Compressible Flow TME085

One-Dimensional Steady Flow

Speed of Sound

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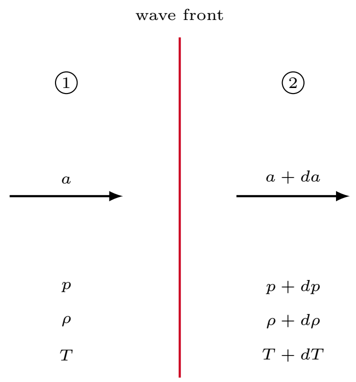
Sound Wave



Figur 1: sound wave

In Fig. 1, station 1 represents the flow state ahead of the sound wave and station 2 the flow state behind the sound wave. Set up the continuity equation for one-dimensional flows between 1 and 2. If we could change frame of reference and follow the sound wave, we would see fluid approaching the wave with the propagation speed of the wave, a, and behind the wave, the fluid would have a slightly modified speed, $a + da$. There would also be a slight in all other flow properties. Let's apply the one-dimensional continuity equation between station 1 and station 2.

$$\rho_1 u_1 = \rho_2 u_2$$

$$\rho a = (\rho + d\rho)(a + da)$$

$$\cancel{\rho a} = \cancel{\rho a} + \rho da + a d\rho + \underbrace{d\rho da}_{\sim 0} \Rightarrow$$

$$a = -\rho \frac{da}{d\rho} \tag{1}$$

The one-dimensional momentum equation between station 1 and station 2 gives

$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$$

$$\rho a^2 + p = (\rho + d\rho)(a + da)^2 + (p + dp)$$

$$\cancel{\rho a^2} + \cancel{p} = \cancel{\rho a^2} + 2\rho a da + \underbrace{\rho da^2}_{\sim 0} + d\rho a^2 + \underbrace{2d\rho a da}_{\sim 0} + \underbrace{d\rho da^2}_{\sim 0} + \cancel{p} + dp \Rightarrow$$

$$dp = -2\rho a da - d\rho a^2 \Rightarrow$$

$$da = -\frac{dp + d\rho a^2}{2\rho a} = -\frac{d\rho}{2a\rho}\left(\frac{dp}{d\rho} + a^2\right) \Rightarrow$$

$$\frac{da}{d\rho} = -\frac{1}{2a\rho}\left(\frac{dp}{d\rho} + a^2\right) \tag{2}$$

Eqn. 2 in 1 gives

$$a = \frac{1}{2a}\left(\frac{dp}{d\rho} + a^2\right) \Rightarrow$$

$$a^2 = \frac{dp}{d\rho} \tag{3}$$

Sound wave:

- gradients are small
- irreversible (dissipative effects are negligible)
- no heat addition

Thus, the change of flow properties as the sound wave passes can be assumed to be an isentropic process

$$a^2 = \left(\frac{dp}{d\rho}\right)_s \tag{4}$$

$$a = \sqrt{\left(\frac{dp}{d\rho}\right)_s} = \sqrt{\frac{1}{\rho\tau_s}} \tag{5}$$

where τ_s is the compressibility of the gas. Eqn. 5 is valid for all gases. It can be seen from the equation, that truly incompressible flow ($\tau_s = 0$) would imply infinite speed of sound.

Since the process is isentropic, we can use the isentropic relations if we also assume the gas to be calorically perfect

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma \Rightarrow p = C\rho^\gamma$$

$$a^2 = \left(\frac{dp}{d\rho}\right)_s = \gamma C\rho^{\gamma-1} = \gamma \underbrace{\left[C\rho^\gamma\right]}_{=p} \rho^{-1} = \frac{\gamma p}{\rho} \Rightarrow$$

$$a = \sqrt{\frac{\gamma p}{\rho}} \tag{6}$$

or

$$a = \sqrt{\gamma R T} \tag{7}$$