Compressible Flow TME085

Compressible Inviscid Fluid Flow Equations

The Entropy Equation

Division of Fluid Dynamics
Department of Mechanics and Maritime Sciences
Chalmers University of Technology

From the second law of thermodynamics

$$\frac{De}{Dt} = T\frac{Ds}{Dt} - p\frac{D}{Dt}\left(\frac{1}{\rho}\right) \tag{1}$$

From the energy equation on differential non-conservation form internal energy formulation

$$\frac{De}{Dt} = \dot{q} - \frac{p}{\rho}(\nabla \cdot \mathbf{v})$$

The continuity equation on differential non-conservation form

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{v}) = 0 \Rightarrow \nabla \cdot \mathbf{v} = -\frac{1}{\rho}\frac{D\rho}{Dt}$$

and thus

$$\frac{De}{Dt} = \dot{q} + \frac{p}{\rho^2}\frac{D\rho}{Dt}$$

$$\frac{D\rho}{Dt} = -\frac{1}{\nu^2}\frac{D\nu}{Dt}$$

$$\rho\frac{De}{Dt} = \rho\dot{q} - \frac{p}{\rho\nu^2}\frac{D\nu}{Dt} = \rho\dot{q} - \rho p\frac{D\nu}{Dt}$$

$$\rho\left[\frac{De}{Dt} + p\frac{D\nu}{Dt} - \dot{q}\right] = 0 \Rightarrow \frac{De}{Dt} = \dot{q} - p\frac{D\nu}{Dt}$$

Insert De/Dt in Eqn. 1

$$\dot{q} - p\frac{D}{Dt}\left(\frac{1}{\rho}\right) = T\frac{Ds}{Dt} - p\frac{D}{Dt}\left(\frac{1}{\rho}\right) \Rightarrow$$

$$T\frac{Ds}{Dt} = -\dot{q} \tag{2}$$

Adiabatic flow:

$$T\frac{Ds}{Dt} = 0 \tag{3}$$

In an adiabatic, steady-state, inviscid flow, entropy is constant along a streamline.