# Compressible Flow - TME085

## Lecture 1

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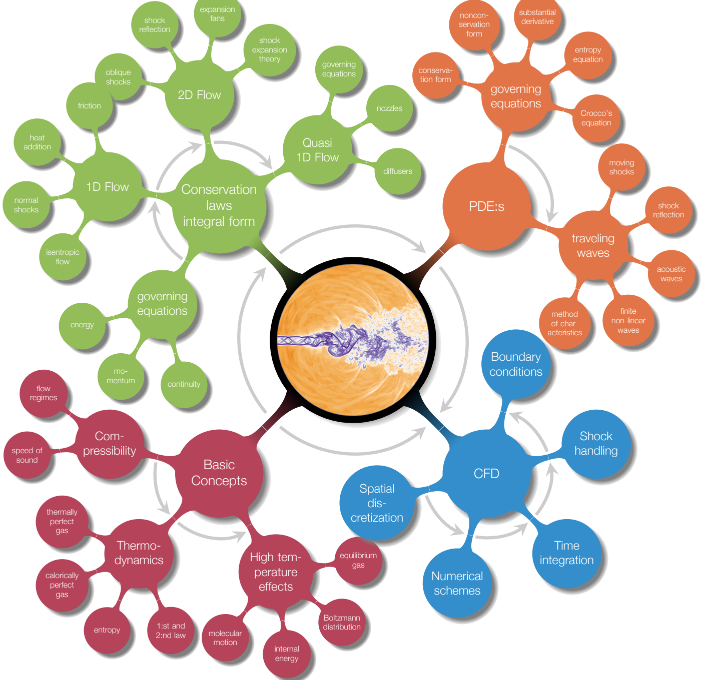
# Overview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shock expansion theory
oblique shocks
2D Flow
friction
governing equations
nozzles
heat addition
Quasi 1D Flow
1D Flow
diffusers
normal shocks
Conservation laws integral form
isentropic flow
governing equations
energy
mo-mentum
continuity
noncon-servation form
substantial derivative
entropy equation
conserva-tion form
governing equations
Crocco's equation
moving shocks
PDE:s
shock reflection
traveling waves
acoustic waves
method of char-acteristics
finite non-linear waves
Boundary conditions
Shock handling
CFD
Spatial dis-cretization
Time integration
Numerical schemes
flow regimes
Com-pressibility
speed of sound
Basic Concepts
thermally perfect gas
Thermo-dynamics
calorically perfect gas
entropy
1:st and 2:nd law
High tem-perature effects
equilibrium gas
Boltzmann distribution
molecular motion
internal energy

# Compressible Flow

*”Compressible flow (gas dynamics) is a branch of fluid mechanics that deals with flows having significant changes in fluid density”*

Wikipedia

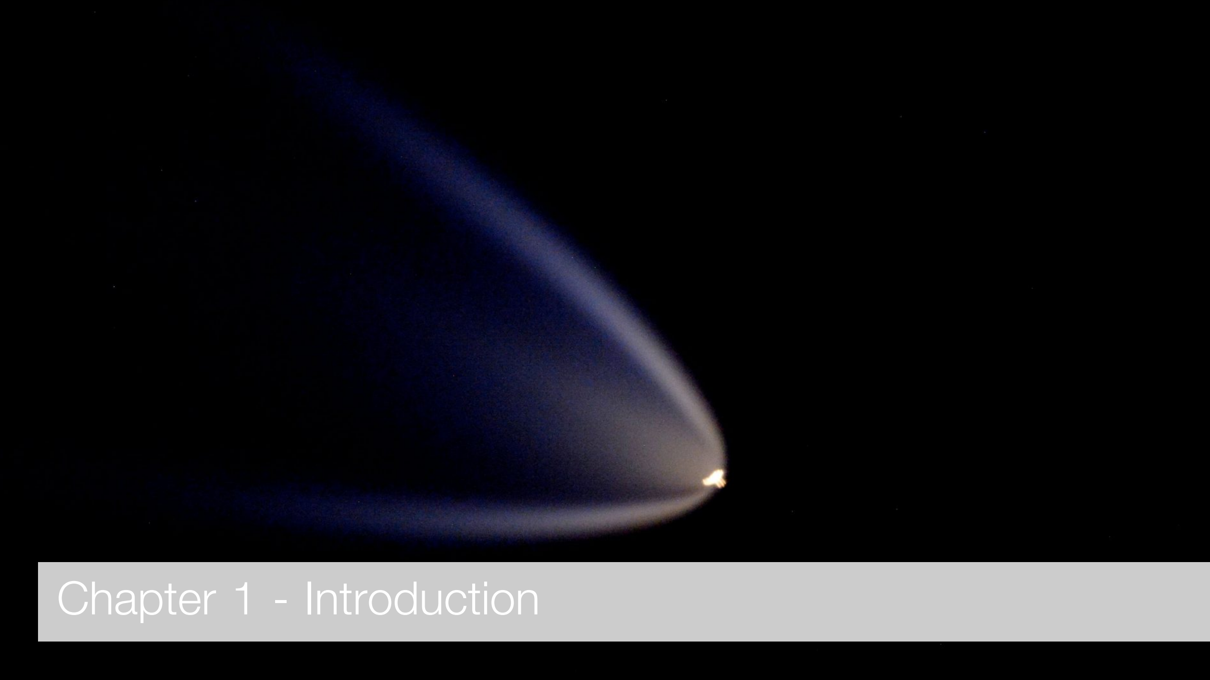
# Gas Dynamics

*”... the study of motion of gases and its effects on physical systems ...”*

*”... based on the principles of fluid mechanics and thermodynamics ...”*

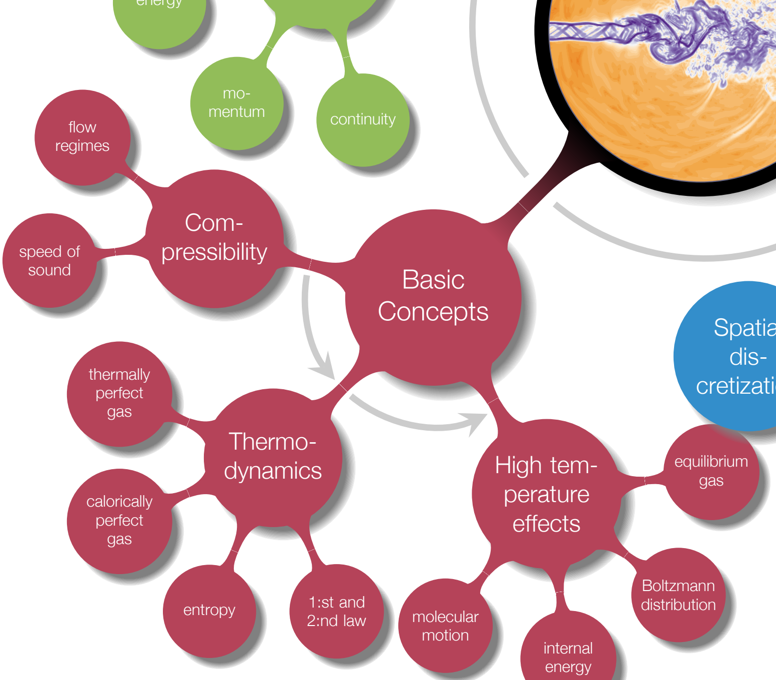
*”... gases flowing around or within physical objects at speeds comparable to the speed of sound ...”*

Wikipedia



# Chapter 1 - Introduction

# Overview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gas
Thermo-
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entropy
1:st and
2:nd law
High tem-
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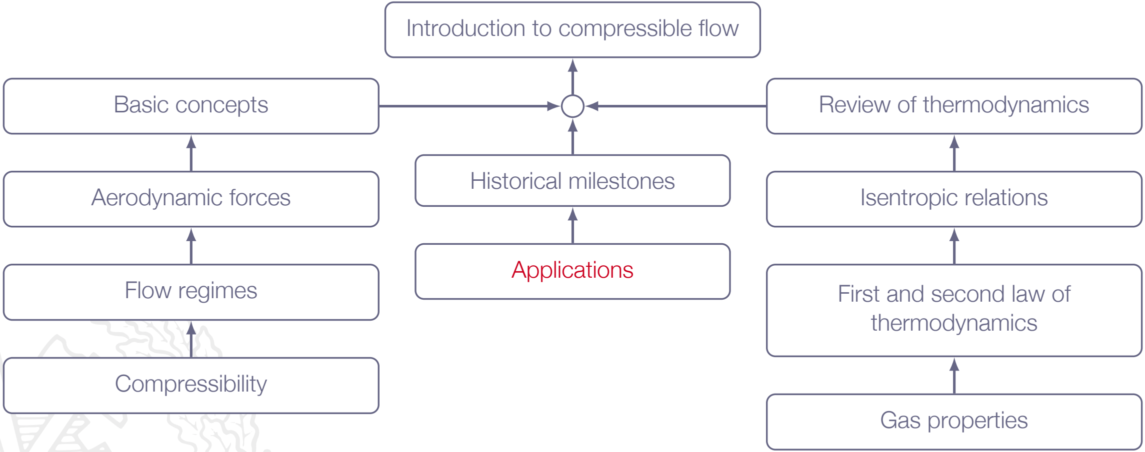
# Learning Outcomes

1. **Define** the concept of compressibility for flows
2. **Explain** how to find out if a given flow is subject to significant compressibility effects
3. **Describe** typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)

6. **Define** the special cases of calorically perfect gas, thermally perfect gas and real gas and **explain** the implication of each of these special cases

*in this lecture we will find out what compressibility means and do a brief review of thermodynamics*

# Roadmap - Introduction to Compressible Flow



Introduction to compressible flow

Basic concepts

Aerodynamic forces

Flow regimes

Compressibility

Historical milestones

Applications

Review of thermodynamics

Isentropic relations

First and second law of thermodynamics

Gas properties

# Applications - Classical

Treatment of calorically perfect gas

Exact solutions of inviscid flow in 1D

Shock-expansion theory for steady-state 2D flow

Approximate closed form solutions to linearized equations in 2D and 3D

Method of Characteristics (MOC) in 2D and axi-symmetric inviscid supersonic flows

# Applications - Modern

- Computational Fluid Dynamics (CFD)
- Complex geometries (including moving boundaries)
- Complex flow features (compression shocks, expansion waves, contact discontinuities)
- Viscous effects
- Turbulence modeling
- High temperature effects (molecular vibration, dissociation, ionization)
- Chemically reacting flow (equilibrium & non-equilibrium reactions)

# Applications - Examples

Turbo-machinery flows:
- Gas turbines, steam turbines, compressors
- Aero engines (turbojets, turbofans, turboprops)

Aeroacoustics:
- Flow induced noise (jets, wakes, moving surfaces)
- Sound propagation in high speed flows

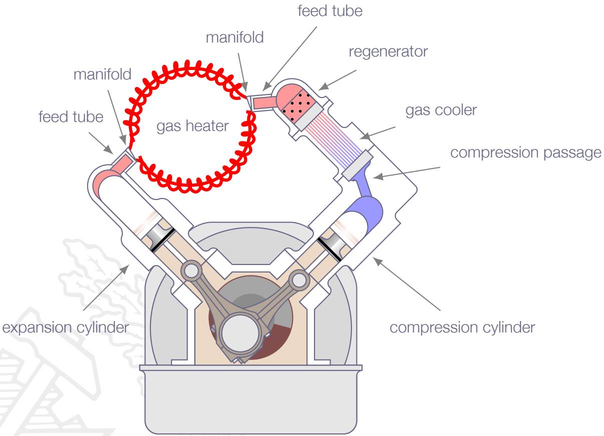
External flows:
- Aircraft (airplanes, helicopters)
- Space launchers (rockets, re-entry vehicles)

Internall flows:
- Nozzle flows
- Inlet flows, diffusers
- Gas pipelines (natural gas, bio gas)

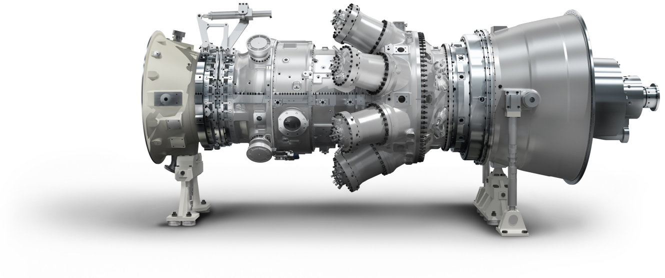
Free-shear flows:
- High speed jets

Combustion:
- Internal combustion engines (valve flow, in-cylinder flow, exhaust pipe flow, mufflers)
- Combustion induced noise (turbulent combustion)
- Combustion instabilities

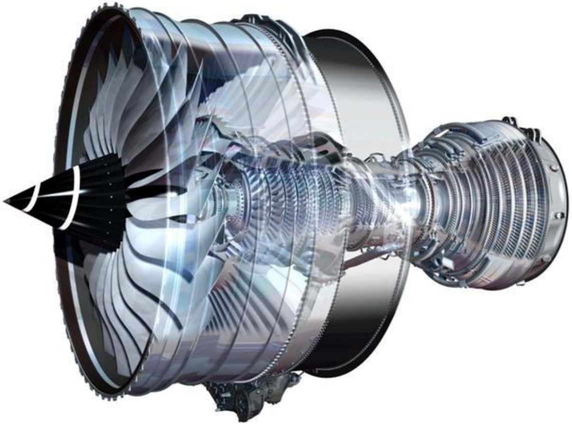
# Applications - Stirling Engine



# Applications - Siemens GT750



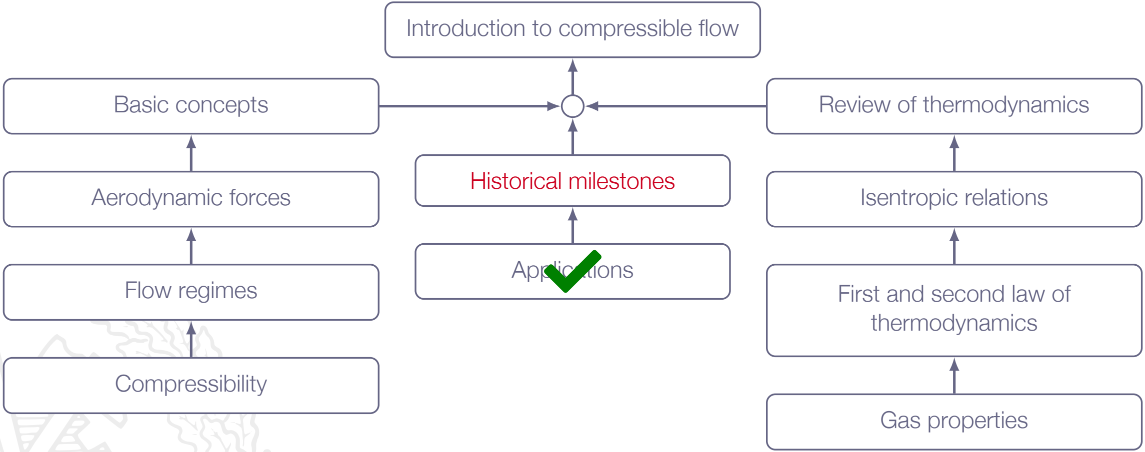
# Applications - Rolls-Royce Trent XWB



# Applications - Airbus A380

# Applications - Vulcain Nozzle

# Roadmap - Introduction to Compressible Flow



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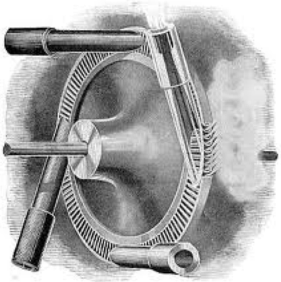
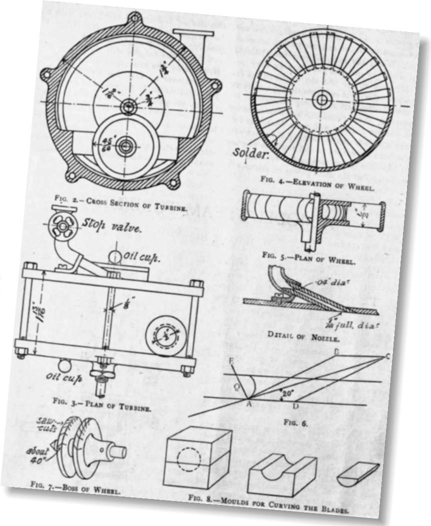
# Historical Milestones

1893 C.G.P. de Laval, first steam turbine with supersonic nozzles (convergent-divergent). At this time, the significance was not fully understood, but it worked!

1947 Charles Yeager, flew first supersonic aircraft (XS-1), M 1.06

# Historical Milestones - C.G.P. de Laval (1893)

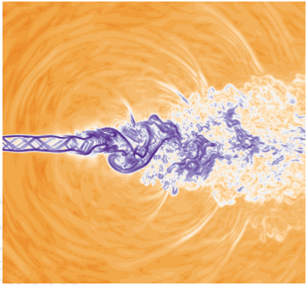


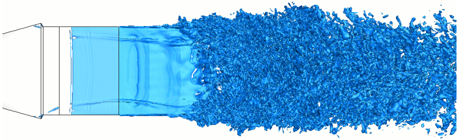


# Historical Milestones - Charles Yeager (1947)

# Modern Compressible Flow

## Screeching rectangular supersonic jet





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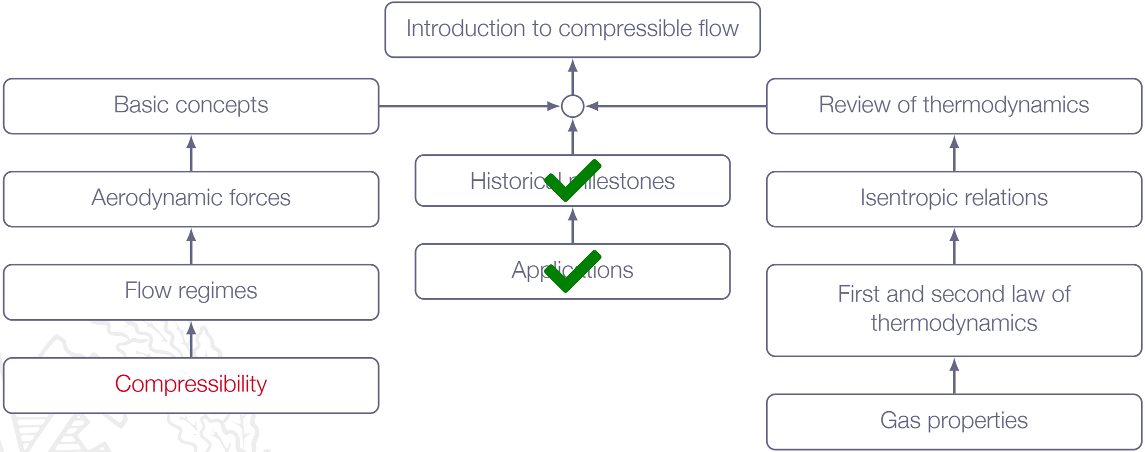

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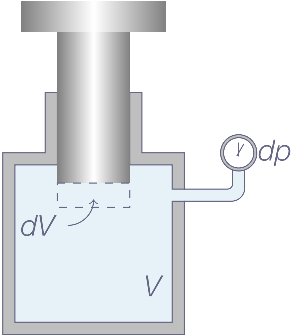
# Chapter 1.2
# Compressibility

# Compressibility

$$\tau = -\frac{1}{\nu}\frac{\partial \nu}{\partial p}, \ (\nu = \frac{1}{\rho})$$

Not really precise!

Is $T$ held constant during the compression or not?



# Compressibility

Two fundamental cases:

**Constant temperature**

Heat is cooled off to keep $T$ constant inside the cylinder

**Adiabatic process**

Thermal insulation prevents heat exchange

# Compressibility

Isothermal process:

$$\tau_T = -\frac{1}{\nu}\left(\frac{\partial \nu}{\partial p}\right)_T$$

Adiabatic reversible (**isentropic**) process:

$$\tau_S = -\frac{1}{\nu}\left(\frac{\partial \nu}{\partial p}\right)_S$$

| | | |
|---|---|---|
| Air at normal conditions: | $\tau_T \approx 1.0 \times 10^{-5}$ | $[m^2/N]$ |
| Water at normal conditions: | $\tau_T \approx 5.0 \times 10^{-10}$ | $[m^2/N]$ |

# Compressibility

$$\tau = -\frac{1}{\nu}\frac{\partial \nu}{\partial p} \text{ where } \nu = \frac{1}{\rho} \text{ and thus}$$

$$\tau = -\rho\frac{\partial}{\partial p}\left(\frac{1}{\rho}\right) = -\rho\left(-\frac{1}{\rho^2}\right)\frac{\partial \rho}{\partial p} = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$$

$$\boxed{\tau_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T}$$

$$\boxed{\tau_S = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_S}$$

# Compressibility

## Definition of compressible flow:

If $p$ changes with amount $\Delta p$ over a characteristic length scale of the flow, such that the corresponding change in density, given by $\Delta\rho \sim \rho\tau\Delta$ p, is **too large to be neglected**, the flow is compressible (*typically* $\Delta\rho/\rho > 0.05$)

**Note!** Bernoulli´s equation is restricted to incompressible flow, *i.e.* it is **not valid** for compressible flow!

# Compressibility - Mach Number

The freestream Mach number is defined as

$$M_\infty = \frac{U_\infty}{a_\infty}$$

where $U_\infty$ is the freestream flow speed and $a_\infty$ is the speed of sound at freestream conditions

# Compressibility

Assume incompressible flow and estimate the maximum pressure difference using

$$\Delta p \approx \frac{1}{2}\rho_\infty U_\infty^2$$

For air at normal conditions we have

$$\tau_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T = \left\{ p = \rho R T \Rightarrow \left(\frac{\partial \rho}{\partial p}\right)_T = \frac{1}{RT} \right\} = \frac{1}{\rho R T} = \frac{1}{p}$$

*(ideal gas law for perfect gas* $p = \rho R T$*)*

# Compressibility

Using the relations on previous slide we get

$$\frac{\Delta\rho}{\rho} \approx \tau_T \Delta p \approx \frac{1}{p_\infty}\frac{1}{2}\rho_\infty U_\infty^2 = \frac{\frac{1}{2}\rho_\infty U_\infty^2}{\rho_\infty R T_\infty}$$

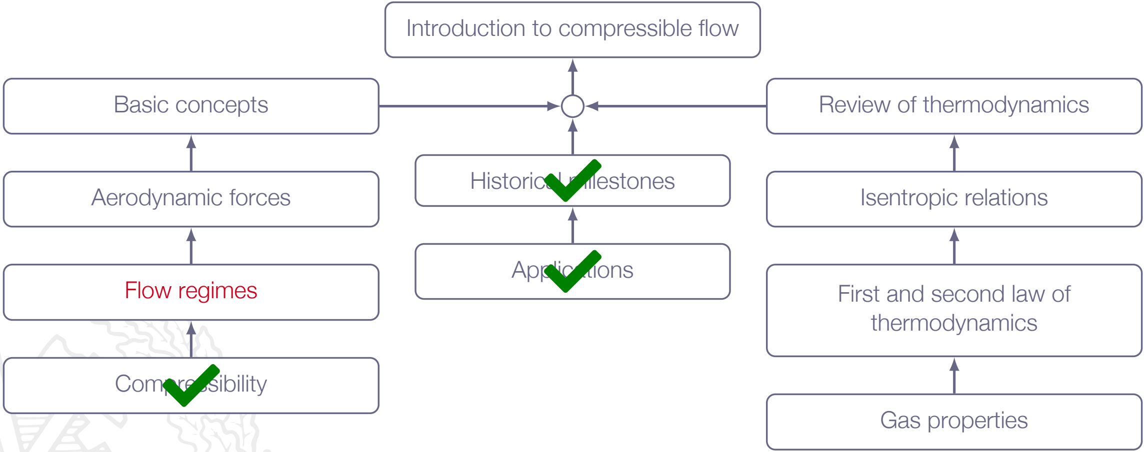
for a calorically perfect gas we have $a = \sqrt{\gamma R T}$

which gives us $\dfrac{\Delta\rho}{\rho} \approx \dfrac{\gamma U_\infty^2}{2a_\infty^2}$

now, using the definition of Mach number we get:

$$\boxed{\frac{\Delta\rho}{\rho} \approx \frac{\gamma}{2} M_\infty^2}$$

# Roadmap - Introduction to Compressible Flow



Introduction to compressible flow

Basic concepts

Aerodynamic forces

Flow regimes

Compressibility ✓

Historical milestones ✓

Applications ✓

Review of thermodynamics

Isentropic relations

First and second law of thermodynamics

Gas properties

# Chapter 1.3
# Flow Regimes

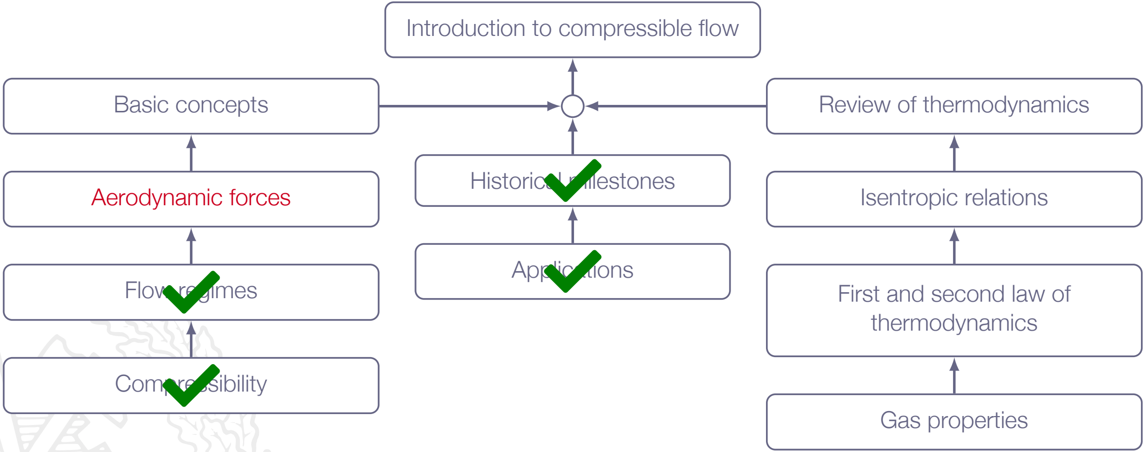
# Flow Regimes

| | |
|---|---|
| Incompressible | $M_\infty < 0.1$ |
| Subsonic | $M_\infty < 1$ and $M < 1$ everywhere |
| Transonic | case 1: $M_\infty < 1$ and $M > 1$ locally<br>case 2: $M_\infty > 1$ and $M < 1$ locally |
| Supersonic | $M_\infty > 1$ and $M > 1$ everywhere |
| Hypersonic | supersonic flow with high-temperature effects |

Compressible

Local Mach number $M$ is based on local flow speed, $U = |\mathbf{U}|$, and local speed of sound, $a$

# Roadmap - Introduction to Compressible Flow



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Flow regimes ✓

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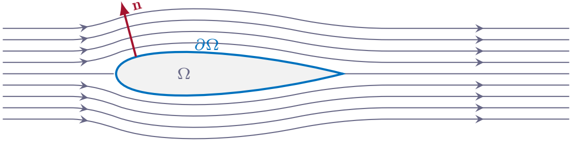
# Chapter 1.5
# Aerodynamic Forces

# Aerodynamic Forces



| | |
|---|---|
| $\Omega$ | region occupied by body |
| $\partial\Omega$ | surface of body |
| $\mathbf{n}$ | outward facing unit normal vector |

# Aerodynamic Forces

Overall forces on the body du to the flow

$$\mathbf{F} = \oiint (-p\mathbf{n} + \tau \cdot \mathbf{n}) dS$$

where $p$ is static pressure and $\tau$ is a stress tensor

# Aerodynamic Forces

**Drag** is the component of $\mathbf{F}$ which is **parallel** with the freestream direction:

$$D = D_p + D_f$$

where $D_p$ is drag due to pressure and $D_f$ is drag due to friction

**Lift** is the component of $\mathbf{F}$ which is **normal** to the free stream direction:

$$L = L_p + L_f$$

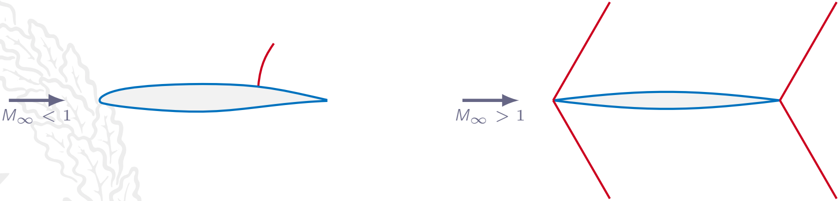
($L_f$ is usually negligible)

# Aerodynamic Forces

Inviscid flow around slender body (*attached flow*)

subsonic flow: $D = 0$
transonic or supersonic flow: $D > 0$

Explanation: **Wave drag**



# Aerodynamic Forces

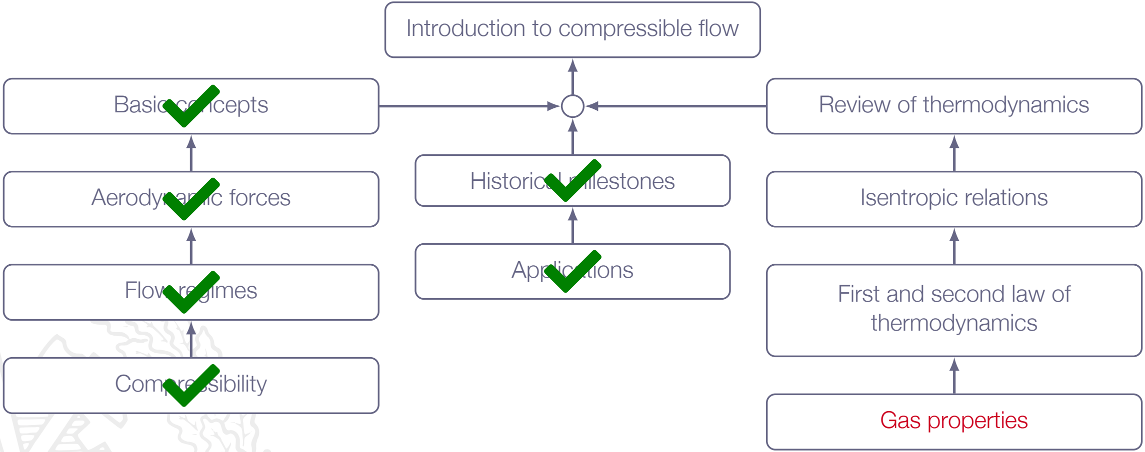
**Wave drag** is an **inviscid phenomena**, connected to the formation of compression shocks and entropy increase

Viscous effects are present in all Mach regimes

At transonic and supersonic conditions a particular phenomena named **shock/boundary-layer interaction** may appear

- shocks trigger flow separation
- usually leads to unsteady flow

# Roadmap - Introduction to Compressible Flow



Introduction to compressible flow

Basic concepts ✓

Aerodynamic forces ✓

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# Chapter 1.4
# Review of Thermodynamics

# Thermodynamic Review

Compressible flow:

> *” strong interaction between flow and thermodynamics ... ”*

# Perfect Gas

All intermolecular forces negligible

Only elastic collitions between molecules

$p\nu = RT$ or $\dfrac{p}{\rho} = RT$

where $R$ is the gas constant $[R] = J/kgK$

Also, $R = R_{univ}/M$ where $M$ is the molecular weight of gas molecules (in $kg/kmol$) and $R_{univ} = 8314\,J/kmol\ K$

# Internal Energy and Enthalpy

Internal energy $e$ ($[e] = J/kg$)

Enthalpy $h$ ($[h] = J/kg$)

$$h = e + p\nu = e + \frac{p}{\rho} \text{ (valid for all gases)}$$

For any gas in thermodynamic equilibrium, e and h are functions of only two thermodynamic variables (*any two variables may be selected*) *e.g.*

$e = e(T, \rho)$ or $h = h(T, p)$

# Internal Energy and Enthalpy

Special cases:

**Thermally perfect gas:**

$e = e(T)$ and $h = h(T)$

OK assumption for air at near atmospheric conditions and $100K < T < 2500K$

**Calorically perfect gas:**

$e = C_v T$ and $h = C_p T$ ($C_v$ and $C_p$ are constants)

OK assumption for air at near atmospheric pressure and $100K < T < 1000K$

# Specific Heat

For thermally perfect (and calorically perfect) gas

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p, \quad C_v = \left(\frac{\partial e}{\partial T}\right)_v$$

since $h = e + p/\rho = e + RT$ we obtain:

$$C_p = C_v + R$$

The ratio of specific heats, $\gamma$, is defined as:

$$\gamma \equiv \frac{C_p}{C_v}$$

# Specific Heat

**Important!**

calorically perfect gas:

$C_v$, $C_p$, and $\gamma$ are **constants**

thermally perfect gas:

$C_v$, $C_p$, and $\gamma$ will **depend on temperature**

# Specific Heat

$$C_p - C_v = R$$

$$C_p - C_v = R$$

# Specific Heat

$$C_p - C_v = R$$

divide by $C_v$

$$C_p - C_v = R$$

divide by $C_p$

# Specific Heat

$$C_p - C_v = R$$

divide by $C_v$

$$\gamma - 1 = \frac{R}{C_v}$$

$$C_p - C_v = R$$

divide by $C_p$

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

# Specific Heat

$$C_p - C_v = R$$

divide by $C_v$

$$\gamma - 1 = \frac{R}{C_v}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p - C_v = R$$

divide by $C_p$

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

# Specific Heat

$$C_p - C_v = R$$

divide by $C_v$

$$\gamma - 1 = \frac{R}{C_v}$$

$$\boxed{C_v = \frac{R}{\gamma - 1}}$$

$$C_p - C_v = R$$

divide by $C_p$

$$1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{R}{C_p}$$

$$\boxed{C_p = \frac{\gamma R}{\gamma - 1}}$$

valid for both thermally perfect and calorically perfect gas!

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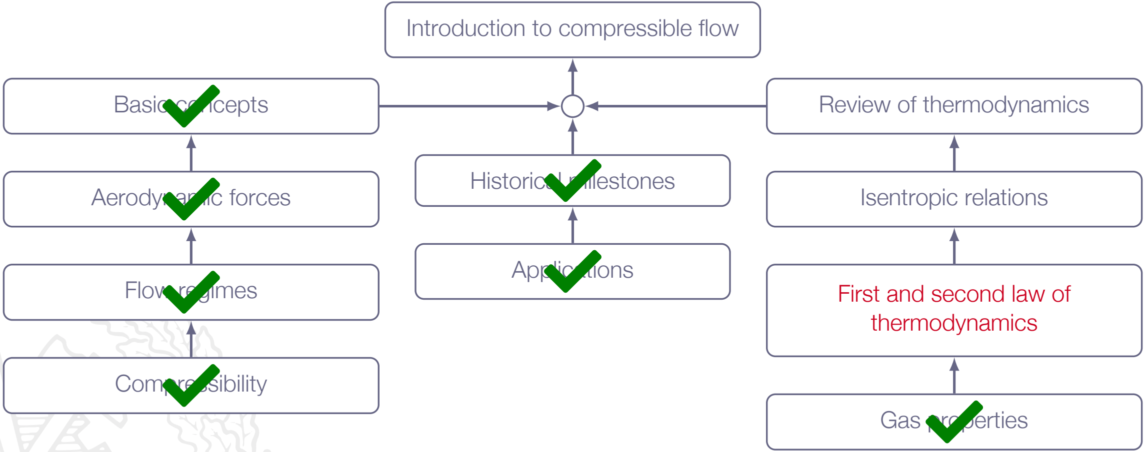

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Basic concepts ✓

Aerodynamic forces ✓

Flow regimes ✓

Compressibility ✓

Historical milestones ✓

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First and second law of thermodynamics

Gas properties ✓

# First Law of Thermodynamics

A fixed mass of gas, separated from its surroundings by an imaginary flexible boundary, is defined as a **system**. This system obeys the relation

$$de = \delta q - \delta w$$

where

$de$ is a change in internal energy of system

$\delta q$ is heat added to the system

$\delta w$ is work done by the system (on its surroundings)

**Note!** $de$ only depends on starting point and end point of the process while $\delta q$ and $\delta w$ depend on the actual process also

# First Law of Thermodynamics

Examples:

**Adiabatic process:**

$\delta q = 0.$

**Reversible process:**

no dissipative phenomena (*no flow losses*)

**Isentropic process:**

a process which is both adiabatic and reversible

# First Law of Thermodynamics

Reversible process:

$$\delta w = p d\nu = p d(1/\rho)$$

$$de = \delta q - p d(1/\rho)$$

Adiabatic & reversible process:

$$\delta q = 0.$$

$$de = -p d(1/\rho)$$

# Entropy

Entropy $s$ is a property of all gases, uniquely defined by any two thermodynamic variables, *e.g.*

$s = s(p, T)$ or $s = s(\rho, T)$ or $s = s(\rho, p)$ or $s = s(e, h)$ or ...

# Second Law of Thermodynamics

Concept of entropy $s$:

$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{ir}$ where $ds_{ir} > 0$. and thus

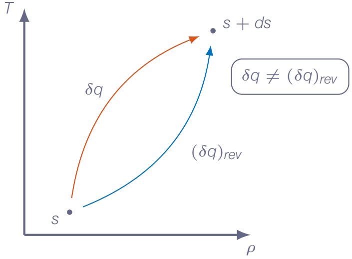
$$\boxed{ds \geq \frac{\delta q}{T}}$$

# Second Law of Thermodynamics

Concept of entropy $s$:

$$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{ir} \text{ where } ds_{ir} > 0. \text{ and thus}$$

$$\boxed{ds \geq \frac{\delta q}{T}}$$



# Second Law of Thermodynamics

In general:

$$ds \geq \frac{\delta q}{T}$$

For adiabatic processes:

$$ds \geq 0.$$

# Second Law of Thermodynamics

> *"In this house, we obey the laws of thermodynamics!"*

Homer Simpson, after Lisa constructs a perpetual motion machine whose energy increases with time

# Calculation of Entropy

For reversible processes ($\delta w = pd(1/\rho)$ and $\delta q = Tds$):

$$de = Tds - pd\left(\frac{1}{\rho}\right) \Leftrightarrow Tds = de + pd\left(\frac{1}{\rho}\right)$$

from before we have $h = e + p/\rho \Rightarrow$

$$dh = de + pd\left(\frac{1}{\rho}\right) + \left(\frac{1}{\rho}\right)dp \Leftrightarrow de = dh - pd\left(\frac{1}{\rho}\right) - \left(\frac{1}{\rho}\right)dp$$

# Calculation of Entropy

For thermally perfect gases, $p = \rho RT$ and $dh = C_p dT \Rightarrow ds = C_p \frac{dT}{T} - R\frac{dp}{p}$

Integration from starting point (1) to end point (2) gives:

$$s_2 - s_1 = \int_1^2 C_p \frac{dT}{T} - R \ln\left(\frac{p_2}{p_1}\right)$$

and for calorically perfect gases

$$\boxed{s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)}$$

# Calculation of Entropy

If we instead use $de = C_v dT$ we get

for thermally perfect gases

$$s_2 - s_1 = \int_1^2 C_v \frac{dT}{T} - R \ln\left(\frac{\rho_2}{\rho_1}\right)$$

and for calorically perfect gases

$$\boxed{s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{\rho_2}{\rho_1}\right)}$$

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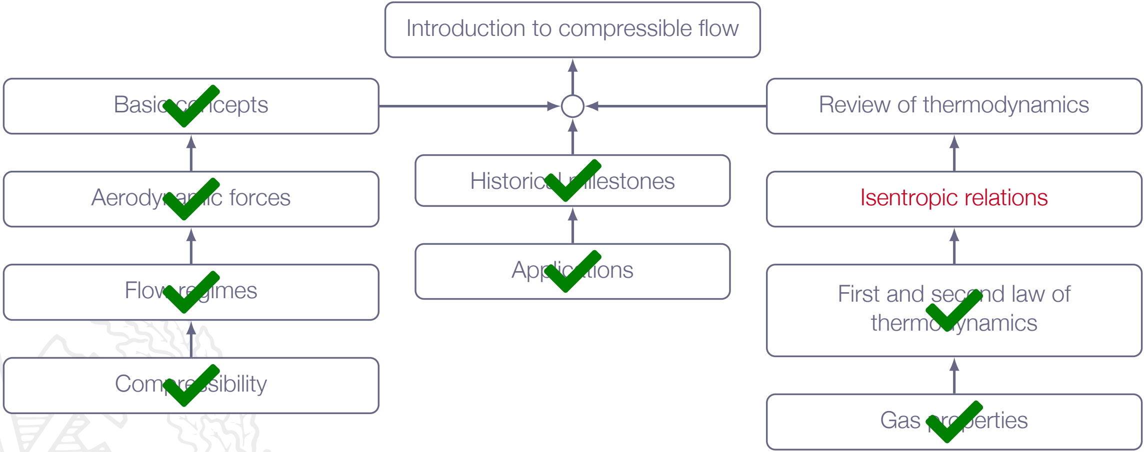

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# Isentropic Relations

For calorically perfect gases, we have

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

For adiabatic reversible processes:

$$ds = 0. \Rightarrow s_1 = s_2 \Rightarrow C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) = 0 \Rightarrow$$

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{C_p}{R}\ln\left(\frac{T_2}{T_1}\right)$$

# Isentropic Relations

with $\frac{C_p}{R} = \frac{C_p}{C_p - C_v} = \frac{\gamma}{\gamma - 1} \Rightarrow$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$

# Isentropic Relations

Alternatively, using $s_2 - s_1 = 0 = C_v \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{\rho_2}{\rho_1}\right) \Rightarrow$

$$\boxed{\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}}$$

# Isentropic Relations - Summary

For an isentropic process and a calorically perfect gas we have

$$\boxed{\frac{p_2}{p_1}=\left(\frac{\rho_2}{\rho_1}\right)^{\gamma}=\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$

A.K.A. the **isentropic relations**

# Thermodynamic Relations and Process Diagrams

Many times it's process diagrams makes it easier to understand physics

Examples of process diagrams: $Ts$-diagram and $p\nu$-diagram

We will use process diagrams in the following chapters to give insights into physical processes such as shocks, heat addition and friction

# Thermodynamic Relations and Process Diagrams

From before:

$$\boxed{ds = C_v \frac{dT}{T} + R\frac{d\nu}{\nu}}$$

$$\boxed{ds = C_p \frac{dT}{T} - R\frac{dp}{p}}$$

# $Ts$-diagram

$$ds = C_v\frac{dT}{T} + R\frac{d\nu}{\nu}$$

$$d\nu = \frac{\nu}{R}ds - C_v\frac{\nu}{RT}dT$$

$$d\nu = \frac{\nu}{R}ds - \frac{C_v}{p}dT$$

$ds = 0 \Rightarrow d\nu < 0$ for positive $dT$

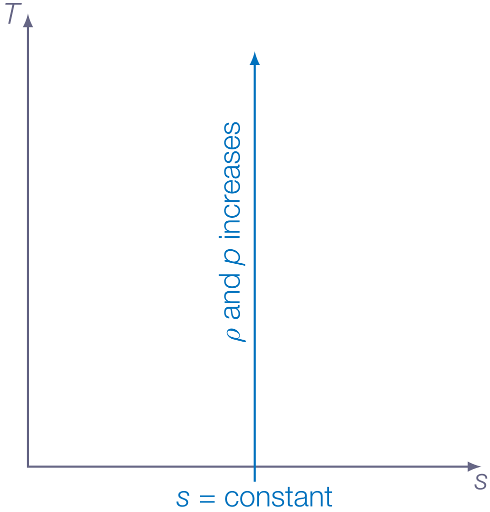
$$ds = C_p\frac{dT}{T} - R\frac{dp}{p}$$

$$dp = -\frac{p}{R}ds + C_p\frac{p}{RT}dT$$

$$dp = -\frac{p}{R}ds + C_p\rho dT$$

$ds = 0 \Rightarrow dp > 0$ for positive $dT$

# $Ts$-diagram



# $Ts$-diagram - Isochoric process

$$d\nu = \frac{\nu}{R}ds - \frac{C_v}{p}dT$$

From before: $\nu$ decreases with $T$ and $p$ increases with $T$ and thus for a given $dT$, $d\nu$ will be greater at lower $T$ than at higher $T$

$\nu$=constant lines will be closely spaced at low $T$ and more sparse at high $T$

$\nu$=constant $\Rightarrow d\nu = 0$:

$$0 = \frac{\nu}{R}\left(ds - C_v\frac{dT}{T}\right) \Rightarrow \frac{dT}{ds} = \frac{T}{C_v}$$

slope is positive and increases with temperature

# $Ts$-diagram - Isobaric process

$$ds = C_p \frac{dT}{T} - R\frac{dp}{p}$$

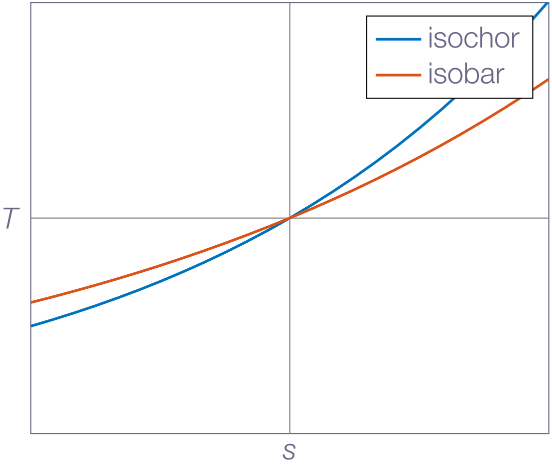
$p$=constant $\Rightarrow dp = 0$:

$$0 = \frac{p}{R}\left(C_p\frac{dT}{T} - ds\right) \Rightarrow \frac{dT}{ds} = \frac{T}{C_p}$$

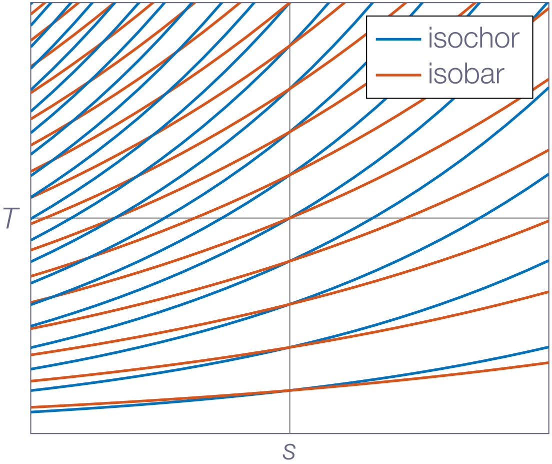
slope is positive and increases with temperature

$C_p > C_v \Rightarrow$ isobars are less steep than isochors

# *Ts*-diagrams



# *Ts*-diagrams



# $p\nu$-diagrams

subtract

$$C_p\left[ds = C_v\frac{dT}{T} + R\frac{d\nu}{\nu}\right]$$

from

$$C_v\left[ds = C_p\frac{dT}{T} - R\frac{dp}{p}\right]$$

gives

$$ds\underbrace{(C_p - C_v)}_{R} = RC_p\frac{d\nu}{\nu} + RC_v\frac{dp}{p} \Rightarrow ds = C_p\frac{d\nu}{\nu} + C_v\frac{dp}{p}$$

# $p\nu$-diagrams

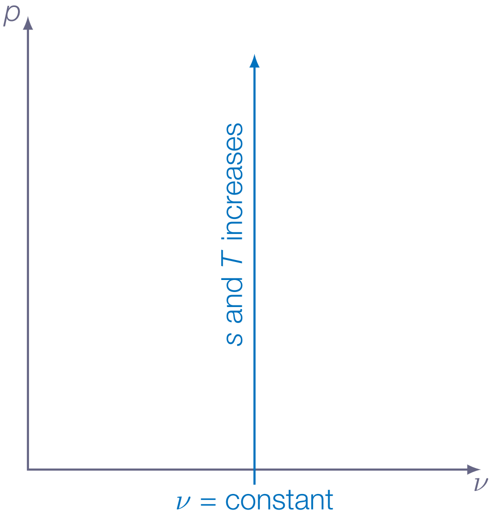
$$ds = C_p \frac{d\nu}{\nu} + C_v \frac{dp}{p}$$

$$d\nu = 0 \text{ (isochoric process)} \Rightarrow ds = C_v \frac{dp}{p}$$

entropy increases with increasing pressure

from before: temperature increases with increasing pressure

# $p\nu$-diagrams



# $p\nu$-diagrams - isentropic process

$$ds = C_p\frac{d\nu}{\nu} + C_v\frac{dp}{p}$$

$s$=constant ($ds = 0$):

$$C_p\frac{d\nu}{\nu} + C_v\frac{dp}{p} = 0 \Rightarrow \frac{dp}{d\nu} = -\gamma\frac{p}{\nu}$$

negative slope

slope becomes steeper with increased pressure and decreased $\nu$

# $p\nu$-diagrams - isothermal process

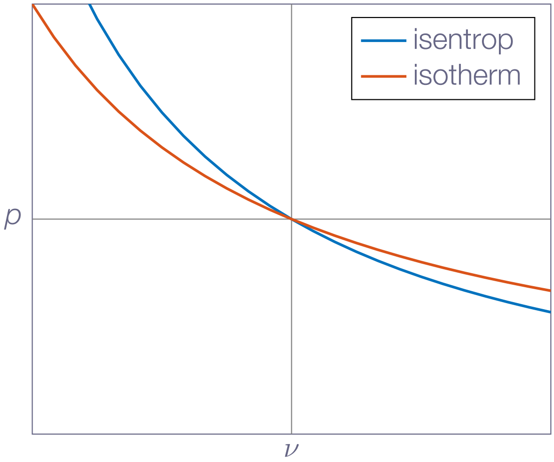
$$ds = C_v \frac{dT}{T} + R\frac{d\nu}{\nu} = C_p \frac{dT}{T} - R\frac{dp}{p}$$

$T$=constant ($dT = 0$):

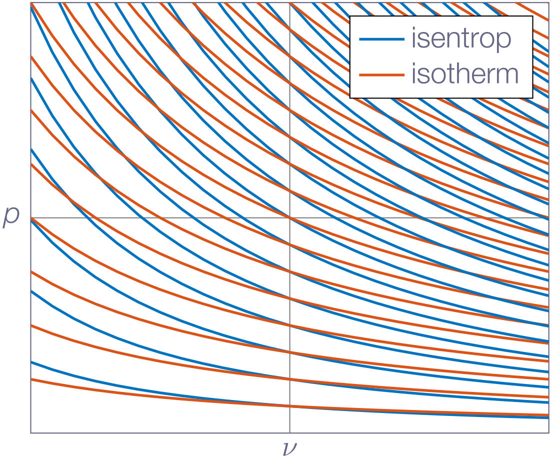
$$\frac{d\nu}{\nu} = -\frac{dp}{p} \Rightarrow \frac{dp}{d\nu} = -\frac{p}{\nu}$$

$\gamma > 0 \Rightarrow$ isentropes are steeper than isotherms

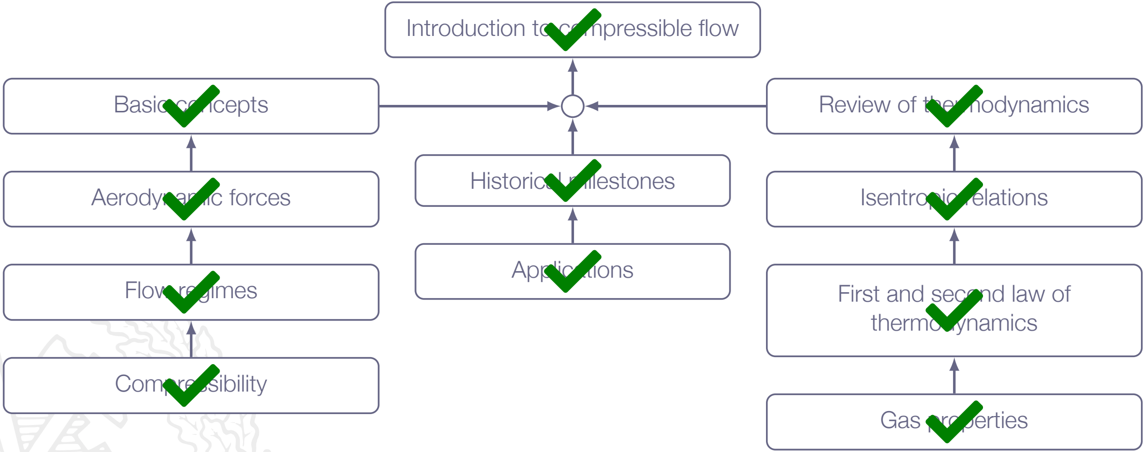
# $p\nu$-diagrams - isothermal process

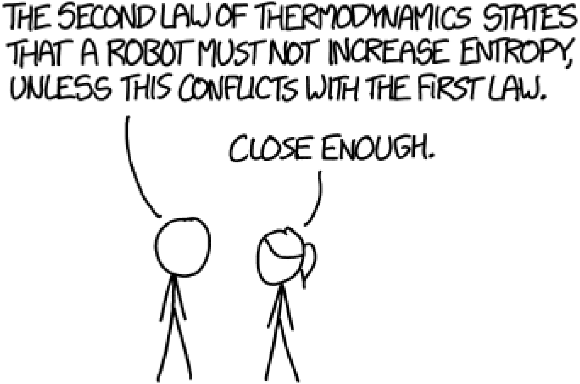


# $p\nu$-diagrams - isothermal process



# Roadmap - Introduction to Compressible Flow




THE SECOND LAW OF THERMODYNAMICS STATES THAT A ROBOT MUST NOT INCREASE ENTROPY, UNLESS THIS CONFLICTS WITH THE FIRST LAW.
CLOSE ENOUGH.