Compressible Flow TME085

One-Dimensional Steady Flow

Flow with Friction

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Governing Equations

The starting point is the governing equations for one-dimensional steady-state flow

Continuity

$$\rho_1 u_1 = \rho_2 u_2 \tag{1}$$

Momentum

$$\rho_1 u_1^2 + p_1 - \bar{\tau}_w \frac{bL}{A} = \rho_2 u_2^2 + p_2 \tag{2}$$

where $\bar{\tau}_w$ is the average wall-shear stress

$$\bar{\tau}_w = \frac{1}{L}\int_0^L \tau_w dx \tag{3}$$

b is the tube perimeter, and L is the tube length. For circular cross sections

$$\frac{bL}{A} = \left\{ A = \frac{\pi D^2}{4}, b = \pi D \right\} = \frac{4L}{D}$$

and thus

$$\rho_1 u_1^2 + p_1 - \frac{4}{D}\int_0^L \tau_w dx = \rho_2 u_2^2 + p_2 \tag{4}$$

Energy

$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2 \tag{5}$$

Differential Form

In order to remove the integral term in the momentum equation, the governing equations are written in differential form

Continuity

$$\rho_1 u_1 = \rho_2 u_2 = const \Rightarrow$$

$$\frac{d}{dx}(\rho u) = 0 \tag{6}$$

Momentum

$$(\rho_2 u_2^2 + p_2 - \rho_1 u_1^2 + p_1) = -\frac{4}{D}\int_0^L \tau_w dx \Rightarrow$$

$$\frac{d}{dx}(\rho u^2 + p) = -\frac{4}{D}\tau_w \tag{7}$$

$$\frac{d}{dx}(\rho u^2 + p) = \rho u \frac{du}{dx} + u\frac{d}{dx}(\rho u) + \frac{dp}{dx} = \left\{\frac{d}{dx}(\rho u) = 0\right\} = \rho u \frac{du}{dx} + \frac{dp}{dx}$$

$$\rho u \frac{du}{dx} + \frac{dp}{dx} = -\frac{4}{D}\tau_w \tag{8}$$

The wall shear stress is often approximated using a shear-stress factor, f, according to

$$\tau_w = f\frac{1}{2}\rho u^2 \tag{9}$$

and thus

$$\rho u \frac{du}{dx} + \frac{dp}{dx} = -\frac{2}{D} f \rho u^2 \tag{10}$$

Energy

$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2 = const$$

$$h_{o_1} = h_{o_2} = const$$

$$\frac{d}{dx}h_o = 0 \tag{11}$$

Summary

continuity:

$$\frac{d}{dx}(\rho u) = 0$$

momentum:

$$\rho u\frac{du}{dx} + \frac{dp}{dx} = -\frac{2}{D}f\rho u^2$$

energy:

$$\frac{d}{dx}h_o = 0$$