



# Compressible Flow TME085

Study Guide

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## Course Material

### Lecture Notes

Chapter 1	Compressible Flow – Some History and Introductory Thoughts	<a href="#">TME085_C01.pdf</a>
Chapter 2	Integral Forms of the Conservation Equations for Inviscid Flows	<a href="#">TME085_C02.pdf</a>
Chapter 3	One-Dimensional Flow	<a href="#">TME085_C03.pdf</a>
Chapter 4	Oblique Shock and Expansion Waves	<a href="#">TME085_C04.pdf</a>
Chapter 5	Quasi-One-Dimensional Flow	<a href="#">TME085_C05.pdf</a>
Chapter 6	Differential Conservation Equations for Inviscid Flows	<a href="#">TME085_C06.pdf</a>
Chapter 7	Unsteady Wave Motion	<a href="#">TME085_C07.pdf</a>
Chapter 12	The Time-Marching Technique	<a href="#">TME085_C012.pdf</a>
Chapter 16	Properties of High-Temperature Gases	<a href="#">TME085_C016.pdf</a>
Chapter 17	High-Temperature Flows: Basic Examples	<a href="#">TME085_C017.pdf</a>

### Course Documents

1. [TME085\\_Formulas-Tables-and-Graphs.pdf](#)

## Complementary Documents

1. [TME085\\_Basic-Concepts\\_Compressibility.pdf](#)
2. [TME085\\_Basic-Concepts\\_Specific-Heat.pdf](#)
3. [TME085\\_Basic-Concepts\\_Isentropic-Relations.pdf](#)
4. [TME085\\_Governing-Equations\\_Integral-Form.pdf](#)
5. [TME085\\_Governing-Equations\\_Differential-Form.pdf](#)
6. [TME085\\_Governing-Equations\\_Alternative-Forms-of-the-Energy-Equation.pdf](#)
7. [TME085\\_Governing-Equations\\_Entropy-Equation.pdf](#)
8. [TME085\\_Governing-Equations\\_Croccos-Equation.pdf](#)
9. [TME085\\_One-Dimensional-Steady-Flow\\_Speed-Of-Sound.pdf](#)
10. [TME085\\_One-Dimensional-Steady-Flow\\_Normal-Shock-Relations.pdf](#)
11. [TME085\\_One-Dimensional-Steady-Flow\\_Hugoniot-Equation.pdf](#)
12. [TME085\\_One-Dimensional-Steady-Flow\\_Added-Heat.pdf](#)
13. [TME085\\_One-Dimensional-Steady-Flow\\_Friction.pdf](#)
14. [TME085\\_One-Dimensional-Steady-Flow\\_Friction-The-Fanno-Equation.pdf](#)
15. [TME085\\_Oblique-Shocks-and-Expansion-Waves\\_Expansion-Wave-Relations.pdf](#)
16. [TME085\\_Oblique-Shocks-and-Expansion-Waves\\_Prandtl-Meyer-Function.pdf](#)
17. [TME085\\_Quasi-One-Dimensional-Flow\\_Governing-Equations.pdf](#)
18. [TME085\\_Quasi-One-Dimensional-Flow\\_Area-Velocity-Relation.pdf](#)
19. [TME085\\_Quasi-One-Dimensional-Flow\\_Area-Mach-Relation.pdf](#)
20. [TME085\\_Quasi-One-Dimensional-Flow\\_Choked-Massflow.pdf](#)
21. [TME085\\_Unsteady-Waves\\_Shock-Relations.pdf](#)
22. [TME085\\_Unsteady-Waves\\_Reflected-Shocks.pdf](#)
23. [TME085\\_Unsteady-Waves\\_Finite-Non-Linear-Waves.pdf](#)
24. [TME085\\_Unsteady-Waves\\_Acoustic-Wave-Propagation.pdf](#)

# 1 Compressible Flow – Some History and Introductory Thoughts

## Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C01.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

[TME085\\_Basic-Concepts\\_Compressibility.pdf](#)

[TME085\\_Basic-Concepts\\_Isentropic-Relations.pdf](#)

[TME085\\_Basic-Concepts\\_Specific-Heat.pdf](#)

## Theory Questions and Reading Instructions

- 1.1 How are the **compressibility** factors  $\tau_T$  and  $\tau_S$  defined?
- 1.2 For a given flow, how can we determine if **compressibility** effects are important?
- 1.3 Use the Bernoulli equation (even though it's not valid for compressible flows) to obtain an estimate of the Mach number for which compressible effects becomes significant and must be considered.
- 1.4 What are the criteria for the classifications **subsonic/transsonic/supersonic/hypersonic** flow?
- 1.5 What is a **perfect gas** and what relation between pressure, density, temperature may be used for such a gas?
- 1.6 What do we mean by **thermally perfect gas** and **calorically perfect gas** respectively?
- 1.7 When can air be regarded as a **calorically perfect gas**?
- 1.8 Define  $C_p$ ,  $C_v$ , and  $\gamma$
- 1.9 In the first law of thermodynamics, how is a **system** defined?
- 1.10 What is a **reversible** process?
- 1.11 What is an **adiabatic** process?
- 1.12 What are the criteria for an **isentropic** process, i.e. what conditions must be satisfied for a **steady-state** compressible flow to be **isentropic**?
- 1.13 How do we define **entropy** for a gas?
- 1.14 What does the second law of thermodynamics say about **entropy**?
- 1.15 Derive the **isentropic relations** for **calorically perfect** gases starting from the **entropy equation**.
- 1.16 In [TME085\\_Formulas-Tables-and-Graphs.pdf](#), check all thermodynamics formulas for **thermally perfect** gas and **calorically perfect** gas on the first three pages. You should be familiar with these formulas and know when/how to apply them.

## Recommended Problems

Problems solved in class: P1.4b, P1.5, P1.7, P2.1 P2.2

Recommended home exercise: E1.3, E1.4, E1.5, E1.7

*Ex.y and Px.y denotes text book examples and problems respectively*

## 2 Integral Forms of the Conservation Equations for Inviscid Flows

### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C02.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

[TME085\\_Governing-Equations-Integral-Form.pdf](#)

### Theory Questions and Reading Instructions

- 2.1 What is the physical interpretation of each of the terms in the **continuity equation** on integral form

$$\frac{d}{dt} \iiint_{\Omega} \rho d\mathcal{V} + \iint_{\partial\Omega} \rho \mathbf{v} \cdot \mathbf{n} dS = 0$$

- 2.2 What is the physical interpretation of each of the terms in the **momentum equation** on integral form

$$\frac{d}{dt} \iiint_{\Omega} \rho \mathbf{v} d\mathcal{V} + \iint_{\partial\Omega} [\rho(\mathbf{v} \cdot \mathbf{n})\mathbf{v} + p\mathbf{n}] dS = \iiint_{\Omega} \rho \mathbf{f} d\mathcal{V}$$

- 2.3 What is the physical interpretation of each of the terms in the **energy equation** on integral form

$$\frac{d}{dt} \iiint_{\Omega} \rho e_o d\mathcal{V} + \iint_{\partial\Omega} [\rho e_o(\mathbf{v} \cdot \mathbf{n}) + p\mathbf{v} \cdot \mathbf{n}] dS = \iiint_{\Omega} \rho \mathbf{f} \cdot \mathbf{v} d\mathcal{V}$$

- 2.4 How can the control volume formulations of the governing flow equations be used?
- 2.5 Check through the three conservation theorems (the integral forms for the conservation of mass, momentum and energy) and make sure you understand how to apply them for a specific case.
- 2.6 In [TME085 Formulas-Tables-and-Graphs.pdf](#), check the conservation laws. Make sure you are familiar with the notation.

### 3 One-Dimensional Flow

#### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C03.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

[TME085\\_One-Dimensional-Steady-Flow\\_Speed-Of-Sound.pdf](#)

[TME085\\_One-Dimensional-Steady-Flow\\_Normal-Shock-Relations.pdf](#)

[TME085\\_One-Dimensional-Steady-Flow\\_Hugoniot-Equation.pdf](#)

[TME085\\_One-Dimensional-Steady-Flow\\_Added-Heat.pdf](#)

[TME085\\_One-Dimensional-Steady-Flow\\_Friction.pdf](#)

[TME085\\_One-Dimensional-Steady-Flow\\_Friction-The-Fanno-Equation.pdf](#)

#### Theory Questions and Reading Instructions

##### Auxiliary Relations

- 3.1 How is the **speed of sound** defined, for any type of gas?
- 3.2 Derive the special formula for the **speed of sound**  $a$  for a **calorically perfect** gas:

$$a = \sqrt{\gamma RT}$$

starting from the general formula:

$$a^2 = \left( \frac{\partial p}{\partial \rho} \right)_s$$

- 3.3 What is the definition of **Mach number**?
- 3.4 How do we define **total conditions** in a steady-state isentropic flow?
- 3.5 How do we define **critical conditions** in a steady-state isentropic flow?
- 3.6 For a steady-state **isentropic** flow of a **calorically perfect** gas, derive the formula for  $T_0/T$ , making use of the fact that the total enthalpy  $h_0$  is constant along the streamlines.

$$\frac{T_0}{T} = 1 + \frac{1}{2}(\gamma - 1)M^2$$

- 3.7 How is the transformed Mach number  $M^*$  defined?



- 3.8 How is  $M^*$  related to the flow Mach number  $M$ ?
- 3.9 For a steady-state **adiabatic** compressible flow of **calorically perfect** gas, which of the variables  $p_0$  (total pressure) and  $T_0$  (total temperature) is/are constant along streamlines? Why?
- 3.10 What is the general definition (valid for any gas) of the **total conditions**  $p_0$ ,  $T_0$ ,  $\rho_0, \dots$  etc at some location in the flow?
- 3.11 In [TME085 Formulas-Tables-and-Graphs.pdf](#), check formulas on pages 5 – 6. You should recognize the formulas except for the “*chemically reacting mixture...*” case.

### Normal Shocks

- 3.12 How do we apply the control volume approach, for conservation of mass, momentum and energy, to this case in order to derive the three main equations?
- 3.13 How many additional relations are needed to solve for the conditions downstream of the shock, given the upstream conditions?
- 3.14 Is it possible to obtain an analytic solution to the shock problem for any type of gas?
- 3.15 The equation actually allow two solutions, one with an entropy increase across the shock (compression shock) and one with an entropy decrease across the shock (expansion shock). What thermodynamic principle guides us in the choice of the physically correct solution, and which solution is the correct one?
- 3.16 Describe how different variables (*e.g.* pressure, density, temperature, velocity, Mach number, total pressure, total temperature, entropy) change across a stationary normal shock, when going from the upstream side to the downstream side (following the flow).
- 3.17 Assume a steady-state 1D flow with a stationary normal shock. The fluid particles crossing the shock are subjected to
- (a) a pressure drop
  - (b) a density increase
  - (c) an entropy increase
  - (d) a temperature drop
  - (e) a deceleration

Which statements are true and which are false?

- 3.18 Are the normal-shock relations mathematically and physically valid for upstream Mach numbers lower than one? Justify your answer.
- 3.19 How come that the control volume approach applied to the governing equations for **adiabatic** form gives us the normal-shock relations? *i.e.*, how do the equations “know” that there is a shock inside of the control volume?
- 3.20 Explain in words what the **Hugoniot** equation is. How does this equation differ from the other normal-shock relations derived in Chapter 3?
- 3.21 In [TME085 Formulas-Tables-and-Graphs.pdf](#), check formulas on page 7. You should recognize the formulas and be familiar with the notation.

### One-Dimensional Flow with Heat Addition

- 3.22 How do we apply the control volume approach, for conservation of mass, momentum and energy, to this case in order to derive the three main equations?
- 3.23 Is it possible to obtain an analytic solution for any type of gas?
- 3.24 Derive the relation between the change in total temperature between point 1 and point 2 and the added heat per unit mass.
- 3.25 What is the **Rayleigh** curve and what does it tell us?
- 3.26 The general solution for **calorically perfect** gas is given in terms of conditions at point 1 and point 2. Describe how these formulas can be simplified by introducing the concept of **sonic point** conditions.
- 3.27 Describe how different variables (*e.g.* pressure, density, velocity, Mach number, total pressure, total temperature, entropy) change from point 1 to point 2 when heat is added.
- 3.28 How could heat addition theoretically be used to generate a supersonic flow, *i.e.* resemble the flow in a convergent-divergent nozzle?
- 3.29 Looking at the **Rayleigh** curve it's evident that removing heat leads to reduced entropy - how come that this is possible?
- 3.30 In one-dimensional flow with heat addition, what is  $q^*$ ?
- 3.31 What happens in the flow when heat is added if the flow is initially supersonic and subsonic, respectively
- 3.32 In [TME085\\_Formulas-Tables-and-Graphs.pdf](#), check formulas on pages 8 – 9. You should recognize the formulas and be familiar with the notation.

### One-Dimensional Flow with Friction

- 3.33 How do we apply the control volume approach, for conservation of mass, momentum and energy, to this case in order to derive the three main equations?
- 3.34 Is it possible to obtain an analytic solution for any type of gas?
- 3.35 What is the **Fanno** curve and what does it tell us?
- 3.36 The general solution for **calorically perfect** gas is given in terms of conditions at point 1 and point 2. Describe how these formulas can be simplified by introducing the concept of **sonic point** conditions.
- 3.37 Describe how different variables (*e.g.* pressure, density, velocity, Mach number, total pressure, temperature, entropy) change from point 1 to point 2 due to friction.
- 3.38 Does the total temperature  $T_0$  change due to friction?
- 3.39 Describe the **choking** of flow that occurs for pipe flow with friction. What happens if the real length  $L$  of a pipe is longer than  $L^*$  (for either subsonic flow or supersonic flow)?
- 3.40 In [TME085\\_Formulas-Tables-and-Graphs.pdf](#), check formulas on pages 10 – 11. You should recognize the formulas and be familiar with the notation.

## Recommended Problems

Problems solved in class: P3.8, P3.9, P3.10, P1 (exam 2009), P3.12, P3.13

Recommended home exercise: E3.5, E3.9, P3.4, E3.13, E3.17

*Ex.y and Px.y denotes text book examples and problems respectively*

## 4 Oblique Shock and Expansion Waves

### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C04.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

[TME085\\_Oblique-Shocks-and-Expansion-Waves\\_Expansion-Wave-Relations.pdf](#)

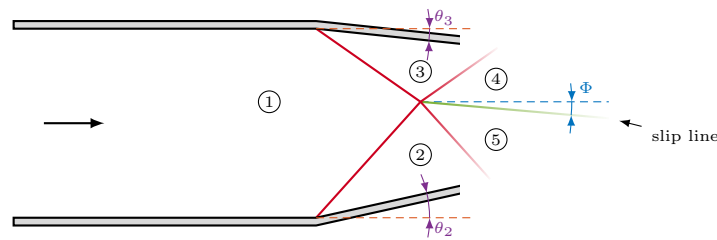
[TME085\\_Oblique-Shocks-and-Expansion-Waves\\_Prandtl-Meyer-Function.pdf](#)

### Theory Questions and Reading Instructions

#### Oblique Shocks

- 4.1 How is the control volume theorem applied to the stationary oblique shock?
- 4.2 How do the resulting equations differ from those of the stationary normal shock?
- 4.3 How can you apply the formulas for a stationary normal shock to compute a stationary oblique shock? How can you use Table A.2 in the book for an oblique shock?
- 4.4 How come that relations including total conditions derived for normal shocks must not be used for oblique shocks?
- 4.5 What is a **shock polar**?
- 4.6 What is the  $\theta$ - $\beta$ - $M$  relation?
- 4.7 The oblique shock generated by a two-dimensional wedge in a supersonic steady-state flow can be either of the **weak** type or the **strong** type. What is the main difference between these two shock types and which type is usually seen in reality?
- 4.8 How does the absolute Mach number change after a **weak/strong** stationary oblique shock?
- 4.9 What happens if there is no possible solution? What is the reason for the no-solution situation?
- 4.10 What kind of shock is obtained for a blunt body in supersonic flow?
- 4.11 For a **detached shock**, indicate where you will find a normal shock, a **strong** oblique shock, a **weak** oblique shock, the **sonic line**.
- 4.12 What is a **slip line**, and what conditions must be fulfilled across it? How can it be generated?
- 4.13 Draw a schematic **pressure-deflection diagram** and explain how it is obtained.
- 4.14 In steady-state 2D supersonic flow there are two types of shock reflection at solid walls. Name these two reflection types and describe the difference between them.

- 4.15 When an oblique is reflected at a wall (**regular reflection**), will the reflection angle be specular? Justify your answer.
- 4.16 What is a Mach wave and what is the angle of a Mach wave relative to the flow?
- 4.17 What are the constraints that lead to the generation of the separating line between regions 4 and 5 (represented by a green line in the figure below)? What is the reason for the need for this separating line?



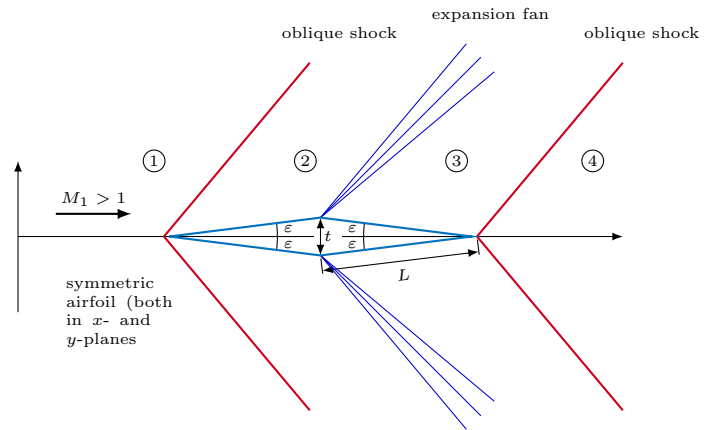
- 4.18 In [TME085\\_Formulas-Tables-and-Graphs.pdf](#), check formulas on pages 12 – 13. You should recognize the formulas for oblique shocks.

### Expansion Waves

- 4.19 What is a centered simple wave, also known as a **Prandtl-Meyer expansion**?
- 4.20 Is an expansion wave **isentropic** or **non-isentropic**?
- 4.21 Describe how you can use the **Prandtl-Meyer function** to compute the change in Mach number due to a given flow deflection.
- 4.22 In [TME085\\_Formulas-Tables-and-Graphs.pdf](#), check formulas on page 13. You should recognize the formulas for **Prandtl-Meyer expansion waves**. Also, look at Table A.5 and make sure that you know how to use the **Prandtl-Meyer function**.

### Shock-Expansion Theory

- 4.23 Describe how oblique shocks and expansion fans may be combined to compute the steady-state inviscid supersonic flow around airfoils that consist of straight edges, *e.g.* the diamond-wedge airfoil or the flat plate airfoil. Explain also how and where slip lines may develop.
- 4.24 Why do we always obtain a non-zero drag, even though the flow is inviscid?
- 4.25 In the figure below, at what angle of attack will the upper shock starting at the leading edge be replaced by an expansion fan?



## Recommended Problems

Problems solved in class: P4.1, P4.6, P1 (exam 2014-03-10),  
P4.10, E4.14, E4.15, P4 (exam 2009)

Recommended home exercise: E4.1, E4.6, E4.12, E4.13

*Ex.y and Px.y denotes text book examples and problems respectively*

## 5 Quasi One-Dimensional Flow

### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C05.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

[TME085\\_Quasi-One-Dimensional-Flow\\_Governing-Equations.pdf](#)

[TME085\\_Quasi-One-Dimensional-Flow\\_Area-Velocity-Relation.pdf](#)

[TME085\\_Quasi-One-Dimensional-Flow\\_Area-Mach-Relation.pdf](#)

[TME085\\_Quasi-One-Dimensional-Flow\\_Choked-Massflow.pdf](#)

### Theory Questions and Reading Instructions

- 5.1 How do we apply the control volume theorems to this type of flow?
- 5.2 How do the resulting equations differ from those of standard 1D flow?
- 5.3 What further assumption do we need to derive the **area-velocity relation**?
- 5.4 What are the implications of the **area-velocity relation** for quasi-one-dimensional flow?
- 5.5 What additional assumptions do we need to derive the **area-Mach relation**?
- 5.6 Check the shape of the **area-Mach relation** as given in Table A.1
- 5.7 Describe the **sub-critical** flow in a **converging-diverging** nozzle, in terms of how we move along the area-Mach curve.
- 5.8 Describe the **critical flow** in the same terms.
- 5.9 What is meant by **choked** flow in a **converging-diverging** nozzle?
- 5.10 Describe how the flow in a **converging-diverging** nozzle develops as we gradually increase the nozzle pressure ratio  $p_0/p_e$ .
- 5.11 What do we mean by an **overexpanded** nozzle flow?
- 5.12 What do we mean by an **underexpanded** nozzle flow?
- 5.13 What do we mean by a **pressure-matched** nozzle flow?
- 5.14 Under what conditions will we get a normal shock standing at the exit plane of a **convergent-divergent** nozzle?
- 5.15 Explain the consequence of **free-boundary reflection** for the external flow of a nozzle operating at **overexpanded** conditions.
- 5.16 Assume a steady-state flow through a pipe with varying cross-section area. If the pipe has negligible heat transfer and wall friction and there are no shocks, then the flow is

- (a) adiabatic
- (b) isentropic
- (c) isobaric (constant pressure)
- (d) isenthalpic (constant enthalpy)

Which statements are true and which are false?

5.17 Assume a steady-state flow in a **convergent-divergent** nozzle. Describe what characterizes the following operating conditions:

- (a) Sub-critical nozzle flow
- (b) Over-expanded nozzle flow
- (c) Under-expanded nozzle flow

5.18 Derive the area-velocity relation in quasi-1D flow starting from the mass conservation relation

$$d(\rho u A) = 0.,$$

Euler's equation

$$dp = -\rho u du,$$

and the definition of the speed of sound

$$a^2 = \left( \frac{\partial p}{\partial \rho} \right)_s$$

5.19 What simplifications are made when analyzing a convergent-divergent nozzle flow using a quasi-1D approach? What are the main limitations of such an analysis?

5.20 Assume that we would have a **Nozzle Pressure Ratio** (NPR) between the **normal-shock-at-exit** NPR and the NPR defining lower limit of **choked nozzle flow**, would it be possible to use the **area-Mach-number** relation throughout the nozzle? Justify and explain why or why not.

5.21 In [TME085 Formulas-Tables-and-Graphs.pdf](#), check formulas on pages 14 – 15. You should recognize the area-velocity and area-Mach relations, the choked mass flow relation and the equation giving the exit Mach number for cases with an internal normal shock.

## Recommended Problems

Problems solved in class: P5.1

Recommended home exercise: P5.2, P5.5, E5.7, P5.11

*Ex.y and Px.y denotes text book examples and problems respectively*



## 6 Differential Conservation Equations for Inviscid Flows

### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C06.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

[TME085\\_Governing-Equations\\_Differential-Form.pdf](#)

[TME085\\_Governing-Equations\\_Alternative-Forms-of-the-Energy-Equation.pdf](#)

[TME085\\_Governing-Equations\\_Entropy-Equation.pdf](#)

[TME085\\_Governing-Equations\\_Croccos-Equation.pdf](#)

### Theory Questions and Reading Instructions

- 6.1 How can you derive (describe in words only) the PDE:s in conservation form from the control volume formulations for the conservation of mass, momentum and energy?
- 6.2 What is the criterium for classifying a PDE as being in **conservation form**?
- 6.3 Derive the continuity equation in **non-conservation** form from the corresponding **conservation form**.
- 6.4 How can the **substantial derivative** operator be interpreted physically?
- 6.5 Which of the alternative versions of the energy equation in **non-conservation form** is simply an expression of the **first law of thermodynamics**?
- 6.6 Prove, by using one of the **non-conservation** forms of the energy equation, that for steady-state, adiabatic flow with no body forces the total enthalpy is preserved along stream-lines.
- 6.7 Derive Crocco's relation starting from the momentum equation and the energy equation (the first and second law of thermodynamics)
- 6.8 Describe in words the significance of **Crocco's equation**.
- 6.9 What does Crocco's relation say about the flow behind a curved shock?
- 6.10 Prove, by using a suitable equation, that a steady-state irrotational flow with constant total enthalpy must also be isentropic.

## 7 Unsteady Wave Motion

### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C07.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

[TME085\\_Unsteady-Waves\\_Shock-Relations.pdf](#)

[TME085\\_Unsteady-Waves\\_Reflected-Shocks.pdf](#)

[TME085\\_Unsteady-Waves\\_Acoustic-Wave-Propagation.pdf](#)

[TME085\\_Unsteady-Waves\\_Finite-Non-Linear-Waves.pdf](#)

### Theory Questions and Reading Instructions

#### Moving Normal Shocks

- 7.1 What is the connection between a stationary and a moving normal shock?
- 7.2 Think about the oblique shock from an object flying through the air at supersonic velocity. How will an observer on the ground experience the shock wave?
- 7.3 Suppose you are given a problem in which there is a moving normal shock but the gas velocities are non-zero on both sides of the shock. How would you transform the problem so as to make it simpler to solve?
- 7.4 Is the **Hugoniot equation** valid for a moving normal shock?
- 7.5 For normal shock moving into a stagnant **calorically perfect** gas, how can we apply the formulas for a stationary normal shock?
- 7.6 Derive an the following expression for the induced flow velocity behind a moving shock

$$u_p = \frac{a_1}{\gamma} \left( \frac{p_2}{p_1} - 1 \right) \left[ \frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}} \right]^{1/2}$$

- 7.7 Describe what happens when a moving normal shock hits a solid wall.
- 7.8 A stationary normal shock with upstream Mach number  $M_1$  ( $M_1 > 1$ ) is compared to a moving normal shock, traveling with Mach number  $M_S$  into quiescent (non-moving) air. If  $M_1 = M_S$ , is there any physical difference between the two shock waves apart from the fact that they have different speeds relative to the observer?
- 7.9 The moving shock is an adiabatic process in the same way as a stationary normal shock is adiabatic. Does this mean that total enthalpy is constant over the moving shock? Explain why/why not.

- 7.10 Can a moving normal shock travel at a speed lower than the speed of sound? Explain why/why not.
- 7.11 What is the maximum Mach number of the induced flow behind a moving shock?
- 7.12 In [TME085.Formulas-Tables-and-Graphs.pdf](#), check formulas on pages 19 – 20. You should recognize the relation between the shock Mach number  $M_s$  and the shock pressure ratio  $p_2/p_1$ , the formula for the induced velocity up and the formulas for the temperature and density ratios. Also the formula for the Mach number  $M_R$  of a reflected shock and how its velocity  $W_R$  is computed.

### Finite Non-Linear Waves

- 7.13 Which two equations are used to derive the **Riemann Invariants**?
- 7.14 What are the assumptions in this case?
- 7.15 How are the two **characteristic** curves  $C^+$  and  $C^-$  defined?
- 7.16 Describe the **Method of Characteristics (MOC)** as a means of computing the time evolution of inviscid flow.
- 7.17 Can we apply **MOC** when there are shocks present?
- 7.18 Describe a left-going simple expansion wave in terms of the **characteristics**  $C^+$ ,  $C^-$  and **Riemann invariants**  $J_+$ ,  $J_-$ .
- 7.19 Derive the expansion wave relations
- 7.20 An unsteady expansion wave is traveling inside a tube in which viscous effects are found to be negligible. Which of the following variables are constant throughout the expansion wave?
- (a) pressure
  - (b) temperature
  - (c) entropy
  - (d) density
- 7.21 In [TME085.Formulas-Tables-and-Graphs.pdf](#), check formulas on pages 21 – 22. You should recognize the relations governing a left-going simple expansion wave where condition 4 is the original stagnant condition on the left side (the head of the expansion wave) and condition 3 is the tail of the expansion wave.

### The Shock Tube

- 7.22 Which types of waves or discontinuities are generated in shock tubes?
- 7.23 Which is the **driver section** and which is the **driven section**?
- 7.24 How does the shock tube solution depend on  $x$  and  $t$ ?
- 7.25 Draw a typical solution in an  $x-t$  diagram, where you describe the phenomena and how they are placed in relation to each other and in relation to the **driver section** and **driven section**.

- 7.26 In shock tubes, unsteady **contact discontinuities** are sometimes found. Describe in words what they are and under which circumstances they may be formed. Which of the variables  $p$ ,  $T$ ,  $\rho$ ,  $u$ ,  $s$  is/are necessarily continuous across such a **contact discontinuity**?
- 7.27 Is the **expansion fan** an isentropic or non-isentropic wave?
- 7.28 What is the shock tube used for in laboratories around the world?
- 7.29 What types of waves or discontinuities are generated in a shock tube with two initially stagnant regions at different pressure (separated by a thin membrane which is removed very quickly)?
- 7.30 In [TME085 Formulas-Tables-and-Graphs.pdf](#), check formulas on page 22. You should recognize the shock tube solution for calorically perfect gas. Note that since conditions 1 and 4 usually are given, the equation must be solved iteratively to obtain the intermediate pressure  $p_2$  ( $= p_3$ ). Once this is known, the expansion fan, the contact discontinuity and the moving shock are all defined.

### Acoustic Theory

- 7.31 Which flow equations and what assumptions are used to derive the acoustic equations?
- 7.32 How can the acoustic equations be combined into one single (classical) wave equation?
- 7.33 Give the general solution to the classical wave equation in one space dimension and time.
- 7.34 Describe how the relations between pressure, density and velocity amplitudes may be obtained.
- 7.35 What is the difference between acoustic waves and other types of waves such as shock waves and expansion waves?

### Recommended Problems

Problems solved in class: P7.2, P7.3, P7.5, P7.8, P3 (exam 2009)

Recommended home exercise: E7.1, E7.2, E7.5, P7.10

*Ex.y and Px.y denotes text book examples and problems respectively*

## 12 The Time-Marching Technique

### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C12.pdf](#)

[TME085\\_C06.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

### Theory Questions and Reading Instructions

- 12.1 Explain the concepts zone of influence and zone of dependence?
- 12.2 Describe in words how a **finite-volume spatial discretization** can be achieved.
- 12.3 Which equations are suitable to use when deriving a **finite-volume spatial discretization** for compressible flow?
- 12.4 What is meant by the term **density-base** when discussing CFD codes for compressible flow?
- 12.5 What is meant by the term **fully coupled** when discussing CFD codes for compressible flow?
- 12.6 When the governing equations are discretized using a **finite-volume** approach, cell face values of flow properties appears in the equations. How are these values approximated?
- 12.7 Derive a **third-order upwind scheme** using a four-cell stencil.
- 12.8 What do we mean when we say that a CFD code for compressible flow is **conservative**?
- 12.9 Why is it important that a CFD code for compressible flow is **conservative**?
- 12.10 When applying a **time-marching** flow solution scheme the so-called **CFL number** is an important parameter. Define the **CFL number** and describe its significance.
- 12.11 What is a typical maximum **CFL number** for stable operation when applying an **explicit time stepping scheme**?
- 12.12 How can we use our knowledge of **characteristics** (and their speed of propagation) to guide us when determining suitable **boundary conditions** for compressible flows?
- 12.13 When applying a CFD code for **unsteady** compressible flow, which of the following choices would you make: **density based** or **pressure based**, **fully coupled** or **segregated**, **conservative** or **non-conservative**, **explicit** or **implicit** time stepping?
- 12.14 An engineer wants to apply a numerical solution scheme for compressible flow. The flow he is interested in contains shocks. He has to choose between two different solution methods – one which is based on the **conservation** form of the governing equations and one which is based on the **non-conservation** form of the governing equations. Which method should he choose and why?

*Hint: go through the lecture notes for Chapter 6.*

## 16 Properties of High-Temperature Gases

### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C16.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

### Theory Questions and Reading Instructions

- 16.1 What are the fundamental modes or forms of energy of a gas molecule?
- 16.2 How does a **mono-atomic** gas differ from a diatomic gas in terms of energy modes?
- 16.3 What is the difference between a **linear polyatomic molecule** and a **nonlinear polyatomic molecule** in terms of **degrees of thermal freedom**?
- 16.4 What does it mean that the energy is **quantized**?
- 16.5 Explain the concept **zero-point energy**.
- 16.6 Explain the concept **energy state**.
- 16.7 What does **macrostate** and **microstate** mean, respectively?
- 16.8 Try to explain what the **Boltzmann distribution** describes and what sparsely populated implies.
- 16.9 Show that the ratio of specific heats ( $\gamma$ ) is 1.4 for a molecule with only **translational** and **rotational energy**. What will the **ratio of specific heats** be for a **monoatomic** gas under the same conditions (explain why)?
- 16.10 What is the difference between a **calorically perfect** gas and a **thermally perfect** gas?
- 16.11 In what temperature range (approximately) can a gas be assumed to be **calorically perfect**?
- 16.12 What happens with the molecules in air at approximately 2500K, 4000K, and 9000K?
- 16.13 A mixture of chemically reacting perfect gases, where the reactions are always in **equilibrium**, may be thermodynamically described as a single-species gas. How does this thermodynamic description differ from that of a calorically perfect or thermally perfect gas?
- 16.14 What is meant by an **equilibrium gas**?
- 16.15 Figure 16.11 on page 629 shows how  $C_v/R$  for a **diatomic** gas develops with temperature. Try to explain the development based on the theory covered in section 16.8. What would the figure look like for a **mono-atomic** gas?

## 17 High-Temperature Flows: Basic Examples

### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_C17.pdf](#)

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

### Theory Questions and Reading Instructions

- 17.1 Assume that you would try to predict the stagnation temperature for a reentry vehicle approaching earth at a Mach number of 32.5 using a model based on a **calorically perfect** gas assumption. At this Mach number there will be a strong shock in front of the vehicle and with the **calorically perfect** gas assumption you will severely overestimate the temperature ratio over the shock - explain why.
- 17.2 Define **thermodynamic** and **chemical equilibrium**.
- 17.3 Explain the concepts **local thermodynamic equilibrium** and **global thermodynamic equilibrium**.
- 17.4 Explain the concepts **local chemical equilibrium** and **global chemical equilibrium**.
- 17.5 What gas model is applicable in each of the cases listed below?
- **local thermodynamic equilibrium** and **local chemical equilibrium**
  - **local thermodynamic equilibrium** and **chemical non-equilibrium**
  - **local thermodynamic equilibrium** and **frozen composition**
  - **thermodynamic non-equilibrium** and **frozen composition**
- 17.6 If the flow is both **chemically** and **vibrationally frozen** we will be able to treat the flow in the same way as if it would have been **calorically perfect** - explain why.
- 17.7 Is equilibrium chemically reacting flow through a **convergent-divergent nozzle** without the presence of normal shocks **isentropic**?
- 17.8 Can we still use the **area-velocity relation** for **convergent-divergent nozzle** flows derived in Chapter 5 at elevated temperatures?
- 17.9 Consider the normal shock relations derived in Chapter 3. What are the differences when analyzing a normal shock strong enough for **high-temperature effects** to be important?
- 17.10 High temperature effects in compressible flows are found when analyzing for example very strong shocks or nozzle flows with extremely high total pressure and total enthalpy. What is the root cause of these effects and what do we mean by **equilibrium gas**? What kind of thermodynamic relations are needed to compute the flow of **equilibrium gas**?
- 17.11 Using **equilibrium gas** assumption in the analysis of chemically reacting nozzle flow will lead to higher exhaust temperatures than if calorically perfect gas assumption is used for the same analysis. Explain why.

## 18 Aircraft Aerodynamics

### Related Documents

Answers to the questions can be found in the course book (*J. D. Anderson Modern Compressible Flow 4<sup>th</sup> ed.*) and in the complementary course material:

[TME085\\_Formulas-Tables-and-Graphs.pdf](#)

### Theory Questions and Reading Instructions

QUESTIONS ON AIRCRAFT AERODYNAMICS WILL BE ADDED HERE