



Compressible Flow TME085

Oblique Shocks and Expansion Waves

Expansion Wave Relations

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The aim is to derive relations of temperature, pressure and density over an expansion wave. Total temperature upstream and downstream of the expansion wave is calculated as

$$\frac{T_{o1}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \quad (1)$$

$$\frac{T_{o2}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2 \quad (2)$$

The temperature ratio over the expansion wave may now be calculated as

$$\frac{T_2}{T_1} = \frac{T_2}{T_{o2}} \frac{T_{o1}}{T_1} = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} = \frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2}$$

The expansion is isentropic and thus total temperature and both total pressure are unaffected by the expansion. Therefore, $T_{o1} = T_{o2}$ and thus

$$\frac{T_2}{T_1} = \frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2} \quad (3)$$

The pressure and density ratios can be obtained from Eqn. 3 using the isentropic relations

$$\frac{p_2}{p_1} = \left[\frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2} \right]^{\gamma/(\gamma-1)} \quad (4)$$

$$\frac{\rho_2}{\rho_1} = \left[\frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2} \right]^{1/(\gamma-1)} \quad (5)$$