Compressible Flow TME085

Unsteady Wave Motion

Acoustic Wave Propagation

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Acoustic Wave Equation

Combining the linearized continuity and momentum equations we get the wave propagation equation for acoustic waves (small perturbations)

$$\frac{\partial^2}{\partial t^2}(\Delta\rho) = a_\infty^2 \frac{\partial^2}{\partial x^2}(\Delta\rho) \tag{1}$$

which is a one-dimensional form of the classic wave equation with the solution

$$\Delta\rho = F(x - a_\infty t) + G(x + a_\infty t) \tag{2}$$

$$\frac{\partial}{\partial t}(\Delta\rho) = \frac{\partial F}{\partial(x - a_\infty t)}\frac{\partial(x - a_\infty t)}{\partial t} + \frac{\partial G}{\partial(x + a_\infty t)}\frac{\partial(x + a_\infty t)}{\partial t}$$

$$\frac{\partial}{\partial t}(\Delta\rho) = -a_\infty F' + a_\infty G' \tag{3}$$

$$\frac{\partial^2}{\partial t^2}(\Delta\rho) = -a_\infty \frac{\partial F'}{\partial(x - a_\infty t)}\frac{\partial(x - a_\infty t)}{\partial t} + a_\infty \frac{\partial G'}{\partial(x + a_\infty t)}\frac{\partial(x + a_\infty t)}{\partial t}$$

$$\frac{\partial^2}{\partial t^2}(\Delta\rho) = a_\infty^2 F'' + a_\infty^2 G'' \tag{4}$$

$$\frac{\partial}{\partial x}(\Delta\rho) = \frac{\partial F}{\partial(x - a_\infty t)}\frac{\partial(x - a_\infty t)}{\partial x} + \frac{\partial G}{\partial(x + a_\infty t)}\frac{\partial(x + a_\infty t)}{\partial x}$$

$$\frac{\partial}{\partial x}(\Delta\rho) = F' + G' \tag{5}$$

$$\frac{\partial^2}{\partial x^2}(\Delta\rho) = \frac{\partial F'}{\partial(x - a_\infty t)}\frac{\partial(x - a_\infty t)}{\partial x} + \frac{\partial G'}{\partial(x + a_\infty t)}\frac{\partial(x + a_\infty t)}{\partial x}$$

$$\frac{\partial^2}{\partial x^2}(\Delta\rho) = F'' + G'' \tag{6}$$

Eqns. 4 and 6 inserted in the acoustic wave equation (Eqn. 1)

$$a_\infty^2 F'' + a_\infty^2 G'' = a_\infty^2 (F'' + G'')$$

which shows that Eqn. 2 is a valid solution to the wave equation