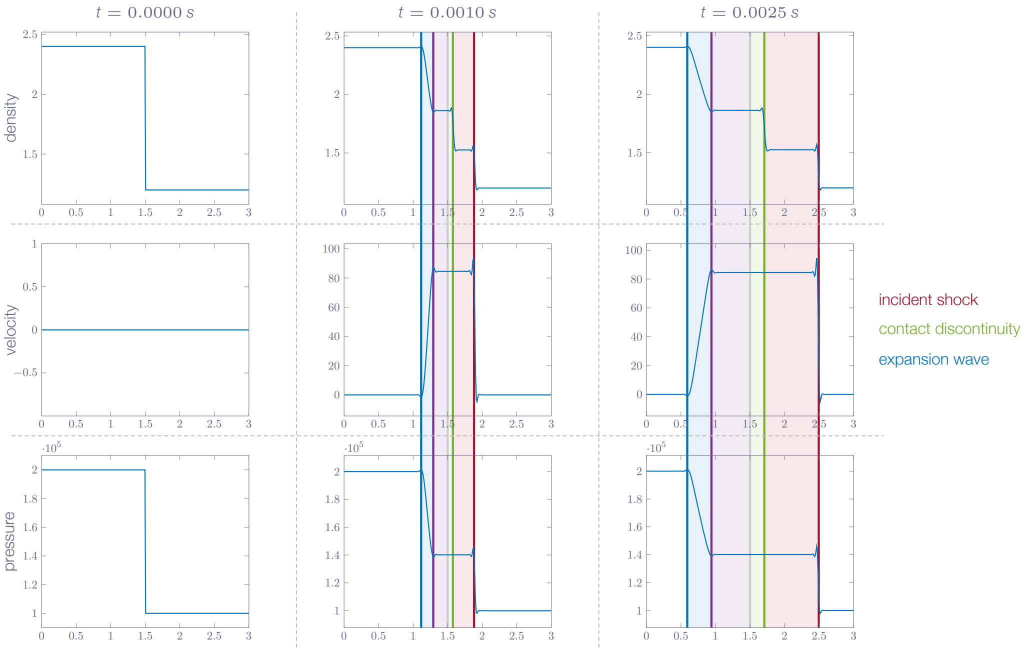
$t = 0.0000\,s$
$t = 0.0010\,s$
$t = 0.0025\,s$
density
velocity
pressure
$\cdot 10^5$



$t = 0.0000\,s$
$t = 0.0010\,s$
$t = 0.0025\,s$
density
velocity
pressure
$\cdot 10^5$
incident shock

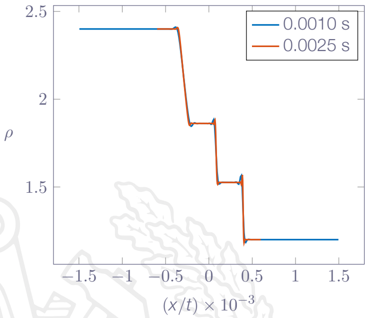


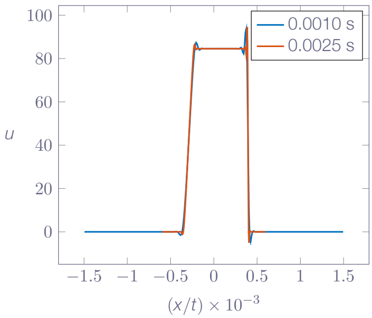
t = 0.0000 s
t = 0.0010 s
t = 0.0025 s
density
velocity
pressure
·10^5
incident shock
contact discontinuity

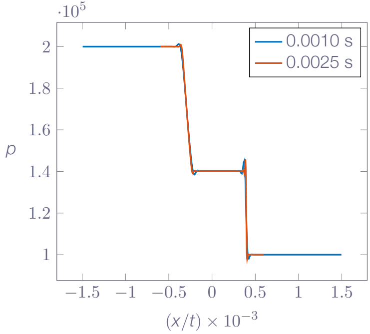


t = 0.0000 s
t = 0.0010 s
t = 0.0025 s
density
velocity
pressure
·10^5
incident shock
contact discontinuity
expansion wave

Riemann Problem - Shock Tube







The solution can be made self similar by plotting the flow field variables as function of x/t

Roadmap - Unsteady Wave Motion