Compressible Flow TME085

Basic Concepts

Compressibility

Division of Fluid Dynamics
Department of Mechanics and Maritime Sciences
Chalmers University of Technology

Compressible Effects

If density fluctuations are significant, compressible effects needs to be accounted for, but the question is what significant means. Anderson suggests that fluctuations in the order of 5% ($\Delta\rho/\rho > 0.05$) could be used as a threshold value. The question is still what that means.

Let's try to figure out...

The compressibility for isothermal process is defined as

$$\tau_T = \left(\frac{1}{\rho}\right)\left(\frac{\partial\rho}{\partial p}\right)_T \tag{1}$$

$$\tau_T = \left(\frac{1}{\rho}\right)\left(\frac{\partial\rho}{\partial p}\right)_T = \{p = \rho RT,\ T = const\} = \left(\frac{RT}{p}\right)\left(\frac{\partial}{\partial p}\left(\frac{p}{RT}\right)\right)_T$$

and thus

$$\tau_T = \frac{1}{p} \tag{2}$$

We are trying to find an estimate of $\Delta\rho/\rho$. Using the equation of state with constant temperature, $\Delta\rho/\rho$ can be expressed in terms of pressure

$$\frac{\Delta\rho}{\rho} = \frac{\Delta p}{RT}\frac{RT}{p} = \left\{\tau_T = \frac{1}{p}\right\} = \tau_T\Delta p \tag{3}$$

If we assume that compressible effects are not significant and use Bernoulli's equation to get an estimate of the pressure fluctuations generated by a flow

$$\Delta p \approx \frac{1}{2}\rho_\infty U_\infty^2 \tag{4}$$

With $\tau_T = 1/p_\infty$, we can rewrite Eqn. 4 as

$$\frac{\Delta\rho}{\rho_\infty} \approx \frac{1}{p_\infty}\left(\frac{1}{2}\rho_\infty U_\infty^2\right) = \{p_\infty = \rho_\infty RT_\infty\} = \frac{\rho_\infty U_\infty^2}{2\rho_\infty RT_\infty} = \frac{U_\infty^2}{2RT_\infty} \tag{5}$$

The speed of sound in the freestream is obtained as $a_\infty = \sqrt{\gamma RT_\infty}$, which gives

$$\frac{\Delta\rho}{\rho_\infty} \approx \frac{\gamma U_\infty^2}{2a_\infty^2} = \frac{\gamma}{2} M_\infty^2 \tag{6}$$

For air ($\gamma = 1.4$) and $\Delta\rho/\rho_\infty < 0.05$ we get $M_\infty < 0.27$