Compressible Flow TME085

Quasi-One-Dimensional Flow

The Area-Velocity Relation

Division of Fluid Dynamics
Department of Mechanics and Maritime Sciences
Chalmers University of Technology

Governing Equations

The governing equations for quasi-one-dimensional form are the continuity, momentum, and energy equations on differential form where a term accounting for the variation in cross-section area is added in the momentum equation.

$$d(\rho u A) = 0 \tag{1}$$

$$dp = -\rho u du \tag{2}$$

$$dh + udu = 0 \tag{3}$$

The Area-Velocity Relation

Starting point - the continuity equation (Eqn. 1):

$$d(\rho u A) = 0 \Rightarrow \rho u dA + \rho A du + u A d\rho = 0$$

divide by $\rho u A$ gives

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0 \tag{4}$$

As the name suggests, the area-velocity relation is a relation including the area and the flow velocity. Therefore, the next step is to replace the density terms.

This can be achieved using the momentum equation (Eqn. 2)

$$dp = -\rho u du \Leftrightarrow \frac{dp}{\rho} = -udu$$

$$\frac{dp}{\rho} = \frac{dp}{d\rho}\frac{d\rho}{\rho} = -udu$$

If we assume adiabatic and reversible flow processes, i.e., isentropic flow

$$\frac{dp}{d\rho} = \left(\frac{dp}{d\rho}\right)_s = a^2 \Rightarrow a^2 \frac{d\rho}{\rho} = -udu$$

$$a^2 \frac{d\rho}{\rho} = -udu = -u^2 \frac{du}{u}$$

$$\frac{d\rho}{\rho} = -M^2 \frac{du}{u} \tag{5}$$

Eqn. 5 inserted in Eqn. 4 gives

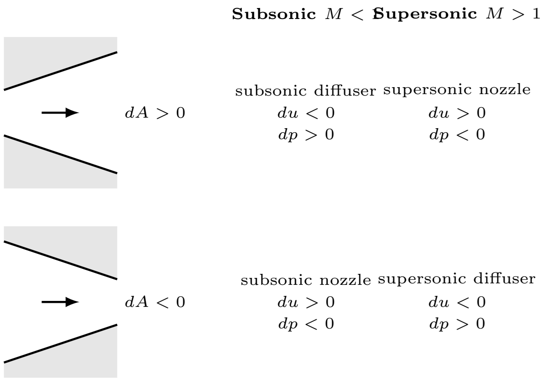
$$-M^2 \frac{du}{u} + \frac{du}{u} + \frac{dA}{A} = 0$$

or

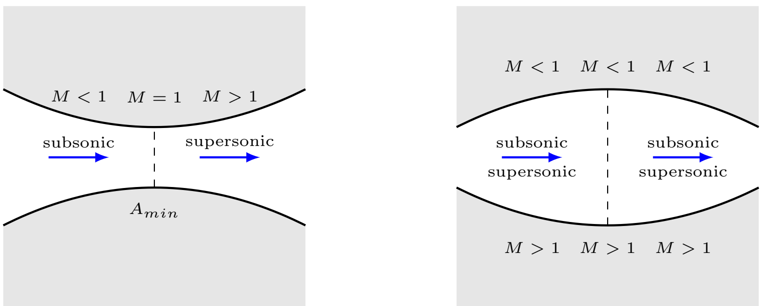
$$\frac{dA}{A} = (M^2 - 1) \frac{du}{u} \tag{6}$$

which is the area-velocity relation.

From the area-velocity relation (Eqn. 6), we can learn that in a subsonic flow, the flow will accelerate if the cross-section area is decreased and decelerate if the cross-section area is increased. It can also be seen that for supersonic flow, the relation between flow velocity and cross-section area will be the opposite of that for subsonic flows, see Fig. 1. For sonic flow, $M = 1$, the relation shows that $dA = 0$, which means that sonic flow can only occur at a cross-section area maximum or minimum. From the subsonic versus supersonic flow discussion, it can be understood that sonic flow at the minimum cross section area is the only valid option (see Fig. 2).



Figur 1: Area-velocity relation - subsonic flow vs. supersonic flow



Figur 2: Area-velocity relation - sonic flow