Compressible Flow TME085

One-Dimensional Steady Flow

Flow with Added Heat

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Added Heat Relations

The aim is to derive relations for pressure ratio and temperature ratio as a function of Mach numbers. We will do that starting from the momentum equation.

$$p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2 \tag{1}$$

Assuming calorically perfect gas

$$\rho u^2 = \rho a^2 M^2 = \rho \frac{\gamma p}{\rho} M^2 = \gamma p M^2$$

which inserted in Eqn. 1 gives

$$p_2 - p_1 = \gamma p_1 M_1^2 - \gamma p_2 M_2^2$$

$$p_2(1 + \gamma M_2^2) = p_1(1 + \gamma M_1^2)$$

and thus

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \tag{2}$$

From the equation of state $p = \rho R T$, we get

$$\frac{T_2}{T_1} = \frac{p_2}{\rho_2 R} \frac{\rho_1 R}{p_1} = \frac{p_2}{p_1} \frac{\rho_1}{\rho_2} \tag{3}$$

Using the continuity equation, we can get ρ_1/ρ_2

$$\rho_1 u_1 = \rho_2 u_2 \Rightarrow \frac{\rho_1}{\rho_2} = \frac{u_2}{u_1}$$

Inserted in Eqn. 3 gives

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{u_2}{u_1} \tag{4}$$

$$\frac{u_2}{u_1} = \frac{M_2 a_2}{M_1 a_1} = \frac{M_2}{M_1}\frac{\sqrt{\gamma R T_2}}{\sqrt{\gamma R T_1}} = \frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}} \tag{5}$$

Eqn. 5 in Eqn. 4 gives

$$\sqrt{\frac{T_2}{T_1}} = \frac{p_2}{p_1}\frac{M_2}{M_1} \tag{6}$$

With p_2/p_1 from Eqn. 2, Eqn 6 becomes

$$\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \left(\frac{M_2}{M_1}\right)^2 \tag{7}$$