Compressible Flow TME085

Compressible Inviscid Fluid Flow Equations

Crocco's Equation

Division of Fluid Dynamics
Department of Mechanics and Maritime Sciences
Chalmers University of Technology

The momentum equation without body forces

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p$$

Expanding the substantial derivative

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p$$

The first and second law of thermodynamics gives

$$T\nabla s = \nabla h - \frac{\nabla p}{\rho}$$

Insert ∇p from the momentum equation

$$T\nabla s = \nabla h + \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}$$

Definition of total enthalpy (h_o)

$$h_o = h + \frac{1}{2}\mathbf{v} \cdot \mathbf{v} \Rightarrow \nabla h = \nabla h_o - \nabla \left(\frac{1}{2}\mathbf{v} \cdot \mathbf{v} \right)$$

The last term can be rewritten as

$$\nabla \left(\frac{1}{2}\mathbf{v} \cdot \mathbf{v} \right) = \mathbf{v} \times (\nabla \times \mathbf{v}) + \mathbf{v} \cdot \nabla \mathbf{v}$$

which gives

$$\nabla h = \nabla h_o - \mathbf{v} \times (\nabla \times \mathbf{v}) - \mathbf{v} \cdot \nabla \mathbf{v}$$

Insert ∇h in the entropy equation gives

$$T\nabla s = \nabla h_o - \mathbf{v} \times (\nabla \times \mathbf{v}) - \cancel{\mathbf{v} \cdot \nabla \mathbf{v}} + \frac{\partial \mathbf{v}}{\partial t} + \cancel{\mathbf{v} \cdot \nabla \mathbf{v}}$$

$$T\nabla s = \nabla h_o - \mathbf{v} \times (\nabla \times \mathbf{v}) + \frac{\partial \mathbf{v}}{\partial t}$$