



# Compressible Flow TME085

Basic Concepts

Compressibility

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## Compressible Effects

If density fluctuations are significant, compressible effects needs to be accounted for, but the question is what significant means. Anderson suggests that fluctuations in the order of 5% ( $\Delta\rho/\rho > 0.05$ ) could be used as a threshold value. The question is still what that means.

Let's try to figure out...

The compressibility for isothermal process is defined as

$$\tau_T = \left(\frac{1}{\rho}\right) \left(\frac{\partial\rho}{\partial p}\right)_T \quad (1)$$

$$\tau_T = \left(\frac{1}{\rho}\right) \left(\frac{\partial\rho}{\partial p}\right)_T = \{p = \rho RT, T = const\} = \left(\frac{RT}{p}\right) \left(\frac{\partial}{\partial p} \left(\frac{p}{RT}\right)\right)_T$$

and thus

$$\tau_T = \frac{1}{p} \quad (2)$$

We are trying to find an estimate of  $\Delta\rho/\rho$ . Using the equation of state with constant temperature,  $\Delta\rho/\rho$  can be expressed in terms of pressure

$$\frac{\Delta\rho}{\rho} = \frac{\Delta p}{RT} \frac{RT}{p} = \left\{ \tau_T = \frac{1}{p} \right\} = \tau_T \Delta p \quad (3)$$

If we assume that compressible effects are not significant and use Bernoulli's equation to get an estimate of the pressure fluctuations generated by a flow

$$\Delta p \approx \frac{1}{2} \rho_\infty U_\infty^2 \quad (4)$$

With  $\tau_T = 1/p_\infty$ , we can rewrite Eqn. 4 as

$$\frac{\Delta\rho}{\rho_\infty} \approx \frac{1}{p_\infty} \left(\frac{1}{2} \rho_\infty U_\infty^2\right) = \{p_\infty = \rho_\infty RT_\infty\} = \frac{\rho_\infty U_\infty^2}{2\rho_\infty RT_\infty} = \frac{U_\infty^2}{2RT_\infty} \quad (5)$$

The speed of sound in the freestream is obtained as  $a_\infty = \sqrt{\gamma RT_\infty}$ , which gives

$$\frac{\Delta\rho}{\rho_\infty} \approx \frac{\gamma U_\infty^2}{2a_\infty^2} = \frac{\gamma}{2} M_\infty^2 \quad (6)$$

For air ( $\gamma = 1.4$ ) and  $\Delta\rho/\rho_\infty < 0.05$  we get  $M_\infty < 0.27$